

Compensation of asymmetry of diffraction characteristics of holographic chirped photonic structures in PDLC by the impact of spatially non-uniform electric field

A O Semkin, S N Sharangovich

Tomsk state university of control systems and radioelectronics, 40 Lenina av., Tomsk, Russia

E-mail: shr@tusur.ru

Abstract. In this work we have proposed the method of asymmetry compensation of diffraction characteristics of chirped photonic structures in PDLC by the impact of electric field, which has different forms of spatial inhomogeneity. It's shown that for each area of external electric field values the function of inhomogeneity has its optimal form.

1. Introduction

Recently many of practical challenges of photonics, non-linear optics and optical telecommunication systems may be solved by chirped photonic structures (ChPS) using. Earlier in [1] the possibility to increase the efficiency of light input to the polymer optical fiber by using ChPS, holographically formed in photopolymer materials, was shown. Besides, in [2] the possibility to compensate the inhomogeneity of holographic chirped diffraction grating by amplitude profiles of recording beams was shown. The liquid crystal (LC) molecules introduction to photopolymer material allows us to make a composite material – polymer-dispersed liquid crystal (PDLC). Optical properties of this material are manageable due to possibility of LC molecules to reorient in external electric field. In [3] we have shown, that the impact of external electric field on ChPS in PDLC causes the distortion of its diffraction characteristics.

The aim of this work is to theoretically investigate the compensation of ChPS diffraction characteristics asymmetry by the impact of external electric field which has the special form of spatial inhomogeneity.

2. Theoretical model

For description of light beam diffraction on holographic ChPS in PDLC we will use results of [4-7]. We will consider a two-dimensional Bragg diffraction of quasi-monochromatic beam $E_p(\mathbf{r})$, corresponding to an extra-ordinary wave, on inhomogeneous photonic structure (figure 1a). The direction of a periodic perturbation of the dielectric constant is given by the vector \mathbf{K} , which is due to inhomogeneity of ChPS represents a function of the coordinates $\mathbf{K} = \mathbf{K}(\mathbf{r})$. The spatial inhomogeneity of the external electric field is set by a special topology of the electrode structure (figure 1a). Different forms of external field inhomogeneity we will describe by the following expression:

$$E(y) = \text{ch}[c \cdot (s \cdot y - t)]^{-1} - g, \quad (1)$$

where y – coordinate oriented along the grating vector; c, s, t, g – approximation coefficients.



Form of external field inhomogeneity $E(y)$ for incident beam aperture $w=1$ cm and for different approximation coefficients is shown on figure 1b. Curve 1 corresponds to uniform field, curve 2 – linearly non-uniform, in analogue to [3]. Function (1) takes the form of increasing, downward convex function (curve 3 on figure 1b) at $c=2.7$, $s=-2$, $t=-2.1$, $g=1$. Curve 4 on figure 1b corresponds to increasing, upward convex function, $c=0.46$, $s=-1$, $t=-0.5$, $g=0.1$. The bell-shaped form (curve 5) function (1) takes at $c=1.8$, $s=-0.5$, $t=-0.01$, $g=0.1$.

