

## Euclidean Closed Linear Transformations of Complex Spacetime and generally of Complex Spaces of dimension four endowed with the Same or Different Metric

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**Abstract.** Relativity Theory and the corresponding Relativistic Quantum Mechanics are the fundamental theories of physics. Special Relativity (SR) relates the frames of Relativistic Inertial observers (RIOs), through Linear Spacetime Transformation (LSTT) of linear spacetime. Classic Special Relativity uses real spacetime endowed with Lorentz metric and the frames of two RIOs with parallel spatial axes are always related through Lorentz Boost (LB). This cancels the transitive attribute in parallelism, when three RIOs are related, because LB is not closed transformation, causing Thomas Rotation. In this presentation, we consider closed LSTT of Complex Spacetime, so there is no necessity for spatial axes rotation and all the frames are chosen having parallel spatial axes. The solution is expressed by a 4x4 matrix ( $A$ ) containing components of the complex velocity of one Observer wrt another and two functions depended by the metric of Spacetime. Demanding isometric transformation, it emerges a class of metrics that are in accordance with the closed LSTT and the transformation matrix contains one parameter  $\omega$  depended by the metric of Spacetime. In case that we relate RIOs with steady metric, it emerges one steady number ( $\omega_1$ ) depended by the metric of Spacetime of the specific SR. If  $\omega_1$  is an imaginary number, the elements of the  $A$  are complex numbers, so the corresponding spacetime is necessarily complex and there exists real Universal Speed ( $U_1$ ). The specific value  $\omega_1 = \pm i$  gives Vossos transformation (VT) endowed with Lorentz metric (for  $g_{ii}=1$ ) of complex spacetime and invariant spacetime interval (or equivalently invariant speed of light in vacuum), which produce the theory of Euclidean Complex Relativistic Mechanics (ECRMs). If  $\omega_1$  is a real number ( $\omega_1 \neq 0$ ) the elements of the  $A$  are real numbers, so the corresponding spacetime is real, but there exist imaginary  $U_1$ . The specific value  $\omega_1=0$  gives Galileo Transformation (GT) with the invariant time, in which any other closed LSTT is reduced, if one RIO has small velocity wrt another RIO. Thus, we have infinite number of closed LSTTs, each one with the corresponding SR theory. In case that we relate accelerated

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observers with variable metric of spacetime, we have the case of General Relativity (GR). For being that clear, we produce a generalized Schwarzschild metric, which is in accordance with any SR based on this closed complex LSTT and Einstein equations. The application of this kind of transformations to the SR and GR is obvious. But, the results may be applied to any linear space of dimension four endowed with steady or variable metric, whose elements (four-vectors) have spatial part (vector) with Euclidean metric.

## 1. Introduction

Relativity Theory (RT) is the foundation stone of modern Physics which combined with Quantum Mechanics (QMs), leads to the Relativistic Quantum Mechanics (RQMs). Moreover, there exist many different approaches of RT, each one with the corresponding RQMs. For instance, *Galileo Transformation* (GT) endowed with the corresponding metric, produces *Newtonian Physics* (NP), which is associated with the classic QMs producing *Schrödinger Equation*. Thus, many low velocities phenomena, like the atomic spectra without fine structure, were explained. On the other hand, *Lorentz Transformation* (LT) endowed with the corresponding metric, produces *Classic Special Relativity* (CSR), which is associated with the Classic RQMs, producing *Klein-Gordon Equation*. Thus, many high velocities phenomena and the fine structure of atomic spectra were explained [1,2].

In this presentation, we prove that there exist two types of complex Linear Spacetime Transformation (LSTTs) with common solution the GT, which can be applied not only to the Special Relativity (SR), but also to General Relativity (GR), because the production of the corresponding matrices has become without adapting one specific metric. In addition, any complex *Cartesian Coordinates* (CCs) of the theory may be turned to the corresponding real CCs, in order to can be realized through human senses. Moreover, we produce a *generalized Schwarzschild metric*, which is in accordance with any SR based on this closed complex LSTT and Einstein equations. This new modeling of study gives us the capability studying at the same time *Einstein* RT, NP, or any other Theory of Physics that is in accordance with closed Linear Spacetime Transformations. This is achieved because the coefficients of spacetime metric are contained in the transformation matrix. Besides NP is obtained, not only by the low velocity limit, but also by the zero limit of the space coefficient of spacetime metric.

