The multi-criteria optimization for the formation of the multiple-valued logic model of a robotic agent

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Abstract. The k-valued Allen-Givone algebra is the attractive tool for modeling of a robotic agent, but it requires the consensus method of minimization for the simplification of logic expressions. This procedure substitutes some undefined states of the function for the maximal truth value, thus extending the initially given truth table. This further creates the problem of different formal representations for the same initially given function. The multi-criteria optimization is proposed for the deliberate choice of undefined states and model formation.

1. Introduction

The k-valued Allen-Givone algebra (AGA) [1] is the mathematical tool, which is potentially very attractive to be used in optoelectronic data processing and for modeling of multi-agent robotic systems [2-4]. The multiple-valued logic function (MVLF) or a structured set of such functions can be used as the skeleton of the heterogeneous logic model of a robotic agent [3,4], providing exclusively high information capacity both for precise and approximately given parameters of the model. The appropriate structure of MVLFs is firstly to be given as the truth table for the switching function, whose logic expression is further to be written and simplified. This simplification is based on the method of consensus [1] which is to shorten the number of logic product terms and the computing time. But the minimization procedure in AGA use initially undefined states of the MVLF, thus extending its truth table. The problem is especially actual for multi-parametric MVLFs, as nobody will be able to use on practice all possible states for the MVLF with several dozens of input variables and 256 discrete truth values, because the truth table for such a MVLF will have ~10^{70} possible combinations of input variables, given as separate rows in the truth table [2,4]. Thus any real MVLF will contain a large number of initially undefined states, whose choice during minimization can influence greatly on the final set of product terms in the logic model, thus complicating its interpretation and knowledge search.

The aim of this paper is to show that the multi-criteria optimization procedure can be principally used for the choice of undefined states in the consensus minimization procedure of a MVLF, and this procedure can be done autonomously without the direct use of the computer- human expert dialog, which usually takes place in multi-criteria decision making.

2. Basic definitions and procedures of Allen-Givone algebra

The minimization of logic expression is regarded within the model of discrete k-valued Allen-Givone algebra (AGA) [1], where all input variables $x_1, \dots, x_n$ and the output variable $y = F(x_1, \dots, x_n)$ can have k discrete truth values $\{0,1,\dots,k-1\}$. In Boolean logic there are only two possible truth values $\{0,1\}$, but in AGA one can use arbitrary values of k. For the sake of simplicity examples further are given only for k=4, but the procedures to be used will be the same for $k = 256$ and even higher. The full set of non-Boolean operators is the set $\langle 0,1,\dots,k-1, X(a,b), MIN, MAX\rangle$, where $0,1,\dots,k-1$ are the constants, $MIN(x,y) = (*)$, $MAX(x,y) = (+)$ are respectively the choice of minimal and maximal values in the pair of values (x,y), operator $X(a,b)$ is called Literal and is given by

$$X(a,b) = \{ \begin{array}{ll} 0, & if \;\; b < x < a \\ k-1, & if \;\; a \le x \le b, \end{array} \quad (1)$$

where $b \ge a$ and $a, b \in L = \{0,1,\dots,k-1\}$.

Arbitrary MVLF in AGA can be given by its truth table [1], like as the function $y = F(x_1, x_2)$ was given for $k = 4$ and for the truth table in figure 1 a), thus the output variable y and input variables x_1, x_2 can have only truth values $\{0,1,2,3\}$. In figure 1 a) the function values y are initially undefined states for $(x_1, x_2) = (0,3)$ and $(x_1, x_2) = (2,3)$ as marked by "-".

x_1	x_2	$y=F(x_1,x_2)$
0	0	1
1	0	1
2	0	3
3	0	1
0	1	1
1	1	1
2	1	3
3	1	1
0	2	1
1	2	1
2	2	3
3	2	2
0	3	-
1	3	2
2	3	-
3	3	2

a)

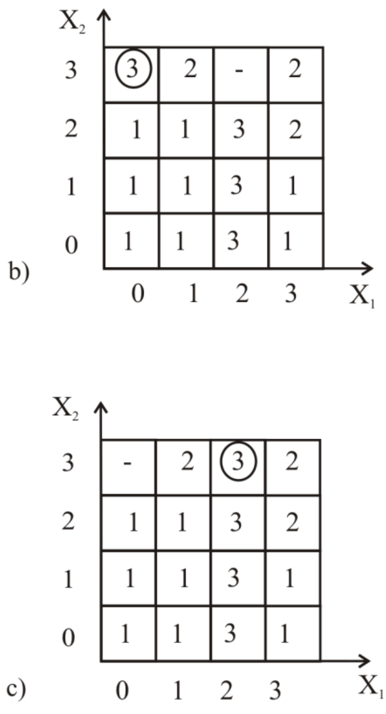

Figure 1. a) The truth table for the MVLF $y=F(x_1, x_2)$, where k=4 and there are initially two undefined function values y marked "- ". Both of them can be taken as the DCS in the process of consensus minimization.
b) The 2D grid representation of the truth table for the new function with the DCS chosen for (x_1, x_2)=(0,3), where "-" was changed for 3 as k-1=3.
c) The grid representation of the truth table for the new function with the DCS chosen for (x_1, x_2)=(2,3), where "-" was also changed for 3.

