Static and dynamic characteristics of angular velocity and acceleration transducers based on optical tunneling effect

V I Busurin, V V Korobkov, Naing Htoo Lwin, Phan Anh Tuan

Moscow Aviation Institute (National Research University), Volokolamskoe Road, 4, Moscow, A-80, GPS-3, 125993, Russia

e-mail: vbusurin@mai.ru

Abstract. Theoretical and experimental analysis of quasi-linear conversion function of angular velocity and acceleration microoptoelectromechnical (MOEM) transducers based on optical tunneling effect (OTE) are conducted. Equivalent oscillating circuit is developed and dynamic characteristics of angular velocity and acceleration MOEM–transducers are investigated.

1. Introduction

Angular velocity and acceleration transducer, constructed with structure «optical components of total internal reflection (TIF) – medium – sensing element (SE)» are suggested to apply for improving effectiveness of navigation and mobile objects control system with satisfied modern requirements [1-2]. Thus optical power, reflected from the structure «TIF – medium –SE», is received by photodetector and supports information about measuring result [3].

While developing such MOEM–transducers it is needed the complete description of actual conversion function of the measuring value (angular velocity and acceleration) in the change of optical output power in static and dynamic modes. The validity of the derived characteristics of MOEM–transducers should be based on the comparison results of theoretical calculations and experimental studies.

2. Correction of static characteristic of angular velocity and acceleration MOEM–transducers based on OTE according to experimental studies results

The operating principle of angular velocity and acceleration MOEM–transducers is based on dependence of optical source reflective coefficient on the value of operating gap between SE and TIF base. Therefore, the reflected part of optical power, reached to photodetector, is an informative value [4].

Experimental studies by using semi-natural setup has been conducted to verify the conversion function of MOEM–transducers based on OTE and determine the correction type of theoretical calculations (figure 1).

Semi–natural setup is comprised of FOT–930 Multifunction Loss Tester *1,* fiber optic cables *2*, TIF – prism made up of quartz *3*, piezoceramic SE in form of round plate *4*, load transferring ball *5*, aluminum beam *7* with hanging cable *6*. FOT-903 is used as optical source in generation mode with wavelengths λ_1=1.3μm and λ_2=0.85μm while supplying constant optical power source P_{OS}=2.045μW. Beam 7, fixed at one end, can change load F_H by changing mass m_H *8* and its hanging position. This varying load F_H, acting on SE center, leads to change of operating gap *d* between SE and TIF base.

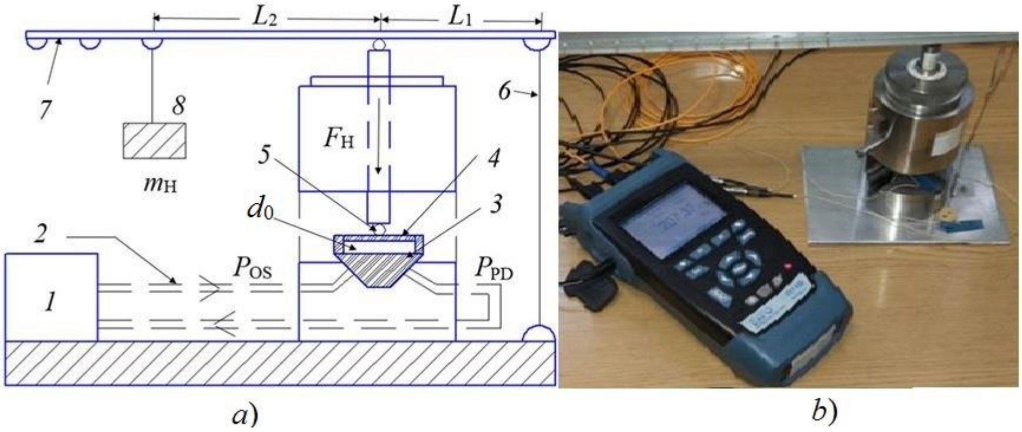

Figure 1. Semi–natural setup for investigation of MOEM–transducers based on OTE: *a* – structure, *b* – external view

From optical source *1*, signal enters to the structure «TIF–medium–SE» via optical fiber cables *2*. While reducing operating gap d from the influence of load F_H most of the optical energy penetrates to SE and is absorbed, which leads to decrease the optical input signal of photodetector P_{PD}.

