

Towards a realistic description of hadron resonances

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Abstract. We report on our attempts of treating excited hadron states as true quantum resonances. Hitherto the spectroscopy of mesons, usually considered as quark-antiquark systems, and of baryons, usually considered as three-quark systems, has been treated through excitation spectra of bound states (namely, confined few-quark systems), corresponding to poles of the quantum-mechanical resolvent at real negative values in the complex energy plane. As a result the wave functions, i.e. the residua of the resolvent, have not exhibited the behaviour as required for hadron resonances with their multiple decay modes. This has led to disturbing shortcomings in the description of hadronic resonance phenomena. We have aimed at a more realistic description of hadron resonances within relativistic constituent-quark models taking into account explicitly meson-decay channels. The corresponding coupled-channels theory is based on a relativistically invariant mass operator capable of producing hadron ground states with real energies and hadron resonances with complex energies, the latter corresponding to poles in the lower half-plane of the unphysical sheet of the complex energy plane. So far we have demonstrated the feasibility of the coupled-channels approach to hadron resonances along model calculations producing indeed the desired properties. The corresponding spectral properties will be discussed in this contribution. More refined studies are under way towards constructing a coupled-channels relativistic constituent-quark model for meson and baryon resonances.

1. Introduction

The description of hadron resonances represents a big challenge in all current approaches to quantum chromodynamics (QCD). Particularly in the framework of constituent-quark models, hadronic resonances are usually treated as excited bound states rather than as resonant states with finite widths. Calculations of strong decays have thus shown short-comings generally producing too small decay widths [1, 2, 3, 4]. To remedy this situation we are investigating a coupled-channels (CC) approach taking into account explicit meson, especially pionic, degrees of freedom.

The CC approach has been tested before within a simple scalar toy model, leaving out all spin and flavor dependences. It turned out that the coupling to a mesonic channel shifts the ground-state mass down and generates the resonant state with a finite width, whose magnitude is dependent essentially on the coupling strength to the meson channel [5, 6]. Recently we have obtained results for the πNN and the $\pi N\Delta$ systems, including all spin and flavor degrees of freedom.

Our theory relies on a relativistically invariant CC mass operator in general coupling a hadron state to n further (decay) channels. We exemplify the approach for a single baryon B , specifically



the N or the Δ , coupled to a one-pion channel by a relativistically invariant mass operator in matrix form:

$$\begin{pmatrix} M_{\tilde{B}} & K \\ K^\dagger & M_{\tilde{B}\pi} \end{pmatrix} \begin{pmatrix} |\psi_B\rangle \\ |\psi_{B\pi}\rangle \end{pmatrix} = m \begin{pmatrix} |\psi_B\rangle \\ |\psi_{B\pi}\rangle \end{pmatrix}. \quad (1)$$

Herein, $M_{\tilde{B}}$ and $M_{\tilde{B}\pi}$ are the invariant mass operators of the bare baryon state \tilde{B} and the $\tilde{B}\pi$ channel, respectively. K contains the transition dynamics. This means that the bare baryon \tilde{B} gets dressed by pion loops (see Fig. 1).

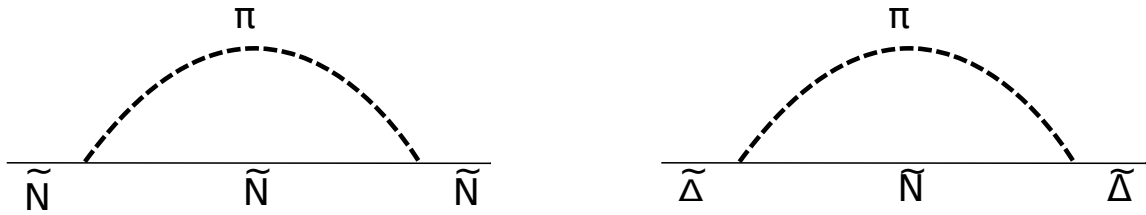


Figure 1. Pion-loop diagrams considered for the mass renormalization of bare baryons \tilde{N} and $\tilde{\Delta}$.

After eliminating the $B\pi$ channel by the Feshbach method one ends up with the following eigenvalue problem for the dressed baryon ground or resonant state $|\psi_B\rangle$:

$$\left[M_{\tilde{B}} - K (m - M_{\tilde{B}\pi} + i0)^{-1} K^\dagger \right] |\psi_B\rangle = m |\psi_B\rangle. \quad (2)$$

Here, evidently an optical potential occurs, which becomes complex above the $\tilde{B}\pi$ threshold. It should be noted that the mass eigenvalue m appears also in the optical-potential term on the left-hand side of Eq. (2). Beyond the resonance threshold it acquires an imaginary part leading to a finite decay width.