Any truth table can be transformed into the formal logic expression [1] written via operators MAX, MIN, LITERAL and constants C:

$$F(x_1, \dots, x_n) = \quad C_1 * X_1(a_{11}, b_{11}) * X_2(a_{12}, b_{12}) * \dots * X_n(a_{1n}, b_{1n}) + \\ \dots \\ C_{k-1} * X_1\big(a_{k-1,1}, b_{k-1,1}\big) * X_2\big(a_{k-1,2}, b_{k-1,2}\big) * \dots * X_n\big(a_{k-1,n}, b_{k-1,n}\big). \tag{2}$$

The function $y = F(x_1, x_2)$ in figure 1 a) can be composed [1] from Literal expressions, given for every row of the truth table

$$y = 1 * X_1(0,0) * X_2(0,0) + 1 * X_1(1,1) * X_2(0,0) + 3 * X_1(2,2) * X_2(0,0) + 1 * X_1(3,3) * X_2(0,0) + \\ 1 * X_1(0,0) * X_2(1,1) + 1 * X_1(1,1) * X_2(1,1) + 3 * X_1(2,2) * X_2(1,1) + 1 * X_1(3,3) * X_2(1,1) + \\ 1 * X_1(0,0) * X_2(2,2) + 1 * X_1(1,1) * X_2(2,2) + 3 * X_1(2,2) * X_2(2,2) + 2 * X_1(3,3) * X_2(2,2) + \\ 2 * X_1(1,1) * X_2(3,3) + 2 * X_1(3,3) * X_2(3,3), \tag{3}$$

where, e.g. the first row with $(x_1, x_2) = (0,0)$ in the truth table in figure 1 a) respond to $1 * X_1(0,0) * X_2(0,0)$. Literal expressions for "-" rows are not to be written into (3).

The method to obtain the minimized logic expression for MVLF`s truth table is given in [1] and is grounded on definitions 1-4, cited from [1] to disclose the principle of subsuming of product terms and the use of consensus method.

Definition 1. One product term $r_1 * X_1(a_1, b_1) * \dots * X_\mathrm{n}(a_\mathrm{n}, b_\mathrm{n})$ **subsumes** another product term $r_2 * X_1(c_1, d_1) * \dots * X_\mathrm{n}(c_\mathrm{n}, d_\mathrm{n})$, if and only if both conditions are true:

1) $r_1 \leq r_2$, (4)
2) $c_i \leq a_i \leq b_i \leq d_i$ for all X_i, $i = 1, \dots, \mathrm{n}$.

Example 1: in exp.(3) the product term $1 * X_1(0,0) * X_2(0,0)$ subsumes term $1 * X_1(0,1) * X_2(0,0)$ as its parameters $a_\mathrm{i}, b_\mathrm{i}$ are between c_i and d_i both for x_1 and x_2. Thus, the first product term is included into the second one and is to be deleted from the formal notation.

Definition 2. **Consensus** $j * U_\mathrm{j} *^i k \cdot U_\mathrm{k}$ in the i –th coordinate for product terms $r * U_1 = r * X_1(a_1, b_1) * \dots * X_\mathrm{n}(a_\mathrm{n}, b_\mathrm{n})$ and $s * U_2 = s * X_1(c_1, d_1) * \dots * X_\mathrm{n}(c_\mathrm{n}, d_\mathrm{n})$ is given by $j * U_\mathrm{j} *^i\ k * U_\mathrm{k} = q * X_1(e_1, f_1) * X_2(e_2, f_2) \dots * X_\mathrm{n}(e_\mathrm{n}, f_\mathrm{n})$ if and only if there exist q, e_k, f_k that

$$q = j * k, \tag{5}$$
$$X_\mathrm{k}(e_\mathrm{k}, f_\mathrm{k}) = X_\mathrm{k}(a_\mathrm{k}, b_\mathrm{k}) + X_\mathrm{k}(c_\mathrm{k}, d_\mathrm{k}) \text{ for } k = i,$$
$$X_\mathrm{k}(e_\mathrm{k}, f_\mathrm{k}) = X_\mathrm{k}(a_\mathrm{k}, b_\mathrm{k}) * X_\mathrm{k}(c_\mathrm{k}, d_\mathrm{k}) \text{ for all } k \neq i.$$