While neglecting the mass of beam *7* (light compared to the suspended mass *8*) load F_H acting on the SE of MOEM–transducer is calculated by formula:

$$F_H = m_H \cdot g \cdot \left(\frac{L_1 + L_2}{L_1} \right). \tag{1}$$

SE of those MOEM–transducers based on OTE is clamped on the perimeter of circular end, and the largest deflection is observed at its center. Under the influence of the central load F_H magnitude of SE deflection $y_F(F_H)$ is function of parameters $y_F(F_H)=f\,(R_{SE}, h_{SE}, v, E_{SE})$ [5],where R_{SE} , h_{SE} – radius and thickness of SE; v – Poisson's ratio; E_{SE} – Young's modulus of SE material.

Magnitude of operating gap $d(F_H)$ between SE and TIF base is defined by initial gap d_0 and deflection of SE $y_F(F_H)$:

$$d_F(F_H) = d_0 - y_F(F_H)\,. \tag{2}$$

The output optical power with the conversion function of such MOEM–transducer based on OTE in account of optical loss is determined by the following formula [6]:

$$P_{PD}(F_H) = P_{OS} \cdot k_{opt} \cdot R\left[d_F\left(F_H\right), \theta, \lambda \right], \tag{3}$$

where k_{opt} – optical loss; λ – wavelength of optical source; θ – angle of incidence, $R[d_F(F_H),\theta,\lambda]$ – reflectivity of structure «TIF – medium –SE».

Experimental studies of conversion function is conducted with following parameters: R_{SE} = 5mm, h_{SE} = 1mm, v = 0.17, E_{SE} = 72.5GPa (figure 2).

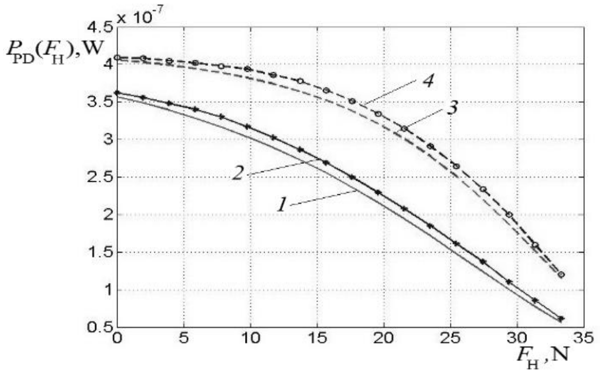

Figure 2. Dependence of conversion function of MOEM–transducer based on OTE on load F_H: *1*, *3* – theoretical calculation results at λ_1=1.3μm, λ_2=0.85 μm; *2*, *4* – experimental results at λ_1=1.3μm, λ_2=0.85 μm accordingly.

From comparing of theoretical calculation and experimental results it should follow that the analytical formula to calculate the conversion function can support to be quarsilinear and can be considered as acceptable for dependent description of output signal on the external influence to MOEM–transducer based on OTE, while ensuring not more than 15% of error.

To improve the accuracy of MOEM–transducer model is necessary to select the correction coefficient of theoretical calculations based on the experimental results, e.g., least squares method. The correction coefficient of conversion function must be satisfied in the condition with n experiments:

$$\sum_{i=1}^{n}\left[P_{\mathrm{PD\,EXP}\,i}(F_{\mathrm{H}\,i})-K_{\mathrm{CORR}}(F_{\mathrm{H}\,i})\cdot P_{\mathrm{PD\,THEO}\,i}(F_{\mathrm{H}\,i})\right]^2\rightarrow\min\,, \qquad (4)$$

where $P_{\mathrm{PD\ EXP}\ i}(F_{\mathrm{H}\ i})$, $P_{\mathrm{PD\ THEO}\ i}(F_{\mathrm{H}\ i})$ – magnitude of optical power by means of experiments and theoretical calculation, corresponding value of load $F_{\mathrm{H}\ i}$; $K_{\mathrm{CORR}}(F_{\mathrm{H}\ i})$ – correction coefficient of conversion function, $K_{\mathrm{CORR}}(F_{\mathrm{H}\,i})=f(F_{\mathrm{H}\,i},f_1,f_2,\ldots,f_{k+1})$, $f_1,f_2,\ldots,f_{k+1}$ – variables of polynomial coefficient in k times.