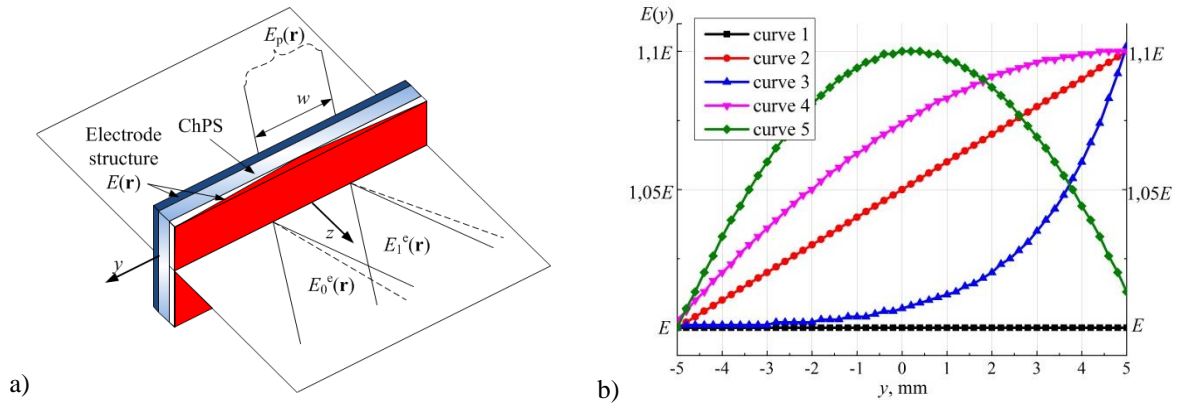


Figure 1. Non-uniform electric field: a) – diffraction geometry and electrode structure; b) – forms of electric field inhomogeneity functions

The perturbation of the sample dielectric tensor because of its smallness in relation to $\hat{\epsilon}_0$ we will present as the following sum:

$$\hat{\epsilon}(\mathbf{r}) = \hat{\epsilon}_0 + \Delta\hat{\epsilon}_s(\mathbf{r}) + \Delta\hat{\epsilon}_e(\mathbf{r}), \quad (2)$$

where $\hat{\epsilon}_0$ – unperturbed dielectric tensor; $\Delta\hat{\epsilon}_s(\mathbf{r})$ – periodical perturbation, caused by the inhomogeneous ChPS recording process; $\Delta\hat{\epsilon}_e(\mathbf{r})$ – smooth perturbation, caused by the impact of spatially non-uniform electric field. $E(y)$ (figure 1b).

$$\Delta\hat{\epsilon}(\mathbf{r}) = 0.5\Delta\hat{\epsilon}[U_0U_m(\mathbf{r})\exp[i\cdot\varphi(\mathbf{r})]], \quad (3)$$

where U_0 , $U_m(\mathbf{r})$ – amplitude of perturbation and normalized amplitude profile; $\varphi(\mathbf{r})$ – phase profile, which in the case of ChPS can be represented as $\varphi(\mathbf{r}) = \mathbf{K}(\mathbf{r}) \cdot \mathbf{r}$; $\Delta\hat{\epsilon}$ – perturbation of $\hat{\epsilon}_0$, corresponding to $U_0 = 1$; \mathbf{r} – radius-vector.

Changing of the dielectric constant caused by the impact of external field is due to the reorientation of LC director in the electric field under the action of Fredericks effect [8-10]. As in the considered medium the statistical distribution of the LC molecules preferred orientation in the "droplets" is observed [9] we will introduce the corresponding changing of the dielectric constant in the form of statistically averaged tensor:

$$\langle\Delta\hat{\epsilon}_e(\mathbf{r}, E)\rangle = -\Delta\epsilon \int_0^1 \int_{-1}^1 \mathbf{C}(\mathbf{r}, E) \mathbf{C}(\mathbf{r}, E) p(\alpha) q(\phi) d\alpha d\phi, \quad (4)$$

where $\mathbf{C}(\mathbf{r}, E)$ – spatial distribution of LC director orientation; $\langle...\rangle$ means statistical averaging.

In (4) we introduced the Gaussian functions of director distribution in the quasi-ellipsoidal LC "droplet" [9]:

$$p(\alpha) = A \exp\left[-(\alpha - \bar{\alpha})^2 / 2\sigma_\alpha^2\right], \quad q(\phi) = B \exp\left[-(\phi - \bar{\phi})^2 / 2\sigma_\phi^2\right], \quad (5)$$

where α , ϕ – orientation angles of LC molecules in the "droplet"; $\bar{\alpha}$, $\bar{\phi}$ – averages; σ_α , σ_ϕ – standard deviations.