## 2. The Matrix ( $A$ ) and the Metric ( $g$ ) of the Closed Linear Transformation

In a 3D complex 'space' endowed with *Euclidean metric*, there exists a frame Oxyz having real CCs. Another real independent variable ('time') and the aforementioned coordinates produce a real four-vector. There exist two types of closed linear 'spacetime' transformation to this real four-vector: **one** with the 'time' being depended by the 'spatial' position where the 'event' happens and **another** with the 'time' being independent by the position. The **first type** has real or imaginary Invariant 'Speed' ( $U$ ), in contrast to the **second type** which has only infinite  $U$ . Moreover, demanding the transformation having **isometry** [3], the **first type** transformation matrix is totally calculated and contains except for the 'velocity' of the frame  $O'x'y'z'$ , a parameter  $\omega$ , with

$$\omega^2 = g_{ii}/g_{00} \quad (1)$$

where  $g_{00}$  and  $g_{ii}$  are the metric's coefficients of time and space respectively. The **second type** is turn to GT. Taking the limit  $\omega \rightarrow 0$  to the **first type**, may emerges GT. So, in case of isometry, the **second type is embedded to the first type** transformation. Besides the **first type** is divided to two cases: **one case** that the 'time' and the 'space' have coefficients of metric of 'spacetime' with different sign, where the transformation leads to complex 3D 'space' with real  $U$  and **the other case** that 'time' and 'space' have coefficients of metric with the same sign, where the transformation leads to real 3D 'space' with imaginary  $U$ . **Time remains real**, in any case. Below, we present the typical matrix ( $A_{\text{typ}}$ ), the general matrix ( $A_{(\beta)}$ ), the covariant matrix of spacetime metric ( $g$ ), the universal speed ( $U$ ) and the domain of spacetime ( $C^4$ ) that corresponds to the transformation of a contravariant four-vector:

$$dX' = A dX \quad (2)$$

$$\begin{array}{c}
 (A_{\tau\nu\pi}, A_{(\beta)}, g, U, \mathbb{C}^4) \\
 | \\
 | \quad \lambda = \omega |\vec{\beta}| b = ? \\
 | \\
 \begin{array}{cc}
 \lambda \neq 0 & \lambda = 0
 \end{array} \\
 | \quad \quad \quad | \\
 \Lambda_{\tau\nu\pi} = b \begin{bmatrix} 1 & \omega^2 \beta & 0 & 0 \\ -\beta & 1 & 0 & 0 \\ 0 & 0 & 1 & \omega \beta \\ 0 & 0 & -\omega \beta & 1 \end{bmatrix}, & \Lambda_{\tau\nu\pi} = \begin{bmatrix} b & 0 & 0 & 0 \\ -h\beta & h & 0 & 0 \\ 0 & 0 & h & 0 \\ 0 & 0 & 0 & h \end{bmatrix}, \\
 \Lambda_{(\beta)} = b \begin{bmatrix} 1 & \omega^2 \beta^T \\ -\beta & \mathbf{I} + \omega A_{(\beta)} \end{bmatrix}, U \in \{\Re, I\}. & \Lambda_{(\beta)} = \begin{bmatrix} b & \mathbf{O} \\ -h\beta & h\mathbf{I} \end{bmatrix}, U = +\infty. \\
 | \quad \quad \quad | \\
 | \text{isometry} & | \text{isometry} \\
 | \quad \quad \quad | \\
 \Lambda_{\tau\nu\pi(\omega, \beta)} = \gamma_{(i\omega\beta)} \begin{bmatrix} 1 & \omega^2 \beta & 0 & 0 \\ -\beta & 1 & 0 & 0 \\ 0 & 0 & 1 & \omega \beta \\ 0 & 0 & -\omega \beta & 1 \end{bmatrix}, & \Lambda_{\Gamma\tau\nu\pi(\beta)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\beta & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\
 \Lambda_{(\beta)} = \gamma_{(i\omega\beta)} \begin{bmatrix} 1 & \omega^2 \beta^T \\ -\beta & \mathbf{I} + \omega A_{(\beta)} \end{bmatrix}, & \Lambda_{\Gamma(\beta)} = \begin{bmatrix} 1 & \mathbf{O} \\ -\beta & \mathbf{I} \end{bmatrix}, \\
 g = g_{ii} \begin{bmatrix} \frac{1}{\omega^2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. & g_{\Gamma} = \begin{bmatrix} g_{00} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, U = +\infty, \mathbb{R}^4. \\
 | \quad \quad \quad | \\
 | \omega = ? & \\
 \begin{array}{cc}
 \omega = \zeta \mathbf{i} & \omega = \zeta \in \Re
 \end{array} \\
 | \quad \quad \quad | \\
 \Lambda_{\tau\nu\pi(i\zeta, \beta)} = \gamma_{(\zeta\vec{\beta})} \begin{bmatrix} 1 & -\zeta^2 \beta & 0 & 0 \\ -\beta & 1 & 0 & 0 \\ 0 & 0 & 1 & i\zeta \beta \\ 0 & 0 & -i\zeta \beta & 1 \end{bmatrix}, & \Lambda_{\tau\nu\pi(\zeta, \beta)} = \gamma_{(i\zeta\beta)} \begin{bmatrix} 1 & \zeta^2 \beta & 0 & 0 \\ -\beta & 1 & 0 & 0 \\ 0 & 0 & 1 & \zeta \beta \\ 0 & 0 & -\zeta \beta & 1 \end{bmatrix}, \\
 \Lambda_{(i\zeta, \vec{\beta})} = \gamma_{(\zeta\vec{\beta})} \begin{bmatrix} 1 & -\zeta^2 \beta^T \\ -\beta & \mathbf{I} + i\zeta A_{(\beta)} \end{bmatrix}, & \Lambda_{(\zeta, \vec{\beta})} = \gamma_{(i\zeta\vec{\beta})} \begin{bmatrix} 1 & \zeta^2 \beta^T \\ -\beta & \mathbf{I} + \zeta A_{(\beta)} \end{bmatrix}, \\
 g = g_{ii} \begin{bmatrix} -\frac{1}{\zeta^2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, U = \frac{1}{|\zeta|} c, X \in \mathbb{R}C^3. & g = g_{ii} \begin{bmatrix} \frac{1}{\zeta^2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, U \in I, X \in \mathbb{R}^4.
 \end{array}$$

where

$$\beta = \begin{bmatrix} \beta_x \\ \beta_y \\ \beta_z \end{bmatrix}, \delta = \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}, A_{(\beta)} = \begin{bmatrix} 0 & \beta_z & -\beta_y \\ -\beta_z & 0 & \beta_x \\ \beta_y & -\beta_x & 0 \end{bmatrix} \quad (3)$$

We observe that

$$A_{(\beta)} \delta = [-\vec{\beta} \times \vec{\delta}] = [\vec{\delta} \times \vec{\beta}] \quad (4)$$

Using vectors, the transformation becomes

$$cdt' = \gamma_{(i\omega\vec{\beta})} (cdt + \omega^2 \vec{\beta} \cdot d\vec{x}), d\vec{x}' = \gamma_{(i\omega\vec{\beta})} (d\vec{x} - \vec{\beta} cdt - \omega \vec{\beta} \times d\vec{x}) \quad (5)$$

The norm of the position four-vector for observers **with the same  $\omega$**  is the corresponding **invariant quantity**:

$$dS^2 = dX^T g dX = g_{00} c^2 dt^2 + g_{ii} d\vec{x}^2 = g_{ii} \left[ \left( \frac{g_{00}}{g_{ii}} \right) c^2 dt^2 + (1) d\vec{x}^2 \right] = g_{ii} \left[ \left( \frac{1}{\omega^2} \right) c^2 dt^2 + (1) d\vec{x}^2 \right] \quad (6)$$