Theoretical calculation of conversion function uses correction coefficient, described polynomial with 3rdorder: $K_{\mathrm{CORR}}(F_{\mathrm{H}}) = (1.0137+0.00424\cdot F_{\mathrm{H}}-0.00017\cdot F_{\mathrm{H}}^2 +0.00001\cdot F_{\mathrm{H}}^3)\cdot 10^{-8}$ to provide the error not more than 2%.

3. Dynamic behaviors of angular velocity and acceleration MOEM–transducers based on OTE

Block diagram is composed to study the dynamic mode of angular velocity and acceleration MOEM–transducers based on OTE (figure 3.), which includes: a movable part - SE, which converts the measuring value Ψ (angular velocity Ω_z or acceleration a_z) according to varying of operating gap d_Ψ; modulator (M), which converts optical power source P_{OS} to output optical power of photodetector P_{PD}; converter "current-voltage" (I-U), which converts current I_{PD} into voltage, supplied to low pass filter (LPF) and generate output voltage; У- amplifier in feedback circuit (OC); PA - piezoactuator that converts the voltage U_{OC} with movement of d_{U}.

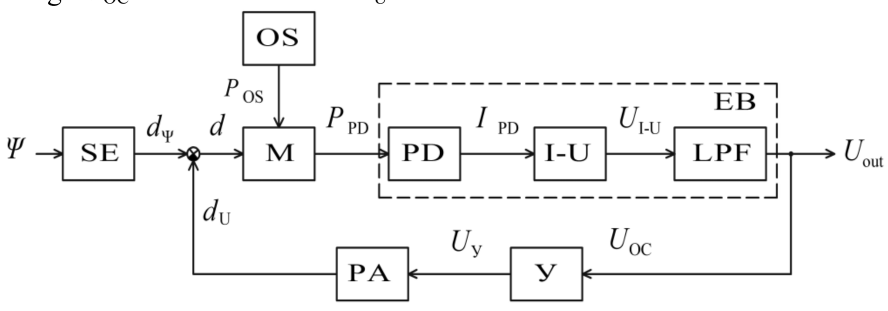

Figure 3. Block diagram of angular velocity and acceleration MOEM–transducers based on OTE with feedback circuit

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Using feedback circuit can reduce the transfer ratio of conversion, which allows to increase the maximum measuring value to desired measuring range by adjusting operating gap of transducers [7]. In contravention of this part sensitivity and linearity of the MOEM–transducers conversion function can be decreased.

According to the block diagram of MOEM–transducers a transfer function $W_{\Sigma}(p)$ in form of operators is:

$$W_{\Sigma}(p) = W_{\text{SE}}(p) \cdot \frac{W_{\text{M}}(p) \cdot W_{\text{EB}}(p)}{1 + W_{\text{M}}(p) \cdot W_{\text{EB}}(p) \cdot W_{\text{У}}(p) \cdot W_{\text{PA}}(p)}, \tag{5}$$