Definition 3. The operator **union** of Literals is defined as $X(a,b) = X(c,d) + X(e,f)$, and it exists if $a = MIN(c,e),\ b = MAX(d,f),\ e - 1 \leq d, c - 1 \leq \mathrm{f}$. (6)

Definition 4. The **intersection** of Literals is defined as $X(a,b) = X(c,d) * X(e,f)$, which exist if $a = MAX(c,e),\ b = MIN(d,f),\ e \leq d, c \leq \mathrm{f}$. (7)

Definitions given above provide the calculations results given in *Example 2* and further in Table 1.

Example 2: Consensus $1 * X_1(0,0) * X_2(0,0) *^1 1 * X_1(1,1) * X_2(0,0) =\ 1 * \mathrm{X1}(0,1) * \mathrm{X2}(0,0)$,
Consensus $1 * X_1(0,0) * X_2(0,1) *^2 1 * X_1(1,1) * X_2(0,0)$ does not exist.

Namely undefined states (i.e. "-" values of y) can be used for further minimization [1].

3. The problem of choosing undefined states of the MVLF for its minimization

Using definitions of subsuming (4) and consensus (5) the minimization procedure can be done, which demonstrate the dependence of minimization results from the choice of undefined states. According to [1] the new modified MVLF should be received by substitution of $k - 1$ instead of "-" for one or more undefined states, and adequate product terms are to be added to exp.(3). Authors of [1] named undefined states of y as "don`t care states" (DCSs) because additional values for the modified MVLF does not change the initial truth table. For simplicity the authors further use only DCS abbreviation.

For the example in figure 1 a) DCSs can be chosen both for $(x_1, x_2) = (0,3)$ and $(0,3)$, thus product terms $3 * X_1(0,0) * X_2(3,3)$ or $3 * X_1(2,2) * X_2(3,3)$ should be added to exp. (3) providing function modification. The new MVLF can be simplified quite efficiently. In order to show this more visible, lets firstly re-write truth table in figure 1 a) as two grids b) and c), containing two different variants of DCSs for the function $y = F(x_1, x_2)$. Product terms $3 * X_1(0,0) * X_2(3,3)$ and $3 * X_1(2,2) * X_2(3,3)$ were added respectively into rows N15 of the left and right parts of Table1.

Table 1. Minimization results are given for two possible variants of DCS choice for the MVLF from figure1 a). Left part of the table respond to the figure 1 b) and to the DCS taken at $(x_1, x_2) = (0,3)$. Right part (marked cursive) respond to figure 1 c) and to the DCS taken at $(x_1, x_2) = (2,3)$. Both left and right columns "Consensus $j *^i k$" demonstrate what tags j and k from columns "N" were substituted into consensus expression for the variable i.