LC director orientation changing in each spatial point under the influence of an electric field will be given by the angle of declination from its initial direction. As the considered medium is a set of LC "droplets" suspended in the polymer, to determine the distribution of the "droplets" rotation angle in the sample we will use the theoretical model, developed in [8]:

$$\gamma(\mathbf{r}, E) = \psi(\mathbf{r}) + \frac{1}{2} \cdot \arctg \left[\frac{\sin[2\vartheta_0(\mathbf{r})]}{e^2 + \cos[2\vartheta_0(\mathbf{r})]} \right], \quad (6)$$

where $\psi(\mathbf{r})$ – orientation angle of external electric field vector \mathbf{E} ; $\vartheta_0(\mathbf{r}) = \gamma_0 - \psi(\mathbf{r})$ – initial angle between \mathbf{E} and \mathbf{C} vectors; $e = |\mathbf{E}|R\sqrt{\Delta\bar{\epsilon}/K(5.7\delta^2 + 2.1\lambda)}$ – parameter, which characterizes the impact of electric field on bipolar LC droplet; R – radius of the droplet; δ – eccentricity of the droplet; $\lambda = RW_a/K$ – parameter of surface adhesion; W_a – azimuth surface adhesion coefficient; K – Frank's elasticity coefficient; $\Delta\bar{\epsilon}$ – effective dielectric anisotropy of bipolar droplet.

Critical electric field at which the Fredericks effect will occur is given by [8]:

$$Ec = \frac{1}{R} \cdot \left(\frac{5.7K\delta^2 + 2.1W_aR}{\Delta\bar{\epsilon}} \right)^{1/2}. \quad (7)$$

In the considered case of Bragg diffraction of the extraordinary waves on grating (2), the total optical field in the sample can be represented as [4]:

$$E_s(\mathbf{r}, E) = \sum_{i=0,1} \mathbf{e}_i E_i^e(\mathbf{r}, E) \exp[i(\omega_i t - \int \mathbf{k}_i \cdot d\mathbf{r})], \quad (8)$$

where \mathbf{e}_i , \mathbf{k}_i , ω_i – polarization vectors, wave vectors and frequencies of transmitted $E_0^e(\mathbf{r}, E)$ and diffracted $E_1^e(\mathbf{r}, E)$ beams.

Amplitude distributions $E_{0,1}^e(\mathbf{r}, E)$ can be determined by solving the system of coupled waves equations (CWE) [4]:

$$\mathbf{N}_{r0}^e \cdot \nabla E_0^e(\mathbf{r}, E) = -iC_1^e(\mathbf{r}, E)U_m^*(\mathbf{r})E_1^e(\mathbf{r}, E) \exp(+i \int_0^r \Delta K(\mathbf{r}, E) dr), \quad (9)$$

$$\mathbf{N}_{r1}^e \cdot \nabla E_1^e(\mathbf{r}, E) = -iC_0^e(\mathbf{r}, E)U_m(\mathbf{r})E_0^e(\mathbf{r}, E) \exp(-i \int_0^r \Delta K(\mathbf{r}, E) dr), \quad (10)$$

where $C_i^e(\mathbf{r}, E) = \frac{1}{4} \frac{\omega \cdot U_0}{c_c n_i^e(\mathbf{r}, E)} \mathbf{e}_1^e(\mathbf{r}) \cdot \Delta \hat{\epsilon}(\mathbf{r}, E) \cdot \mathbf{e}_0^e(\mathbf{r})$ – amplitude coupling coefficients; $\mathbf{N}_{r0,1}^e$ – group

normals; c_c – speed of light; $\Delta K(\mathbf{r})$ – length of spatially non-uniform phase mismatch vector $\Delta \mathbf{K}(\mathbf{r}, E) = \mathbf{k}_0(\mathbf{r}, E) - \mathbf{k}_1(\mathbf{r}, E) + \mathbf{K}(\mathbf{r})$, characterizing the diffraction geometry changing by the impact of external electric field and ChPS period varying. Parameter $\Delta K(\mathbf{r})$ can be determined as in [4].

Refraction indexes $n_{0,1}^e(\mathbf{r}, E)$ for the diffracted light beams are given by:

$$n_i^e(\mathbf{r}, E) = n_o n_e \left[n_e^2 \cdot \sin^2(\gamma(\mathbf{r}, E) \pm \theta_i^e(\mathbf{r})) + n_o^2 \cdot \cos^2(\gamma(\mathbf{r}, E) \pm \theta_i^e(\mathbf{r})) \right]^{1/2}, \quad (11)$$

where $\theta_i^e(\mathbf{r})$ – angles of incidence and diffraction of extra-ordinary waves.