where $W_{\text{SE}}(p)$ – transfer function of SE; $W_{\text{M}}(p)=P_{\text{OS}}\cdot k_{\text{M}}(p)$ – transfer function of M; $W_{\text{EB}}(p)=W_{\text{PD}}(p)\cdot W_{\text{I-U}}(p)\cdot W_{\text{F}}(p)$ – transfer function of electronic block (EB); $W_{\text{PD}}(p)=k_{\text{PD}}/(T_{\text{PD}}\cdot s+1)$, $W_{\text{I-U}}(p)=k_{\text{I-U}}/(T_{\text{PD}}\cdot s+1)$, $W_{\text{F}}(p)=k_{\text{F}}\cdot \omega_{\text{F}}/(s+\omega_{\text{F}})$, $W_{\text{У}}(p)=k_{\text{У}}/(T_{\text{У}}\cdot s+1)$, $W_{\text{PA}}(p)=k_{\text{PA}}/(T_{\text{PA}}\cdot s+1)$ – transfer function of PD, I-U, LPF, У, PA respectively; $k_{\text{M}}(p)$ – transfer ratio of M [1/m]; T_{PD}, k_{PD} – time constant, transfer ratio of PD[A/W]; $T_{\text{I-U}}$, $k_{\text{I-U}}$ – time constant, transfer ratio of I-U [Ohms]; $k_{\text{F}}= 1/[1+(\omega/\omega_{\text{F}})^2]$, ω_{F} – transfer ratio, frequency pole of LPF; $T_{\text{У}}$, $k_{\text{У}}=R_2/R_1$ – time constant, gain of У; T_{PA}, k_{PA} – time constant, transfer ratio of PA[m/V].

SE of MOEM–transducers based on OTE can be considered as a mechanical oscillating system which can be described with second order differential equation [8]:

$$J \cdot \frac{d^2 y}{dt^2} + K_{\text{GD}} \cdot \frac{dy}{dt} + G_{\text{stiff}} \cdot y = Q(t), \tag{6}$$

where J, G_{stiff} – inertial magnitude and stiffness of SE, K_{GD} – coefficient of gas dynamic damping due to viscous friction of the medium of structure «TIF – medium –SE», y – oscillating velocity.

Note that an arbitrary external factor $Q(t)$, acting on the SE of MOEM–transducers, can be expanded in Fourier series [9]:

$$Q(t) = \frac{1}{2}a_0 + \sum_{k=1}^{+\infty} A_k \cos(k\omega t + \theta_k) \ . \tag{7}$$

where A_{k}, $k\omega$, θ_{k} – amplitude, circular frequency, initial phase k times harmonics oscillation, a_0 – coefficient of Fourier series.

Let us assume that the investigated system is satisfied by filter condition, which do not effect with higher harmonics significantly the basic, i.e. can be neglected. Thus, the equation (7) can be written as:

$$Q(t) = \frac{1}{2}a_0 + A_1 \cos(\omega t + \theta_1). \tag{8}$$

By following equation (8), the external factor $Q(t)$ according to the principle of superposition can be separately considered as a constant and harmonious action. Then, the external factor $Q(t)$, acting on the acceleration MOEM–transducer can be assumed constant, and for angular velocity MOEM–transducer - harmonic.

Angular velocity and acceleration MOEM–transducers are acoustomechanical oscillating system with distributed parameters, consisting of SE and the air gap between SE and prism base. To analyze the dynamic characteristics of MOEM–transducers, which are defined by properties of acoustomechanical oscillatory system, conveniently and efficiently by using well known methods of electrical engineering and circuit theory. The criteria of equivalence are governed by the laws of energy conservation. In addition, each factor of the differential equation (6) is replaced by combination of equivalent elements of electrical oscillating RLC–circuit, in which the value of inertial J and, gas dynamic damping coefficient K_{GD} and stiffness of SE G_{stiff} are corresponded to inductive L_{E}, capacitive C_{E} and active electrical resistance R_{E}.

Equivalence, acting on external factor of SE corresponds to EMF $U_E(t)$, and oscillating velocity of system y - current $I_E(t)$ (figure 4).