N	IMPLICANT	Consens. $j*^{i}k$	Subsumes	*N*	*IMPLICANT*	*Consens. $j*^{i}k$*	*Subsumes*
1	~~1*X1(0,0) *X2(0,0)~~		Subsumes 16	1	~~1*X1(0,0) *X2(0,0)~~		Subsumes 16
2	~~1*X1(1,1) *X2(0,0)~~		Subsumes 16	2	~~1*X1(1,1) *X2(0,0)~~		Subsumes 16
3	3~~*X1(2,2) *X2(0,0)~~		Subsumes 18	3	~~3*X1(2,2) *X2(0,0)~~		Subsumes 18
4	~~1*X1(3,3) *X2(0,0)~~		Subsumes 17	4	~~1*X1(3,3) *X2(0,0)~~		Subsumes 17
5	~~1*X1(0,0) *X2(1,1)~~		Subsumes 19	5	~~1*X1(0,0) *X2(1,1)~~		Subsumes 19
6	~~1*X1(1,1) *X2(1,1)~~		Subsumes 19	6	~~1*X1(1,1) *X2(1,1)~~		Subsumes 19
7	~~3*X1(2,2) *X2(1,1)~~		Subsumes 18	7	~~3*X1(2,2) *X2(1,1)~~		Subsumes 18
8	~~1*X1(3,3) *X2(1,1)~~		Subsumes 20	8	~~1*X1(3,3) *X2(1,1)~~		Subsumes 20
9	~~1*X1(0,0) *X2(2,2)~~		Subsumes 21	9	~~1*X1(0,0) *X2(2,2)~~		Subsumes 21
10	~~1*X1(1,1) *X2(2,2)~~		Subsumes 21	10	~~1*X1(1,1) *X2(2,2)~~		Subsumes 21
11	3*X1(2,2) *X2(2,2)		Subsumes 23	11	~~3*X1(2,2) *X2(2,2)~~		Subsumes 23
12	~~2*X1(3,3) *X2(2,2)~~		Subsumes 22	12	~~2*X1(3,3) *X2(2,2)~~		Subsumes 22
13	~~2*X1(1,1) *X2(3,3)~~		Subsumes 24	13	~~2*X1(1,1) *X2(3,3)~~		Subsumes 25
14	~~2*X1(3,3) *X2(3,3)~~		Subsumes 26	14	~~2*X1(3,3) *X2(3,3)~~		Subsumes 27
15	**3*X1(0,0)*X2(3,3)**			15	~~3*X1(2,2)*X2(3,3)~~		Subsumes 23
16	~~1*X1(0,1) *X2(0,0)~~	$1*^{1}2$	Subsumes 28	16	~~1*X1(0,1) *X2(0,0)~~	$1*^{1}2$	Subsumes 28
17	~~1*X1(2,3) *X2(0,0)~~	$3*^{1}4$	Subsumes 28	17	1~~*X1(2,3) *X2(0,0)~~	$3*^{1}4$	Subsumes 28
18	~~3*X1(2,2) *X2(0,1)~~	$3*^{2}7$	Subsumes 23	18	~~3*X1(2,2) *X2(0,1)~~	$3*^{1}7$	Subsumes 31
19	~~1*X1(0,1) *X2(1,1)~~	$5*^{1}6$	Subsumes 30	19	~~1*X1(0,1) *X2(1,1)~~	$5*^{1}6$	Subsumes 29
20	~~1*X1(3,3) *X2(1,2)~~	$8*^{2}12$	Subsumes 25	20	~~1*X1(3,3) *X2(1,2)~~	$8*^{2}12$	Subsumes 26
21	1~~*X1(0,1) *X2(2,2)~~	$9*^{1}10$	Subsumes 30	21	~~1*X1(0,1) *X2(2,2)~~	$9*^{1}10$	Subsumes 32
22	**2*X1(2,3) *X2(2,2)**	$11*^{1}12$		22	~~2*X1(2,3) *X2(2,2)~~	$11*^{1}12$	Subsumes 37
23	**3*X1(2,2) *X2(0,2)**	$11*^{2}18$		23	~~3*X1(2,2) *X2(2,3)~~	$11*^{2}15$	Subsumes 31
24	**2*X1(0,1) *X2(3,3)**	$13*^{1}15$		24	1~~*X1(1,1) *X2(2,3)~~	$13*^{2}21$	Subsumes 33
25	~~1*X1(3,3) *X2(1,3)~~	$14*^{2}20$	Subsumes 29	25	~~2*X1(1,2) *X2(3,3)~~	$13*^{1}23$	Subsumes 35
26	**2*X1(3,3) *X2(2,3)**	$14*^{2}22$		26	1~~*X1(3,3) *X2(1,3)~~	$14*^{2}20$	Subsumes 36
27	~~1*X1(0,0) *X2(2,3)~~	$15*^{2}21$	Subsumes 32	27	~~2*X1(3,3) *X2(2,3)~~	$14*^{2}22$	Subsumes 37
28	1~~*X1(0,3) *X2(0,0)~~	$16*^{1}17$	Subsumes 34	28	**1*X1(0,3) *X2(0,0)**	$16*^{1}17$	
29	**1*X1(3,3) *X2(0,3)**	$17*^{2}25$		29	~~1*X1(0,2) *X2(1,1)~~	$18*^{1}19$	Subsumes 38
30	~~1*X1(0,1) *X2(1,2)~~	$19*^{2}21$	Subsumes 32	30	~~2*X1(2,2) *X2(0,2)~~	$18*^{2}22$	Subsumes 31
31	~~1*X1(2,3) *X2(0,2)~~	$23*^{1}29$	Subsumes 34	31	**3*X1(2,2) *X2(0,3)**	$18*^{2}23$	
32	~~1*X1(0,1) *X2(1,3)~~	$24*^{2}30$	Subsumes 33	32	**1*X1(0,3) *X2(2,2)**	$21*^{1}22$	
33	**1*X1(0,1) *X2(0,3)**	$28*^{2}32$		33	~~1*X1(1,1) *X2(1,3)~~	$24*^{2}29$	Subsumes 39
34	**1*X1(0,3) *X2(0,2)**	$31*^{1}33$		34	~~1*X1(1,3) *X2(3,3)~~	$25*^{1}26$	Subsumes 35
				35	**2*X1(1,3) *X2(3,3)**	$25*^{1}27$	
				36	~~1*X1(3,3) *X2(0,3)~~	$26*^{2}28$	Subsumes 40
				37	**2*X1(2,3) *X2(2,3)**	$27*^{1}31$	
				38	~~1*X1(0,2) *X2(0,1)~~	$28*^{2}29$	Subsumes 41
				39	~~1*X1(1,1) *X2(0,3)~~	$28*^{2}33$	Subsumes 43
				40	1~~*X1(2,3) *X2(0,3)~~	$31*^{2}36$	Subsumes 44
				41	~~1*X1(1,2) *X2(0,3)~~	$31*^{2}40$	Subsumes 44
				42	~~1*X1(1,3) *X2(2,3)~~	$32*^{2}35$	Subsumes 44
				43	**1*X1(0,2) *X2(0,2)**	$32*^{2}38$	
				44	**1*X1(1,3) *X2(0,3)**	$40*^{1}41$	