Researching for a possible **Invariant Speed ( $U$ )** for observers **with the same  $\omega$** , it emerges

$$\omega^2 = -\frac{c^2}{U^2} \quad (7)$$

**In case of SR**, the matrices form a group of elements

$$d = (\Lambda_{(\omega_1, \vec{\beta})}, b) \quad (8)$$

with operation

$$d_1 * d_2 = (\Lambda_{(\omega_1, \vec{\beta}_2)} \Lambda_{(\omega_1, \vec{\beta}_1)}, \Lambda_{(\omega_1, \vec{\beta}_2)} b_1 + b_2) \quad (9)$$

where  $b_1^\mu$  is  $\mu$ -coordinate measured by  $O'$ , if  $O$  measures  $x^\nu = 0$  and  $b_2^\mu$  is  $\mu$ -coordinate measured by  $O''$ , if  $O'$  measures  $x_M^\nu = 0$ .

The **Universal Speed ( $U_1$ )** for RIOs, is

$$U_1^2 = -\frac{c^2}{\omega_1^2} \quad (10)$$

### 3. Time – Proper Time

Let have a particle  $P$ , moving with velocity  $\vec{v}_P$  wrt observer  $O$  ( $\vec{v}'_P$  wrt observer  $O'$ ) in a spacetime. The *generalized definition of Proper Time (PT)* is

$$d\tau^2 = \frac{dS'^2}{g_{00} c^2} \quad (11)$$

Using (6) and (1), we have

$$d\tau^2 = \frac{g_{ii}}{g_{00} c^2} \left( \frac{1}{\omega^2} c^2 dt'^2 + d\vec{x}'^2 \right) = \frac{\omega^2}{c^2} \left( \frac{1}{\omega^2} c^2 dt'^2 + d\vec{x}'^2 \right) = dt'^2 + \frac{\omega^2}{c^2} d\vec{x}'^2 = dt'^2 \left( 1 + \frac{\omega^2}{c^2} \vec{v}'_P{}^2 \right) \quad (12)$$

Thus, the relation between the time and the proper time is

$$\frac{dt'}{d\tau} = \gamma'_{(i\omega\vec{\beta}_P)} \quad (13)$$

For GT with  $\omega \rightarrow 0$ , it is  $\gamma'_{(i\omega\vec{\beta}_P)} = 1$ . So  $d\tau = dt' = dt$ .

**In case of RIOs**, (13) becomes

$$\frac{dt'}{d\tau} = \gamma'_{(i\omega_1\vec{\beta}_P)} \quad (14)$$

So, for *Vossos Transformation* (VT) with  $\omega_1 = \pm i$ , it emerges  $\gamma'_{(i\omega_1 \bar{\beta}_p)} = \gamma'_{(\bar{\beta}_p)}$ . Thus, we have the same result as CSR [4].

**In any case** using PT, we can define four-velocity, four-momentum etc building the whole structure of SR and GR.

#### 4. Generalized Schwarzschild metric in accordance with closed complex Linear Spacetime Transformation

For being clear the meaning of this theory, we apply it to the unique solution of *Einstein equations* in vacuum with spherical symmetry that is *Schwarzschild metric*, according to *Birkoff's theorem* [5]. Thus, it emerges a *generalized Schwarzschild metric* that is in accordance with any acceptable SR having real, imaginary or infinite universal speed. We define the relativistic potential  $\Phi$  around a center of gravity as

$$\Phi = \frac{U_1^2}{2} \ln \left( 1 - \frac{r_{SI}}{r} \right) \quad (15)$$

where  $r$  is the distance between the center of the gravity and the spatial position of the event and  $r_{SI}$  is the *generalized Schwarzschild radius*

$$r_{SI} = \frac{2GM}{U_1^2} \quad (16)$$

The definition of *Schwarzschild radius* is

$$r_S = \frac{2GM}{c^2} \quad (17)$$

Thus (16) combined with (17) and (10) gives

$$r_{SI} = -\omega_1^2 r_S \quad (18)$$

and (15) becomes

$$\Phi = \frac{-c^2}{2\omega_1^2} \ln \left( 1 + \frac{\omega_1^2 r_S}{r} \right) = -\frac{c^2}{2} \frac{r_S}{r} + \dots = -\frac{GM}{r} + \dots \quad (19)$$

