Journal of Physics: Conference Series **738** (2016) 012036 doi:10.1088/1742-6596/738/1/012036

Breakdown of the equivalence between active gravitational mass and energy for a quantum body

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Abstract. We determine active gravitational mass operator of the simplest composite quantum body - a hydrogen atom - within the semiclassical approach to the Einstein equation for a gravitational field. We show that the expectation value of the mass is equivalent to energy for stationary quantum states. On the other hand, it occurs that, for quantum superpositions of stationary states with constant expectation values of energy, the expectation values of the gravitational mass exhibit time-dependent oscillations. This breaks the equivalence between active gravitational mass and energy and can be observed as a macroscopic effect for a macroscopic ensemble of coherent quantum states of the atoms. The corresponding experiment could be the first direct observation of quantum effects in General Relativity.

1. Introduction

The notions of active and passive gravitational masses for a classical composite body are not trivial and have been discussed in recent literature by K. Nordtvedt [1] and S. Carlip [2]. In particular, they have stressed that gravitational field is coupled with the following combination: $3K + 2U$, where K is kinetic and U is potential energies of a composite body. They have revealed an important role of the classical virial theorem, which states that averaged over time value of $\langle 2K+U\rangle_t = 0$ and thus guarantees that the averaged over time gravitational masses are equivalent to the total energy,

$$\langle m^g\rangle_t = m_1 + m_2 + \langle 3K + 2U\rangle_t/c^2 = m_1 + m_2 + \langle K + U\rangle_t/c^2 = E/c^2. \tag{1}$$

The author of this article has very recently considered quantum case of the simplest composite quantum body - a hydrogen atom [3-7]. He has shown that the quantum virial theorem [8] is responsible for the fact that the expectation values of active and passive gravitational masses of the atom are equivalent to energy for stationary quantum states. On the other hand, he has found important breakdowns of the above-mentioned equivalence for two cases: (a) for quantum superpositions of stationary states, (b) for stationary quantum states due to quantum fluctuations. In Refs.[3-7], there have suggested two different idealized experiments to detect the above-mentioned breakdowns of the Equivalence Principle [9]. If such experiments are done they will be the first direct observations of quantum effects in General Relativity.

2. Goal

The goal of this paper is to study a quantum problem of active gravitational mass of a composite body in semiclassical theory of gravity [10]. Below, we consider the simplest composite quantum body - a hydrogen atom. We obtain and discuss the following two main results. The first one is that the expectation value of the mass is equivalent to energy for stationary quantum states due to the quantum virial theorem [8]. The second result is the breakdown of the above mentioned equivalence for a macroscopic coherent ensemble of quantum superpositions of stationary states. In particular, we show that the expectation value of active gravitational mass is time dependent value for superpositions of stationary quantum states even in the case, where the expectation value of energy is constant. We also discuss possible experiment to discover this breakdown of the Equivalence Principle.

3. Active gravitational mass in classical physics

In this section, we determine active gravitational mass of a hydrogen atom, provided that we consider its classical model. More precisely, below we consider light negatively charged particle exhibiting a bound motion in the Coulomb field of heavy positively charged particle. Our task is to calculate contributions to the mass from kinetic and potential energies of the light particle.

Let us write gravitational potential at large distances from the atom, $R \gg r_B$, where r_B is the the so-called Bohr radius (i.e., effective "size" of a hydrogen atom). In accordance with general theory of a weak gravitational field [9,11], the gravitational potential can be written as

$$\phi(R,t) = -G\frac{m_e + m_p}{R} - G\int \frac{\Delta T^{kin}_{\alpha\alpha}(t,\mathbf{r}) + \Delta T^{pot}_{\alpha\alpha}(t,\mathbf{r})}{c^2 R} d^3\mathbf{r}, \tag{2}$$

where $\Delta T^{kin}_{\alpha\beta}(t,\mathbf{r})$ and $\Delta T^{pot}_{\alpha\beta}(t,\mathbf{r})$ are contributions to stress-energy tensor density, $T_{\alpha\beta}(t,\mathbf{r})$, due to kinetic and the Coulomb potential energies, respectively, m_e and m_p are electron and proton bare masses. We point out that in Eq.(2) we disregard all retardation effects. Therefore, in the above-mentioned approximation, electron active gravitational mass is equal to

$$m^g_a = m_e + \frac{1}{c^2}\int [\Delta T^{kin}_{\alpha\alpha}(t,\mathbf{r}) + \Delta T^{pot}_{\alpha\alpha}(t,\mathbf{r})] d^3\mathbf{r}. \tag{3}$$