After this the minimization was done for both variants. During it some product terms with numbers in columns "N" subsumed other product terms with numbers given in the column "Subsumes", that is why they are shown crossed out. This procedure for both variants of DCS has given 8 and 7 final product terms, which are not crossed out and are shown bold. Direct substitution of all possible pairs (x_1, x_2) into resulting expressions demonstrates that both grids in figure 1 b) and c) are equivalent to reproduce the initial truth table in figure 1 a).

Resulting sets of product terms are written in the descending order of logic constants in figure 2 c) and d). They respond to grid variants b) and c) in the figure 1. AGA based consensus minimization [1] has provided double reduction of product terms number, and each of minterms respond to the specific rectangular group of cells in the 2D space (multidimensional in general case!). Sets of minterms in figure 2a) and b) are drastically different, what can be seen from the structure of truth values in rectangular segments separated by ellipses in figure 2 c) and d), respectively. As here $k = 4$, then truth values can be only within the range from 0 up to 3. The number signed on the outer part of each ellipse designates the serial number of the product term in columns of the figure 2 a) and b).

It follows from the data given above that one step minimization for one added DCS has formed the set of segments with equal values of truth values, and these rectangular segments also contain "islands" with greater truth values. Thus the choice of DCS modifies not only the rectangular segment for the truth value $k - 1$, but also can change all segments with $\{1,2,\dots,k-2\}$.

a) y=3∗X1(0,0)∗X2(3,3)+
3∗X1(2,2)∗X2(0,2)+
2∗X1(2,3)∗X(2,2)+
2∗X1(0,1)∗X2(3,3)+
2∗X1(3,3)∗X2(2,3)+
1∗X1(3,3)∗X2(0,3)+
1∗X1(0,1)∗X2(0,3)+
1∗X1(0,3)∗X2(0,2).

b) y=3∗X1(2,2)∗X2(0,3)+
2∗X1(1,3)∗X2(3,3)+
2∗X1(2,3)∗X(2,3)+
1∗X1(0,3)∗X2(0,0)+
1∗X1(0,3)∗X2(2,2)+
1∗X1(0,2)∗X2(0,2)+
1∗X1(1,3)∗X2(0,3).

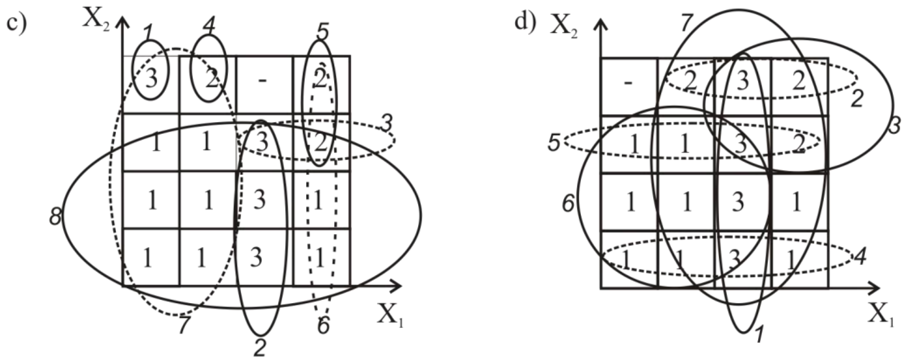

Figure 2. Minimized sets of minterms for DCSs chosen for $(x_1, x_2) = (0, 3)$ and $(x_1, x_2) = (2, 3)$ are given at a) and b), respectively. Grid segments structures produced by them are shown in c) and d). Group of cells produced by specific minterms in cases a) and b) are marked by ellipses in c) and d), where cursive number near the ellipse shows the serial number of this minterm in appropriate columns of minterms in a) and b). 8 ellipses are given for a) and 7 ones are given for b), respectively.