We observe that if  $\omega_1 = \pm i$  (VT), it emerges the original *Schwarzschild potential*. Moreover, if  $\omega_1 \rightarrow 0$  (GT), we compute

$$\lim_{\omega_1 \rightarrow 0} \Phi = \frac{-c^2}{2} \lim_{\omega_1 \rightarrow 0} \left[ \frac{1}{\omega_1^2} \ln \left( 1 + \frac{\omega_1^2 r_S}{r} \right) \right] = \frac{-c^2}{4} \lim_{\omega_1 \rightarrow 0} \left[ \frac{1}{\omega_1} \frac{\frac{2\omega_1 r_S}{r}}{1 + \frac{\omega_1^2 r_S}{r}} \right] = -\frac{c^2}{2} \frac{r_S}{r} = -\frac{GM}{r} \quad (20)$$

In figure 1, we show the parametric plot of the relativistic potential  $\Phi$  wrt  $r/r_S$ , for different values of  $\omega_1$ . We observe that for  $\omega_1 \rightarrow \pm\infty$ , the relativistic potential becomes zero-function.

Now, we examine the case that  $g_{00I} \leq 0$  and  $g_{iiI} \geq 0$ , it is  $g_{rrI} \geq 0$ , too. Besides  $\omega = \zeta i$ , with  $\zeta \in \mathbb{R}$ .

The *generalized Schwarzschild metric* is

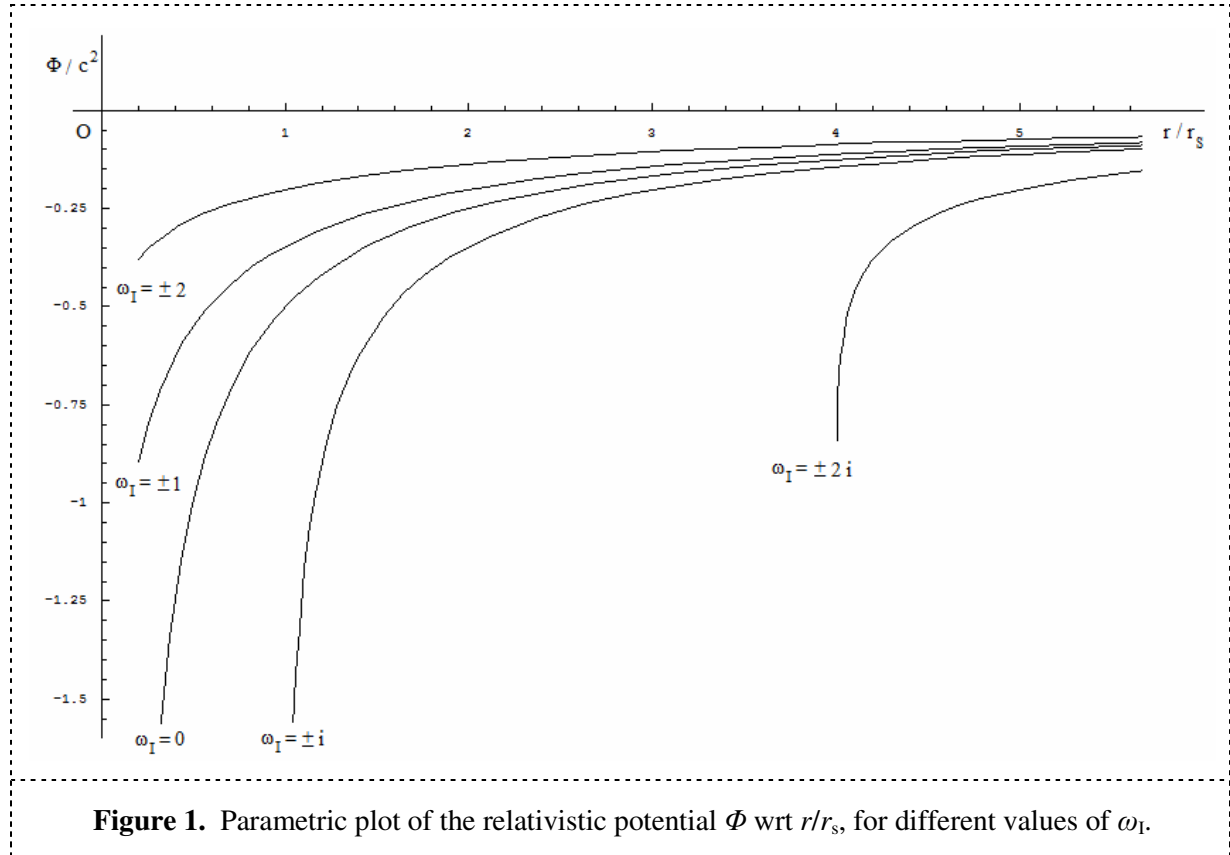
$$dS^2 = dX^T g dX = \begin{bmatrix} cd t & dr & d\theta & d\phi \end{bmatrix} \cdot \begin{bmatrix} -(-g_{00I})e^A & 0 & 0 & 0 \\ 0 & g_{rrI}e^B & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \theta \end{bmatrix} \cdot \begin{bmatrix} cd t \\ dr \\ d\theta \\ d\phi \end{bmatrix} \quad (21)$$