The transition dynamics contained in K is deduced from the following Lagrangian densities

$$\mathcal{L}_{\pi NN} = -\frac{f_{\pi NN}}{m_\pi} \bar{\Psi} \gamma_5 \gamma^\mu \Psi \partial_\mu \Phi, \quad (3)$$

$$\mathcal{L}_{\pi N\Delta} = -\frac{f_{\pi N\Delta}}{m_\pi} \bar{\Psi} \Psi^\mu \partial_\mu \Phi + h.c., \quad (4)$$

where Ψ and Ψ^μ represent the N and Δ fields, which are coupled in pseudovector form by the π field Φ with strengths $f_{\pi NN}$ and $f_{\pi N\Delta}$, respectively. This leads to transition matrix elements from the bare \tilde{N} and $\tilde{\Delta}$ states to the channels including the explicit pions (with mass m_π) for the cases of πNN

$$\langle \tilde{N} | \mathcal{L}_{\pi \tilde{N} \tilde{N}}(0) | \tilde{N}, \pi; \vec{k}_\pi'' \rangle = \sum \frac{i f_{\pi \tilde{N} \tilde{N}}}{m_\pi} \bar{u}(k_{\tilde{N}}, \Sigma_{\tilde{N}}) \gamma^\mu \gamma_5 u(k_{\tilde{N}}'', \Sigma_{\tilde{N}}'') (k_\pi'')_\mu \quad (5)$$

and $\pi N\Delta$

$$\langle \tilde{\Delta} | \mathcal{L}_{\pi \tilde{N} \tilde{\Delta}}(0) | \tilde{N}, \pi; \vec{k}_\pi'' \rangle = \sum \frac{i f_{\pi \tilde{N} \tilde{\Delta}}}{m_\pi} \bar{u}^\mu(k_{\tilde{\Delta}}, \Sigma_{\tilde{\Delta}}) u(k_{\tilde{N}}'', \Sigma_{\tilde{N}}'') (k_\pi'')_\mu. \quad (6)$$

Here $u(k_N, \Sigma_N)$ are the spin- $\frac{1}{2}$ Dirac spinors of the N and $u^\mu(k_\Delta, \Sigma_\Delta)$ the spin- $\frac{3}{2}$ Rarita-Schwinger spinors of the Δ . In the rest frame of the baryon B the eigenvalue equation (2) finally turns into the following explicit form

$$\begin{aligned} & \left(m_{\tilde{B}} + \int \frac{d^3 k_\pi''}{(2\pi)^3} \frac{1}{2\omega_\pi'' 2\omega_N'' 2m_{\tilde{B}}} \mathcal{F}_{\pi\tilde{N}\tilde{B}}(|\vec{k}_\pi''|) < \tilde{B} | \mathcal{L}_{\pi\tilde{N}\tilde{B}}(0) | \tilde{N}, \pi; \vec{k}_\pi'' > \right. \\ & \quad \times \left(m - \sqrt{m_{\tilde{N}}^2 + \vec{k}_\pi''^2} - \sqrt{m_\pi^2 + \vec{k}_\pi''^2} + i0 \right)^{-1} \mathcal{F}_{\pi\tilde{N}\tilde{B}}^*(|\vec{k}_\pi''|) \\ & \quad \left. \times < \tilde{N}, \pi; \vec{k}_\pi'' | \mathcal{L}_{\pi\tilde{N}\tilde{B}}^\dagger(0) | \tilde{B} > \right) < \tilde{B} | \psi_B > = m < \tilde{B} | \psi_B > \end{aligned} \quad (7)$$

with B standing for N or Δ and all quantities with a tilde referring to bare particles. The wave functions of the baryon states $< \tilde{B} | \psi_B >$ are represented by free momentum eigenstates denoted by $< \tilde{B} |$ or equivalently by free velocity states $< \tilde{B}; \vec{v} = 0 |$ (for pertinent details see Ref. [5]). The processes corresponding to the optical potential in Eq. (7) are graphically represented in Fig. 1.

In Eq. (7) we have inserted form factors $\mathcal{F}_{\pi\tilde{N}\tilde{B}}$ for the extended meson-baryon vertices. They are taken from three different models, namely, a relativistic constituent-quark model (RCQM) [7, 8, 9] as well as two phenomenological meson-nucleon models, namely, the one by Sato and Lee (SL) [10] and the one by Polinder and Rijken (PR) [11]. The corresponding parametrizations are all given in Ref. [9] according to the form

$$F(\vec{q}^2) = \frac{1}{1 + \left(\frac{\vec{q}}{\Lambda_1}\right)^2 + \left(\frac{\vec{q}}{\Lambda_2}\right)^4}. \quad (8)$$

The cut-off parameters occurring in Eq. (8) and the values of the coupling constants are summarized in Tab. 1. The functional dependences of the various vertex form factors are shown in Figs. 2 and 3. It should be noted that the RCQM vertex form factors are derived from a microscopic calculation on the quark level, while the SL and PR form factors are determined by a fit to meson-nucleon phenomenology.