It seems reasonable that the obtained segments structure like given above can be potentially used for processing of multi-grade digital images, but its interpretation is difficult in contrast to traditional digital maps, which usually contain united groups of equal parameters. This stimulates to search for a method of intended choice of DCSs within specific rectangular segments or at their boundaries, which would form deliberately specific product terms to depict some object classes. Such a work can be done as a separate step of processing, providing additional pages of the MVLF digital map [2]. But for a model with large $k = 256$, the preparation of such data will need to process the large set of different segments with various truth values. That is why in general case one should regard the choosing of DCS as the multi-criteria problem.

4. The multi-criteria optimization as the potential method to choose DCSs for the MVLF

It follows from the previous section that the consensus minimization [1] can produce not visual structure of product terms even for very simple MVLFs. As this procedure takes into account DCSs position and truth values of nearby cells and segments of the grid, it will need to compute the large number of formal criteria and to apply multi-criteria choosing of DCSs. Now there are different methods of multi-criteria optimization (MCO) [5], where the most difficult cases refers to Pareto optimization methods. The general idea of Pareto method [5] is to find alternatives of $x: x_1, x_2, \ldots, x_n$ which obey the constraints $Q(x) = \left(q_1(x), \ldots, q_{1k}(x)\right) \rightarrow minimum\ or\ maximum,\ h_j(x) \geq 0$, where $q_j(x)$ are quality criteria and $h_j(x)$ are the constraints for the feasible solutions set D. Pareto set is regarded as the set for which the value of any criteria can be improved only at the cost of reduction of another criteria. The solution x_0 is Pareto optimal if there are no solutions $x \in D$, for which $q_i(x) \geq q_i(x_0)$ and the value at least for one criteria is strictly greater than for x_0.

However there is one substantial limitation for the robotic agent applications of MCO, as there are no universal multi-criteria methods for all tasks and mainly such methods involve direct participation of a human decision maker [5,6], whose expert preferences are used to choose the optimal solution.

That is why robotic agents are to use only “autonomous” methods of decision making, e.g. described in [6, 7] and based on axioms and special rules instead of direct dialog PC-human expert. The recent development of this technology was given in [7], where 5 basic axioms have been proven to provide Pareto optimal solutions. MCO problem was regarded in [7] as the choosing of possible solutions x in the arbitrary set X, where $f(x) = (f_1(x), \ldots, f_m(x))$ is the criteria vector, and binary relation $\succ_X$ given for X means preferences of the decision making system, i.e. the fact that solution x_1 is more preferable than solution x_2 is expressed by the notation $x_1 \succ_X x_2$. Solution of the problem is called the set of selected solutions $C(X)$, i.e. the set of best alternatives chosen from X by the decision making system. Also sets of possible vectors $Y = f(X)$ and of chosen vectors $C(Y) = f(C(X))$ were used [7].

Substantial hardship of MCO application for the choice of DCSs is in the fact, that MCO procedures are to be adapted to the specifics of MVLFs and heterogeneous logic model. Earlier proposed heterogeneous logic model [4] was designed to unite together precise (Boolean and multiple valued logic based) and approximate (fuzzy logic based) computing in an robotic agent, as in it all multi-parametric computing processes are to be controlled by means of the special skeleton structure of MVLFs. As AGA uses very specific non-binary operators set, the direct integration of MCO in AGA is impossible, but the architecture of heterogeneous logic modeling [4] provides the mapping of MCO calculations results on the subset $N` = \{0,1,2, \ldots, 256\}$ of the natural numbers set $N = \{0,1,2,3 \ldots\}$. First of all the task is to provide high enough precision for MVLF images of MCO.

The method to provide necessarily high precision for MVLF images was earlier proposed in [2] for high precision excessive description of space coordinates x, y, z by several interrelated parameters. E.g., the coordinate x can be represented as the triple $(x_1, x_{2,} x_3)$ with different scales, where x_1 has kilometer scale, $x_{2,}$ has meter scale and x_3 has millimeter scale. Thus the criteria $f(x) = (f_1(x), \ldots, f_m(x))$ can be mapped into AGA model as the extended set of interrelated functions $f(x) = (f_1^1(x), f_1^2(x), f_1^3(x))$, which are defined on the subset of $N` = \{0,1,2, \ldots, 256\}$. In AGA [1] new variables always can be correctly added to the model as additional Literal operator X(0, k-1) with parameters $a = 0$ and $b = k - 1$ just written into every product term in the model. For example, the product term $1 * X_1(0,0) * X_2(0,0)$ for $k = 4$ always can be extended as $1 * X_1(0,0) * X_2(0,0) * X_3(0,3)$ to enlarge the number of variables in the model.