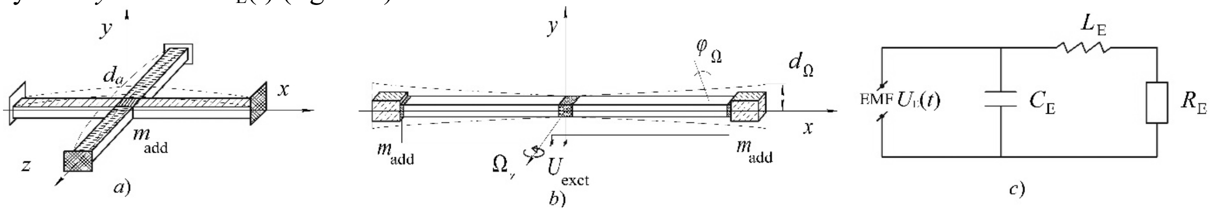

Figure 4. SE of angular velocity (*a*) and acceleration (*b*) MOEM–transducers based on OTE and equivalent schematic (*c*) of oscillating RLC-circuit

Under the influence of acceleration a_y acceleration MOEM–transducer based on OTE (Figure 4*a*) is deflected along the *OY* axis [2], which is corresponding to oscillating RLC-circuit (Figure 4*c*) with equivalent parameters: $U_E(t)=U_a=(m_a+m_{add})\cdot a_y$; $L_E=b_a\cdot h_a^3/12$; $R_E=K_{GD}=2\cdot\mu\cdot h_a^3\cdot b_a^3/[l_a^3\cdot[(h_a^2+b_a^2)]$; $1/C_E=G_{stiff}=(128\cdot E_a\cdot b_a\cdot h_a^3)/l_a^4$; where $\mu=f(p_c,T_0,\rho_c,v_c,k_B)$ – coefficient of dynamic viscosity, depend on density ρ_c, pressure p_c, average flow rate v_c, ambient temperature T_c and Boltzmann constant k_B (at temperature T_0=20°C, here $\mu\approx1{,}83\cdot10^{-6}$kg/m·s can be selected for air); l_a, h_a, b_a, m_a, E_a – length, thickness, width, mass of SE and Young's modulus of four-beam fused silicon SE, m_{add} – additional mass accordingly; and, electrical current $I_E(t)=dd_a(t)/dt$ – rate of central part deflection of SE. By the Kirchhoff's voltage law:

$$L_{\mathrm{E\,a}}\cdot\frac{dI_{\mathrm{E\,a}}(t)}{dt}+R_{\mathrm{E\,a}}\cdot I_{\mathrm{E\,a}}(t)+\frac{1}{C_{\mathrm{E\,a}}}\cdot\int I_{\mathrm{E\,a}}(t)\cdot dt=U_{\mathrm{a}}\,. \tag{9}$$

Differentiating equation (9) with respect to time and multiplying by $1/L_E$, obtain:

$$\frac{d^2I_{\mathrm{E\,a}}(t)}{dt^2}+2\alpha\frac{dI_{\mathrm{E\,a}}(t)}{dt}+\omega_0^2I_{\mathrm{E\,a}}(t)=0\,, \tag{10}$$

where $\alpha=R_{\mathrm{E\,a}}\,/\,(2\cdot L_{\mathrm{E\,a}})$ – attenuation coefficient of oscillation in RLC-circuit, $\omega_0=1/\sqrt{(L_{\mathrm{E\,a}}\cdot C_{\mathrm{E\,a}})}$ – natural oscillating frequency in RLC-circuit.

At the initial condition: current $I_{\mathrm{E\,a}}|_{t=0}=U_a/R_{\mathrm{E\,a}}$ and the rate of change of current is different from zero $L_{\mathrm{E\,a}}\cdot dI_{\mathrm{E\,a}}(t)/dt|_{t=0}=-U_a$, the total current $I_{\mathrm{E\,a}}(t)$ in an oscillatory RLC-circuit varies according to formula [10]:

$$I_{\mathrm{E\,a}}(t)=\frac{U_{\mathrm{a}}}{R_{\mathrm{E\,a}}}-\frac{U_{\mathrm{a}}}{L_{\mathrm{E\,a}}\cdot\sqrt{\alpha^2-\omega_0^2}}e^{-\alpha t}\mathrm{sh}\left(\sqrt{\alpha^2-\omega_0^2}\cdot t\right). \tag{11}$$

Consequently, the transfer function $W_{\mathrm{SE\,a}}(p)$ of SE of acceleration transducer based on OTE may be defined by formula:

$$W_{\mathrm{SE\,a}}(p)=L\left\{\frac{\partial d_{\mathrm{a}}(t)}{\partial a}\right\}=L\left\{\frac{\partial}{\partial a}\left[\frac{U_{\mathrm{a}}}{R_{\mathrm{E\,a}}}-\frac{U_{\mathrm{a}}}{L_{\mathrm{E\,a}}\cdot\sqrt{\alpha^2-\omega_0^2}}e^{-\alpha t}\mathrm{sh}\left(\sqrt{\alpha^2-\omega_0^2}\cdot t\right)\right]\right\}, \tag{12}$$

where L – Laplace transform.