Solution of the diffraction problem we will seek for a uniform amplitude profile $U_m(\mathbf{r})=1$ and quasi-quadratic phase front $\varphi(\mathbf{r})$. We will consider the case when the $\nabla\varphi(\mathbf{r})$ is directed along the grating vector \mathbf{K} (figure 1a), which corresponds to the ChPS. In this case, the phase profile of the ChPS included in (3) we can represent as a Taylor series:

$$\varphi(\mathbf{r}) = \varphi_0 + \nabla\varphi \cdot \mathbf{r} + 0.5\nabla^2\varphi \cdot \mathbf{r}^2, \quad (12)$$

where $\nabla\varphi = K_0$ – average length of vector \mathbf{K} ; $0.5\nabla^2\varphi$ – deviation of \mathbf{K} length, caused by the inhomogeneity of ChPS phase profile.

Substituting (12) to (3), considering a uniform amplitude profile, we will find the solution of CWE system (9)-(10) in analogy with [4] in aperture coordinates ξ_0, ξ_1 , without taking into account the attenuation of light. Then the distribution of the diffracted light field amplitude at the output of the PDL sample can be written as follows:

$$E_1(\eta) = -i \frac{C_1 U_0 L}{2v_1} \int_{-1}^{+1} \left\{ \exp[\delta m(1-y) + \delta^2 n(1-y^2)] \times \right. \\ \left. \times \left[\Phi\left(\frac{d}{a}, 1; a\delta^2 \frac{v_1}{v_0}(1-y)^z\right) E_0\left(\frac{\delta(1-y)}{v_0} - \frac{\eta}{v_1}\right) \right] \right\} dy, \quad (13)$$

where $\Phi(a, c, z)$ – confluent hypergeometric function of the first kind; L – sample thickness; the remaining symbols are shown in [4].

For analysis of observed solution and dependences of diffracted field amplitude on the form of external impact inhomogeneity we will consider the incidence of a plane monochromatic wave of unit amplitude on ChPS. For this, we will introduce the relative phase mismatch $\Delta = f(\delta\theta)$, where $\delta\theta = \theta - \theta_B$ – the deviation of the plane wave components incidence angle from the Bragg angle. For a quantitative estimation of a ChPS characteristic asymmetry compensation компенсации by the different forms of external impact (figure 1b), in dependence on impact value, we will estimate the symmetry of $|E_1(\Delta, E)|^2$ relatively to the line $\Delta = 0$ by the following expression:

$$\delta(E) = \left(\int_{-20}^0 |E_1(\Delta, E)|^2 d\Delta - \int_0^{20} |E_1(\Delta, E)|^2 d\Delta \right) \cdot \left(\int_{-20}^0 |E_1(\Delta, E)|^2 d\Delta \right)^{-1} \cdot 100\%. \quad (14)$$

3. Numerical simulations and discussions

Figure 2a shows the results of numerical simulations of $|E_1(\Delta, E)|^2$ dependencies on the relative phase mismatch for the amplitude-uniform photonic structures with quadratic phase profile relative to $0.5\nabla^2\varphi = 2.5 \cdot 10^5$ for external functions $E(y)$ of different forms at $E = 1.1E_c$. The incident beam lightwave $\lambda = 1.55$ μm . Figure 2b shows the results of numerical simulations of $\delta(E)$ for such ChPS at different functions' form $E(y)$ and different external impact values. Curves numeration on figure 2 corresponds to function types shown on figure 1b.