Table 1. πNN and $\pi N\Delta$ coupling constants as well as cut-off parameters entering into Eq. (8) for the three different form-factor models used in the present work (cf. Ref. [9]).

		RCQM	SL	PR
N	$\frac{f_N^2}{4\pi}$	0.0691	0.08	0.013
	Λ_1	0.451	0.453	0.940
	Λ_2	0.931	0.641	1.102
Δ	$\frac{f_\Delta^2}{4\pi}$	0.188	0.334	0.167
	Λ_1	0.594	0.458	0.853
	Λ_2	0.998	0.648	1.014

By solving the eigenvalue equation (7) with the mass m calibrated to the physical nucleon mass $m_N = 939$ MeV we find the bare nucleon mass $m_{\tilde{N}}$ and thus the influence of the pion loop. Tab. 2 contains the results for the pion dressing of the nucleon ground state. It is seen that all three different form-factor models lead to very similar magnitudes for the mass differences $m_N - m_{\tilde{N}}$ of about 100 MeV.

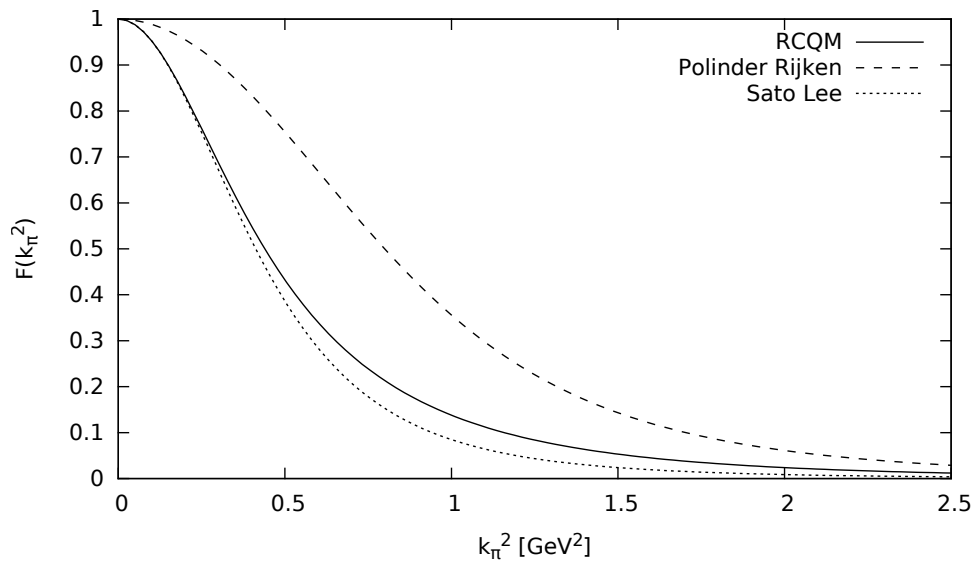


Figure 2. Momentum dependences of the three different form-factor models in case of the πNN system.