Under the influence of measuring angular velocity Ω_z SE of angular velocity MOEM–transducers based on OTE (Figure 4*b*) occurs small deviations $d_\Omega(t)$ due to Coriolis effect [11]. This schematic can be seen as an oscillatory RLC-circuit (Figure 4*c*) with sinusoidal action at equivalent parameters: $Q(t)=U_0\sin(\omega t)=[l_\Omega\cdot F_0(\Omega_z)/(2\omega)]\cdot\sin(\omega t)$, where $F_0(\Omega_z)=\pi\cdot d_b\cdot U_{exct}\cdot\Omega_z\cdot[(\rho_\Omega\cdot l_\Omega\cdot h_\Omega\cdot b_\Omega\cdot\omega)/2+(4\cdot\pi\cdot m_{add})]$ – amplitude of Coriolis force); $R_E=\mu\cdot l_\Omega^4\cdot h_\Omega^3/[8\cdot b_\Omega^2\cdot(l_\Omega^2+4\cdot h_\Omega^2)]$; $1/C_E=2\cdot E_\Omega\cdot h_\Omega\cdot b_\Omega^3/2\cdot l_\Omega^3$, $L_E=J=(\rho_\Omega\cdot h_\Omega\cdot b_\Omega\cdot l_\Omega^3/24)+(m_{add}\cdot l_\Omega^2)/4$; where l_Ω, h_Ω, b_Ω, m_Ω, E_Ω, d_b – length, thickness, width, mass of SE, Young's modulus и piezomodule with feedback effect of piezoceramic SE, U_{exct} – excitation voltage with frequency ω accordingly; and electrical current $I_E(t)=d\varphi_\Omega(t)/dt$ – rate of change of angular displacement of SE along OY axis (for small value $\varphi_\Omega(t)\approx\sin[\varphi_\Omega(t)]$ is accepted, that $\varphi_\Omega(t)=2\cdot d_\Omega(t)/l_\Omega$). Then second order differential equation for such oscillatory RLC-circuit is:

$$\frac{d^2 I_{\mathrm{E}\,\Omega}(t)}{dt^2}+2\alpha\frac{dI_{\mathrm{E}\,\Omega}(t)}{dt}+\omega_0^2 I_{\mathrm{E}\,\Omega}(t)=\frac{\omega U_0}{L_{\mathrm{E}\,\Omega}}\sin(\omega t)\,. \tag{13}$$

Similarly, the general solution of equation (13) can be written as

$$I_{\mathrm{E}\,\Omega}(t)=\frac{U_0\cdot\omega\cdot\sin\left(\omega t-\Delta\varphi\right)}{L_{\mathrm{E}\,\Omega}\cdot\sqrt{\left(\omega_0^2-\omega^2\right)+4\alpha^2\omega^2}}-\frac{U_0}{L_{\mathrm{E}\,\Omega}\cdot\sqrt{\alpha^2-\omega_0^2}}\cdot e^{-\alpha t}\cdot\mathrm{sh}\left(\sqrt{\alpha^2-\omega_0^2}\cdot t\right), \tag{14}$$

where $\Delta\varphi=\arctan[(\omega^2-\omega_0^2)/(2\alpha\omega)]$ – phase shift.