To evaluate $\Delta T^{kin}_{\alpha\alpha}(t,\mathbf{r})$, we make use of the standard expression for stress-energy tensor density of a moving relativistic point mass [9,11]:

$$T^{\alpha\beta}(\mathbf{r},t) = \frac{m_e v^{\alpha}(t) v^{\beta}(t)}{\sqrt{1 - v^2(t)/c^2}}\ \delta^3[\mathbf{r} - \mathbf{r}_e(t)], \tag{4}$$

where v^{α} is a four-velocity and $\mathbf{r}_e$ is three dimensional electron trajectory. As directly follows from Eq.(4),

$$\Delta T^{kin}_{\alpha\alpha}(t) = \int \Delta T^{kin}_{\alpha\alpha}(t,\mathbf{r}) d^3\mathbf{r} = \frac{m_e[c^2 + v^2(t)]}{\sqrt{1 - v^2(t)/c^2}} - m_e c^2. \tag{5}$$

Calculation of the contribution from potential energy to stress energy tensor is done by using the standard formula for stress energy tensor of electromagnetic field [11],

$$T^{\mu\nu}_{em} = \frac{1}{4\pi}[F^{\mu\alpha}F^{\nu}_{\ \alpha} - \frac{1}{4}\eta^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}], \tag{6}$$

where $\eta_{\alpha\beta}$ is the Minkowski metric, $F^{\alpha\beta}$ is the so-called tensor of electromagnetic field [11]. Below, we use approximation, where we disregard magnetic field and take into account only the Coulomb electrostatic field. In this case, we can simplify Eq.(6) and obtain the following expression:

$$\Delta T^{pot}_{\alpha\alpha}(t) = \int \Delta T^{pot}_{\alpha\alpha}(t,\mathbf{r}) d^3\mathbf{r} = -2\frac{e^2}{r(t)}. \tag{7}$$

As follows from Eqs.(5),(7), active electron gravitational mass can be written in the following way

$$m_a^g = \left[\frac{m_e c^2}{(1 - v^2/c^2)^{1/2}} - \frac{e^2}{r}\right]/c^2 + \left[\frac{m_e v^2}{(1 - v^2/c^2)^{1/2}} - \frac{e^2}{r}\right]/c^2. \tag{8}$$

We note that the first term in Eq.(8) is the expected total energy contribution to the mass, whereas the second term is the so-called relativistic virial one [8,12], which depends on time. Therefore, in classical physics, active gravitational mass depends on time too. Nevertheless, it is possible to introduce electron active gravitational mass averaged over time. This procedure restores the expected equivalence between active gravitational mass and energy:

$$m_a^g = \left\langle \frac{m_e c^2}{(1 - v^2/c^2)^{1/2}} - \frac{e^2}{r} \right\rangle_t /c^2 + \left\langle \frac{m_e v^2}{(1 - v^2/c^2)^{1/2}} - \frac{e^2}{r} \right\rangle_t /c^2 = m_e + E/c^2, \tag{9}$$

where the averaged over time virial term is zero due to the classical virial theorem. Note that for non-relativistic particle our Eqs.(8),(9) can be reduced to the results of Refs.[1,2]:

$$m_a^g = m_e + \left[\frac{m_e v^2}{2} - \frac{e^2}{r}\right]/c^2 + \left[2\frac{m_e v^2}{2} - \frac{e^2}{r}\right]/c^2 \tag{10}$$

and

$$< m_a^g >_t = m_e + \left\langle \frac{m_e v^2}{2} - \frac{e^2}{r} \right\rangle_t /c^2 + \left\langle 2\frac{m_e v^2}{2} - \frac{e^2}{r} \right\rangle_t /c^2 = m_e + E/c^2. \tag{11}$$

4. Gravitational mass in quantum physics

In this section, we make use of semiclassical theory of gravity [10], where gravitational field is not quantized but the matter is quantized in the Einstein equation:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} \langle \hat{T}_{\mu\nu} \rangle, \tag{12}$$

where $< \hat{T}_{\mu\nu} >$ stands for the expectation value of quantum operator, corresponding to the stress energy tensor. To this end, we need to rewrite Eq.(10) for electron active gravitational mass using momentum, instead of velocity, and then quantize it:

$$\hat{m}_a^g = m_e + \left(\frac{\hat{\mathbf{p}}^2}{2m_e} - \frac{e^2}{r}\right)/c^2 + \left(2\frac{\hat{\mathbf{p}}^2}{2m_e} - \frac{e^2}{r}\right)/c^2. \tag{13}$$