where

$$A = \frac{2}{U_1^2} \Phi \quad (22)$$

This combined with *Einstein equations* in vacuum gives

$$dS^2 = g_{001} \left( 1 - \frac{r_{SI}}{r} \right) c^2 dt^2 - \frac{1}{g_{001} \left( 1 - \frac{r_{SI}}{r} \right)} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad (23)$$



**Figure 1.** Parametric plot of the relativistic potential  $\Phi$  wrt  $r/r_s$ , for different values of  $\omega_I$ .

The isotropic form of the generalized Schwarzschild metric is

$$dS^2 = g_{00} c^2 dt^2 + g_{ii} (d\tilde{r}^2 + \tilde{r}^2 d\theta^2 + \tilde{r}^2 \sin^2 \theta d\phi^2) \quad (24)$$

with

$$d\tilde{x}^2 = d\tilde{r}^2 + \tilde{r}^2 d\theta^2 + \tilde{r}^2 \sin^2 \theta d\phi^2 \quad (25)$$

$$\tilde{x} = \tilde{r} \sin \theta \cos \phi, \quad \tilde{y} = \tilde{r} \sin \theta \sin \phi, \quad \tilde{z} = \tilde{r} \cos \theta \quad (26)$$

$$\tilde{r} = \frac{|\zeta_1| r_s}{4} \left( \frac{2r}{|\zeta_1|^2 r_s} \left( 1 + \sqrt{1 - |\zeta_1|^2 \frac{r_s}{r}} \right) - 1 \right)^{\frac{1}{\sqrt{-g_{001}}}} \quad (27)$$

$$r = \frac{|\zeta_1|^2 r_s}{4} \left[ 1 + \left( \frac{|\zeta_1| r_s}{4\tilde{r}} \right)^{\sqrt{-g_{001}}} \right]^2 \left( \frac{4\tilde{r}}{|\zeta_1| r_s} \right)^{\sqrt{-g_{001}}} \quad (28)$$

$$g_{00} = g_{001} \left( \frac{1 - \left( \frac{|\zeta_1| r_s}{4\tilde{r}} \right)^{\sqrt{-g_{001}}}}{1 + \left( \frac{|\zeta_1| r_s}{4\tilde{r}} \right)^{\sqrt{-g_{001}}}} \right)^2 \quad (29)$$

$$\tilde{g}_{ii} = \frac{|\zeta_1|^4 r_s^2}{16\tilde{r}^2} \left( 1 + \left( \frac{|\zeta_1| r_s}{4\tilde{r}} \right)^{\sqrt{-g_{001}}} \right)^4 \left( \frac{4\tilde{r}}{|\zeta_1| r_s} \right)^{2\sqrt{-g_{001}}} \quad (30)$$

$$|\zeta| = \frac{4^{\sqrt{-g_{001}-1}} |\zeta_1|^{2-\sqrt{-g_{001}}} \left( 1 + \left( \frac{|\zeta_1| r_s}{4\tilde{r}} \right)^{\sqrt{-g_{001}}} \right)^3 \left( \frac{\tilde{r}}{r_s} \right)^{\sqrt{-g_{001}-1}}}{\sqrt{-g_{001}} \left| 1 - \left( \frac{|\zeta_1| r_s}{4\tilde{r}} \right)^{\sqrt{-g_{001}}} \right|} \quad (31)$$