Another aspect is to appreciate, if switching functions of AGA are “good” enough for mapping of criteria of MCO. Let`s see possible constraints drawn by 5 basic axioms of autonomous MCO procedure, which are cited from [7] and accompanied by commentaries.

<u>Axiom 1</u> (Pareto axiom). *For every pair of vectors* $y`, y`` \in R^m$, *which satisfy inequality* $y` \geq y``$, *the ratio* $y` \succ_Y y``$ *is satisfied too.*

<u>Interpretation</u>. *R* here is the set of real numbers. Due to [7] correct notation $y` \geq y``$ means that $y_i` \geq y_i``$ for all $i = 1,2, \ldots, m$, and the preference relation $y` \succ_Y y``$ is true, which provides the correct choice of one of y in the pair. Besides this the axiom impose that the preference is done even for equal vectors, thus reducing the number of vectors to be further analyzed. Also formal expressions for criteria can`t be changed and there are no other criteria to be used for choosing of solutions. MVLF

mappings on to $N` = \{0,1,2,\dots,256\}$ respond to Pareto axiom as it will always provide comparable values of MVLF criteria images according to inequality $y` \geq y``$, and can be done by means of comparison of output variables for MVLF, written by its truth table. Also it follows that not only vectors y but their MVLF images must be comparable and they can`t contain any DCSs.

Axiom 2. *For every pair of vectors $y`, y`` \in Y$, which satisfy relation $y` \succ_Y y``$, the condition $y`` \notin C(Y)$ is also satisfied.*

Interpretation. If one vector in the pair $y`, y``$ was not chosen, it is to be excluded from the whole set of possible vectors $C(Y)$ and out of all processing procedures. Thus the main task of Pareto set reduction method [7] is to shorten the set $C(Y)$ and to exclude one vector from every analyzed pair during every step of MCO.

Axiom 3**.** *The relation $\succ_Y$ has irreflexive and transitive extention $\succ$ for the whole space R^m.*

Interpretation. It means [7] that for the set Y there is no difference between $\succ_Y$ and $\succ$, what makes possible to work with the whole set R of real numbers. As the set of natural numbers N is the subset of R, then one can also use preference relation for the whole set $N`$. Also if the first solution is preferred to the second solution, and the second solution is preferred to the third one, than the first one is preferred to the third solution mapped on $N`$ [7]. Thus the process of Pareto set reduction can be regarded simply as a consequence of preference selections for all values of MVLF variables.

Axiom 4**.** *Each of criteria $f_1, f_2, \dots, f_m$ is consistent with preference relation $\succ$.*

Interpretation. Due to [7] it means that the criteria vector $f(x) = (f_1(x), \dots, f_m(x))$ is given as the formal expressions set, and if the pair of vectors $y`, y``$ has not equal criteria values for one criterion, but for all other criteria values are equal, than the decision maker system always prefer the maximal criterion value. Note that minimum criterion if necessary can be received from maximum criterion by the reverse function**.** Thus mapping criteria functions can`t have undefined DCS in it, as for each of criteria $f_1, f_2, \dots, f_m$ their images also always to be compared and chosen.

Axiom 5**.** *Preference relation $\succ$ is invariant relative to the positive linear transformation.*

Interpretation. Due to [7] for every pair of vectors $`, y`` \in R^m$, *f*or the relation $y` \succ y``$, for every $\alpha > 0$ and for an arbitrary vector $c \in R^m$ the expression $\alpha y` + c \succ \alpha y`` + c$ is performed. As multiplication and summation are not defined in MVLF, linear transformations can`t be done directly within AGA, but MVLF images should correctly sustain the same preference rule after the mapping procedure. That is why criteria functions are to be mapped on the MVLF switching functions with high enough precision, what needs to use further large k.

Generalizing the requirements of MCO axioms (1-5) [6, 7], one can conclude the following criteria functions limitations actual for AGA models:

- only bijection mapping one-to-one should be used for MVLFs images of criteria functions,
- all criteria values and their MVLF mappings should be comparable and any DCSs are prohibited,
- only single brunch and continuous functions should be selected for criteria mapping,
- criteria functions can`t be changed during minimization, any alternatives are regarded separately.

If MCO axioms (1-5) given above are satisfied, then the method [7] can be applied for further deliberate choosing of DCS, basing on the preference relation $\succ$ and criteria vector $f(x) = (f_1(x), \dots, f_m(x))$. Briefly it can be described as follows.