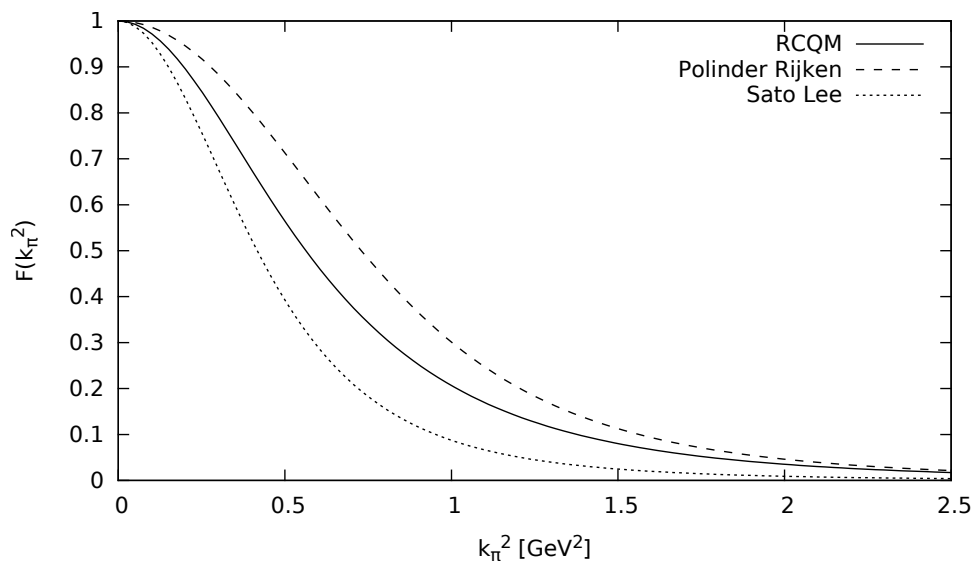


Figure 3. Momentum dependences of the three different form-factor models in case of the $\pi N\Delta$ system.

In the case of the Δ resonance we are interested in the mesonic effects on both the mass as well as the decay width. In the first instance, we employ a bare intermediate nucleon \tilde{N} as is shown in the graph on the r.h.s. of Fig. 1. The corresponding results are given in Tab. 3. Again the pionic effects on the masses are quite similar for the three different form-factor models. The π -decay widths, however, show bigger variations. Still, they are all too small as compared to the phenomenological value.

A more realistic description of the $\Delta \rightarrow N\pi$ decay width is obtained by replacing the bare \tilde{N}

Table 2. One-loop effects on the nucleon mass m_N from coupling to the πNN channel ($m_{\tilde{N}}$ being the bare mass and m_N the dressed one).

	RCQM	SL	PR
$m_{\tilde{N}}$	1.067	1.031	1.051
$m_N - m_{\tilde{N}}$	-0.128	-0.092	-0.112

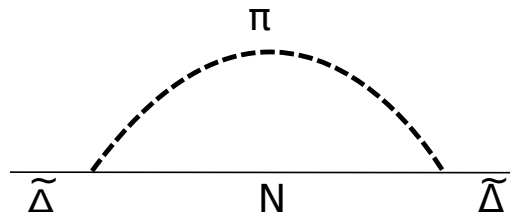


Figure 4. Pion-loop diagram for the $\pi N\Delta$ system with an intermediate physical nucleon with mass $m_N = 939$ MeV.

Table 3. Mesonic effects on the Δ mass $\text{Re}(m_\Delta)$ and π -decay width Γ from coupling to the $\pi N\Delta$ channel, according to the loop diagram on the r.h.s. of Fig. 1. The bare nucleon masses $m_{\tilde{N}}$ are the same as in Tab. 2.

	RCQM	SL	PR
$m_{\tilde{N}}$	1.067	1.031	1.051
$m_{\tilde{\Delta}}$	1.300	1.295	1.336
$\text{Re}(m_\Delta) - m_{\tilde{\Delta}}$	-0.068	-0.062	-0.104
$\Gamma = 2 \text{Im}(m_\Delta)$	0.0026	0.017	0.0048

Table 4. Mesonic effects on the Δ mass $\text{Re}(m_\Delta)$ and π -decay width Γ from coupling to the $\pi N\Delta$ channel, according to the loop diagram in Fig. 4.

	RCQM	SL	PR
$m_{\tilde{N}}$	0.939	0.939	0.939
$m_{\tilde{\Delta}}$	1.318	1.306	1.358
$\text{Re}(m_\Delta) - m_{\tilde{\Delta}}$	-0.086	-0.073	-0.125
$\Gamma = 2 \text{Im}(m_\Delta)$	0.042	0.069	0.039

with mass $m_{\tilde{N}}$ in the intermediate state by the physical nucleon N with mass $m_N = 939$ MeV, as depicted in Fig. 4.

The corresponding results are given in Tab. 4. It is immediately seen that the decay widths get much enhanced, while the effects on the masses are only slightly changed. We expect the larger phase space for the pionic decay to be responsible for the enhancement of the decay widths.

At this stage an open problem is left with regard to dressing the vertex form factors and the coupling strengths in our work. Corresponding studies have earlier been undertaken, e.g., by both Sato and Lee [10] as well as Polinder and Rijken [11]. We may expect a further improvement of our results by following a similar way. However, it constitutes a difficult task to realize such a framework consistently in our approach.

2. Summary

We have considered one-loop pionic effects on the N mass as well as on the resonance energy and the hadronic decay width of the Δ in a relativistically invariant CC approach. Thereby, pionic contributions on these observables are taken into account explicitly. The results show the desired behaviour: The dressed masses turn out to be smaller than the bare ones and in addition a decay width naturally comes about for the resonant Δ state. Further investigations are presently under way for the N^* resonances, and effects of additional mesonic channels are studied.

Acknowledgment

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