Besides

$$g_{ii1} = \begin{cases} 0 & , \quad -1 < g_{001} \leq 0 \\ |\zeta_1|^2 & , \quad g_{001} = -1 \\ +\infty & , \quad g_{001} < -1 \end{cases} \quad (32)$$

The universal speed is

$$U = \frac{1}{|\zeta|} c \quad (33)$$

so, the relative change of  $U$  wrt  $U_{(+\infty)}$  is

$$\Delta U_r = \frac{U_{(r)} - U_{(+\infty)}}{U_{(+\infty)}} = \frac{|\zeta_1|}{|\zeta|} - 1 \quad (34)$$

The transformation inside the infinitesimal area around the surface of a sphere with center the same as the center of the gravity and radius  $r$  is

$$\begin{bmatrix} cd t' \\ d\tilde{x}' \\ d\tilde{y}' \\ d\tilde{z}' \end{bmatrix} = \gamma_{(\zeta\tilde{\beta})} \begin{bmatrix} 1 & -\zeta^2 \beta_x & \zeta^2 \beta_y & \zeta^2 \beta_z \\ -\beta_x & 1 & \zeta \beta_z & -\zeta \beta_y \\ -\beta_y & -\zeta \beta_z & 1 & \zeta \beta_x \\ -\beta_z & \zeta \beta_y & -\zeta \beta_x & 1 \end{bmatrix} \begin{bmatrix} cd t \\ d\tilde{x} \\ d\tilde{y} \\ d\tilde{z} \end{bmatrix} \quad (35)$$

In case of *Lorentz metric* ( $g_{001} = -1$  and  $g_{ii1} = 1$ ), it is  $\zeta_i = \pm 1$  and we have the SR which corresponds to VT with the metric of spacetime

$$\tilde{g}_{00} = - \left( \frac{1 - \frac{r_s}{4\tilde{r}}}{1 + \frac{r_s}{4\tilde{r}}} \right)^2, \quad \tilde{g}_{ii} = \left( 1 + \frac{r_s}{4\tilde{r}} \right)^4 \quad (36)$$

so, it emerges the original *isotropic form* of *Schwarzschild metric* [6] with

$$|\zeta| = \frac{\left( 1 + \frac{r_s}{4\tilde{r}} \right)^3}{\left| 1 - \frac{r_s}{4\tilde{r}} \right|} \quad (37)$$

and

$$\tilde{r} = \frac{r_s}{4} \left( \frac{2r}{r_s} \left( 1 + \sqrt{1 - \frac{r_s}{r}} \right) - 1 \right) \quad (38)$$

## 5. Experimental

Applying the above results of the original *isotropic form* of *Schwarzschild metric*:

- (i) to the **Earth** with mass  $M=5.9742 \times 10^{24}$  kg, on the sea level with  $r=6378140$  m and taking into account that  $G=6.67428(67) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$  and  $c=299792458 \text{ ms}^{-1}$  (exact) [7], it emerges  $r_s=8.873578523 \times 10^{-3} \text{ m} \approx 0.88 \text{ cm}$ . Thus  $r_s/r=1.391248628 \times 10^{-9}$ ,  $\tilde{r}=6378139.995 \text{ m}$ ,  $\zeta=\pm 1.000000001$ ,  $U=299792457.6 \text{ m s}^{-1}$  and  $\Delta U_r=-1.36 \times 10^{-9}$  and
- (ii) to the **Sun**, with mass is  $M=1.9891 \times 10^{30}$  kg, on the surface with radius is  $r=6.9599 \times 10^8 \text{ m}$ ,  $r_s=2954.443279 \text{ m}$ , we have  $r_s/r=4.244950759 \times 10^{-6}$  m,  $\tilde{r}=6.959885228 \times 10^8 \text{ m}$ ,  $\zeta=\pm 1.000004245$ ,  $U=299791185.4 \text{ m s}^{-1}$  and  $\Delta U_r=-4.244912 \times 10^{-6}$ .

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