If there are two groups of criteria $A = (f_1(x), f_2(x), \dots, f_{q-1}(x))$ and $B = (f_q(x), f_{q+1}(x), \dots, f_m(x))$, where:

- the group of criteria $A = (f_1(x), f_2(x), \dots, f_{q-1}(x))$ is preferred to the group of criteria $B = (f_q(x), f_{q+1}(x), \dots, f_m(x))$ for relative increase in criteria parameters $\{w_1^+, w_2^+, \dots, w_{q-1}^+\}$ and relative decrease in criteria parameters $\{w_q^-, w_{q+1}^-, \dots, w_m^-\}$,
- the group of criteria B is preferred to the group A for relative increase in criteria parameters $\{v_1^+, v_2^+, \dots, v_{q-1}^+\}$ and relative decrease in criteria parameters $\{v_q^-, v_{q+1}^-, \dots, v_m^-\}$,
- all these sets of mutually dependent data are not contradictory,
- for two solution vectors x^1 and x^2 the criteria vector takes values $F(x^1) = (r_1, r_2, \dots, r_m)$ and $F(x^2) = (s_1, s_2, \dots, s_m)$,

then the task can be transformed into the problem to choose the solution from the set $X = (x^1, x^2)$ using two new criteria $g(x) = \{g_1(x), g_2(x)\}$, where values $g_1(x)$ and $g_2(x)$ can be calculated via

$$\begin{aligned}
g_1(x^1) &= min\left\{\frac{r_1}{w_1^+}, \frac{r_2}{w_2^+}, \dots, \frac{r_{q-1}}{w_{q-1}^+}\right\} + min\left\{\frac{r_q}{w_q^-}, \frac{r_{q+1}}{w_{q+1}^-}, \dots, \frac{r_m}{w_m^-}\right\},\\
g_2(x^1) &= min\left\{\frac{r_1}{v_1^+}, \frac{r_2}{v_2^+}, \dots, \frac{r_{q-1}}{v_{q-1}^+}\right\} + min\left\{\frac{r_q}{v_q^-}, \frac{r_{q+1}}{v_{q+1}^-}, \dots, \frac{r_m}{v_m^-}\right\},\\
g_1(x^2) &= min\left\{\frac{s_1}{w_1^+}, \frac{s_2}{w_2^+}, \dots, \frac{s_{q-1}}{w_{q-1}^+}\right\} + min\left\{\frac{s_q}{w_q^-}, \frac{s_{q+1}}{w_{q+1}^-}, \dots, \frac{s_m}{w_m^-}\right\},\\
g_2(x^2) &= min\left\{\frac{s_1}{w_1^+}, \frac{s_2}{w_2^+}, \dots, \frac{s_{q-1}}{w_{q-1}^+}\right\} + min\left\{\frac{s_q}{w_q^-}, \frac{s_{q+1}}{w_{q+1}^-}, \dots, \frac{s_m}{w_m^-}\right\}.
\end{aligned} \tag{8}$$

One solution from x^1, x^2 is to be chosen further [7] according to greater values obtained for criteria $g_1(x)$ and $g_2(x)$. Before the practical testing of the method, however, one should also to analyze the method to retrieve non contradicting data from mutual related data [7] for criteria groups of A and B.

5. Conclusions

K-valued Allen -Givone algebra is attractive for modeling of robotic agents and optoelectronic data processing, but it needs to simplify logic expressions by the consensus minimization method. This procedure includes the substitution of arbitrary taken initially undefined states of the MVLF into the truth table. That finally results in alternative formal representations for the same initially given truth table, what is discomfort for knowledge analysis. In order to control the formation of logic model expressions and to compare quickly them with templates, one should deliberately choose these DCS (initially undefined states), but this problem needs to analyze the complicated structure of product terms and segments, obtained for the large number of logic constants and variables. That is why this problem is to be regarded as the MCO task based on the Pareto set methods.

As any autonomous robotic system can`t be based on the direct dialog with the human expert, traditionally used in MCO tasks, than special autonomous decision making method is to be applied here [7], which bases on the special set of axioms and rules. In order to apply this version of MCO within the heterogeneous logic model [2-4] of a robotic agent, one should use only bijection (one-to-one) and high enough precision mapping of MCO criteria functions on to MVLFs, excluding any DCSs for them. Also only one brunch and continuous functions are possible for the criteria set. For this conditions to be fulfilled the MCO problem can be reduced to comparatively simple expressions and to the comparison of relative increase and decrease for different criteria groups.

The MCO application in AGA minimization is supposed to be used in robotic agents computer vision and pattern recognition systems.

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