As follows from Eq.(13), the expectation value of electron active gravitational mass can be expressed as

$$< \hat{m}_a^g >= m_e + \left\langle \frac{\hat{\mathbf{p}}^2}{2m_e} - \frac{e^2}{r} \right\rangle /c^2 + \left\langle 2\frac{\hat{\mathbf{p}}^2}{2m_e} - \frac{e^2}{r} \right\rangle /c^2, \tag{14}$$

where third term is the virial one. Let us consider a macroscopic ensemble of hydrogen atoms with each of them being in ground state. In this case, the expectation value of the mass is

$$< \hat{m}_a^g >= m_e + \frac{E_1}{c^2}, \tag{15}$$

where the expectation value of the virial term in Eq.(14) is equal to zero in stationary quantum states due to the quantum virial theorem [8]. Thus, we make conclusion that, in stationary quantum states, active gravitational mass of a composite quantum body is equivalent to its energy.

Here, we consider the simplest quantum superposition of stationary states in a hydrogen atom,

$$\Psi(r,t) = \frac{1}{\sqrt{2}}[\Psi_1(r)\exp(-iE_1t) + \Psi_2(r)\exp(-iE_2t)], \tag{16}$$

where $\Psi_1(r)$ and $\Psi_2(r)$ are the ground state (1S) and first excited state (2S), respectively. As directly follows from (16), the superposition is characterized by the following constant expectation value of energy:

$$< E >= (E_1 + E_2)/2. \tag{17}$$

Nevertheless, as it follows from (14) the expectation value of electron active mass operator oscillates with time:

$$< \hat{m}^g_a >= m_e + \frac{E_1 + E_2}{2c^2} + \frac{V_{1,2}}{c^2}\cos\left[\frac{(E_1 - E_2)t}{\hbar}\right], \tag{18}$$

where $V_{1,2}$ is matrix element of the virial operator between the above-mentioned two stationary quantum states. Note that these time dependent oscillations directly demonstrate inequivalence between the expectation values of active gravitational mass and energy for superpositions of stationary quantum states. We stress that such quantum mechanical oscillations are very general and are not restricted by a hydrogen atom. To simplify the situation, in the same way as in the previous section, we can introduce the averaged over time expectation value of active gravitational mass, which obeys the Einstein's equation:

$$<< \hat{m}^g_a >>_t = m_e + \frac{E_1 + E_2}{2c^2} = \left\langle \frac{E}{c^2} \right\rangle. \tag{19}$$

5. Suggested experiment

In this short section, we discuss in brief an idealized experiment, which, in principle, allows to observe oscillations of the expectation value of active gravitational mass (18). By using laser, it is possible to create a macroscopic ensemble of coherent superpositions of electron stationary states in some gas. It is important that they are characterized by a feature that each molecule has the same phase difference between two wave function components, $\tilde{\Psi}_1(r)$ and $\tilde{\Psi}_2(r)$. In this case, the ensemble of atoms generates gravitational field, which oscillates in time (18).

Acknowledgments

We are thankful to N.N. Bagmet (Lebed), V.A. Belinski, Steven Carlip, Alex Cronin, Li-Zhi Fang, Arvinder Sandhu, Douglas Singleton, and V.E. Zakharov for useful discussions. This work was partially supported by the NSF under Grant DMR-1104512.

References

[1] Nordtvedt K 1994 *Class. Quantum Grav.* **11**, A119
[2] Carlip S 1998 *Am. J. Phys.* **66**, 409
[3] Lebed A G 2013 *Cent. Eur. J. Phys.* **11**, 969
[4] Lebed A G 2014 *J. Phys.: Conf. Ser.* **490** 012154
[5] Lebed A G 2014 *Adv. High Ener. Phys.* **2014** 678087
[6] Lebed A G 2014 *Proc. of the 13th Marcel Grossmann Meeting on General Relativity*, ed Remo Ruffini, Robert Jantzen, and Kjell Rosquist (World Scientific, Singapore), p. 1953
[7] Lebed A G 2015 *Int. J. Mod. Phys. D* **24** 1530027
[8] Park D 2005 *Introduction to the Quantum Theory*, 3rd edn. (Dover Publications, New York, USA)
[9] Misner C W, Thorne K S, and Wheeler J A 1973 *Gravitation* (W.H. Freeman and Co, New York, USA)
[10] Birrell N D and Davies P C W 1982 *Quantum Fields in Curved Space*, 3rd edn. (Cambridge University Press, Cambridge, UK)
[11] Landau L D and Lifshitz E M 2003 *The Classical Theory of Fields* (Butterworth-Heinemann, Oxford, UK)
[12] Lucha W 1990 *Phys. Rev. Lett.* **64** 2733