Transfer function $W_{\mathrm{SE}\,\Omega}(p)$ of SE of angular velocity MOEM–transducers based on OTE is:

$$W_{\mathrm{SE}\,\Omega}(p)=L\left\{\frac{1}{2\cdot l_\Omega}\cdot\frac{\partial\varphi_\Omega(t)}{\partial\Omega_Z}\right\}=\frac{1}{2\cdot l_\Omega}L\left\{\frac{\partial}{\partial\Omega_Z}\left[\frac{U_0\cdot\omega\cdot\sin\left(\omega t-\Delta\varphi\right)}{L_{\mathrm{E}\,\Omega}\cdot\sqrt{\left(\omega_0^2-\omega^2\right)+4\alpha^2\omega^2}}-\frac{U_0\cdot e^{-\alpha t}}{L_{\mathrm{E}\,\Omega}\cdot\sqrt{\alpha^2-\omega_0^2}}\cdot\mathrm{sh}\left(\sqrt{\alpha^2-\omega_0^2}\cdot t\right)\right]\right\}. \tag{15}$$

Transient process modelling of angular velocity and acceleration MOEM–transducers based on OTE is shown in figure 5 with the following parameters of functional elements: PD T_{PD}=1μs, k_{PD}=0.4A/W; $T_{\mathrm{I\text{-}U}}$=10μs, $k_{\mathrm{I\text{-}U}}$=4kOhm; T_{Y}=10μs, k_{Y}=100; $k_{\mathrm{F}}\approx1$; T_{F}=10μs, T_{PA}=3μs, P_{OS}=0.5mW, λ=1.3μm, θ=47°.

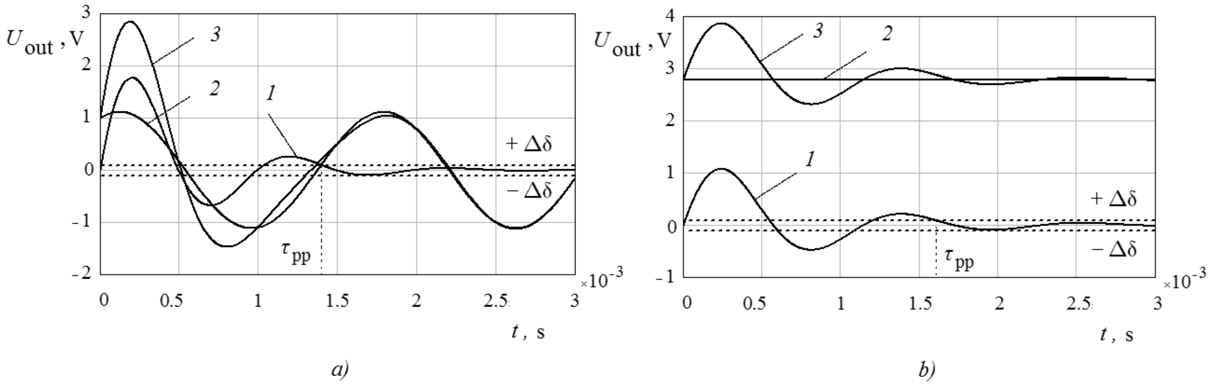

Figure 5. Transient processes of angular velocity (*a*) and acceleration (*b*) MOEM–transducers based on OTE *1*- composed damping, *2*- composed setting, *3* - resultant output voltage

In dynamic mode the considered transducers are stable and possess only small oscillatory transients (line *1*). When the permissible dynamic error $\Delta\delta$ is not exceed 5%, output characteristics (line *3*) of angular velocity and acceleration MOEM–transducers are included in the steady state (lines *2*) with a short duration transient $\tau_{\mathrm{PP}}\sim$ 1ms.

4. Conclusion

In this study angular velocity and acceleration MOEM–transducers based on OTE, providing quasi-linearity conversion function is investigated. An experimental study of this transducer conversion function is described. Correction coefficient of conversion functions based on comparing of theoretical calculations and experimental studies is defined and it supports to improve the accuracy of the calculation of the described model.

The block diagram is determined and equivalent oscillatory RLC-circuit is composed for the study of dynamic characteristics angular velocity and acceleration MOEM–transducers based on OTE. It is shown that the transition process of described transducers have oscillated character with permissible magnitude of transition process duration τ_{PP} is around 1ms, which allows to apply those transducers in mobile objects control systems.

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