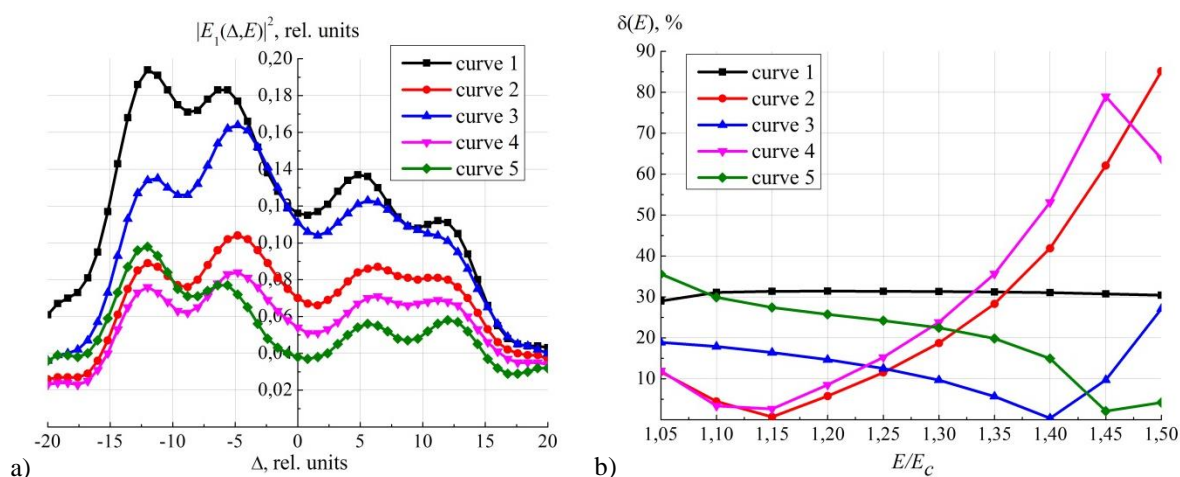


Figure 2. Numerical simulations results: a) – diffraction characteristic of ChPS under the impact of each form of external electric field ($E = 1.1E_c$); b) – the characteristic asymmetry value under the impact of each form of external electric field

Figure 2 analysis shows that uniform impact of different values (curves 1) causes the characteristic asymmetry at about 31% (figure 2b). This asymmetry compensation to the level about 10% provide linearly non-uniform impact (curves 2) and increasing, upward convex function (curves 4). However, this level of compensation is observed only at external field values $E = 1 \div 1.25E_c$. At $E > 1.35E_c$ the degree of inhomogeneity of the ChPS characteristics for these forms of impact increases rapidly. On the other hand, the same level of asymmetry compensation at values $E = 1.3 \div 1.5E_c$ and more provides increasing, downward convex (curves 3) and bell-shaped (curves 5) functions.

4. Conclusion

Thus, in this work we've shown that function of external electric field inhomogeneity $E(y)$ has its optimal form for each area of field values. This effect can be explained by inhomogeneity of ChPS phase profile.

Acknowledgments

The work is performed as a base part of Government Task of Russian Ministry of Education (project № 3642).

References

- [1] Dovolnov E A, Sharangovich S N, Sheridan J T 2005 *Photorefractive Effects, Materials, and Devices 2005 (PR05), OSA Trends in Optics and Photonics Series (TOPS)* **99** 337.
- [2] Dovolnov E A, Sharangovich S N 2006 *Proc. Of SPIE* **6187** 6187L-1.
- [3] Semkin A O, Sharangovich S N 2015 *Proceedings of the IX International Conference of young scientists and professionals "Optics-2015"* St. Petersburg, Russia 245.
- [4] Kushnarev I N, Sharangovich S N 1993 *Technical physics* **63** №2 24
- [5] Semkin A O, Sharangovich S N 2013 *Bulletin of the Russian Academy of Sciences. Physics* **77** №12 1722.
- [6] Semkin A O, Sharangovich S N 2013 *Izvestiya vuzov. Fizika* **56** № 9/2 21.
- [7] Semkin A O, Sharangovich S N 2014 *Doklady TUSURa* **1(31)** 136.
- [8] Afonin O A, Nazvanov V F 1998 *Technical physics letters* **24** 451
- [9] Dovolnov E A, Sharangovich S N 2008 *Izvestiya vuzov. Fizika* **51** № 9/3 171
- [10] Sonin A S *Introduction to the physics of liquid crystals* (Science Press, Moscow) 1983