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Electromagnetic field distribution in a quasi-1D rhombic waveguide array

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Abstract. The exact solution of the coupled mode equations is presented in the case of the continuous electromagnetic wave propagation in the quasi-one-dimensional rhombic array of the waveguides. In general case of the boundary conditions the discrete diffraction is described by these solutions. The flat band mode propagation is found as the special case.

1. Introduction

Recently the optical simulations of the different phenomena of solid state have been developed. There is one interested example. The investigation of two dimensional electron systems demonstrated that presence of the third atom in the elementary cell as well as long range interaction in the lattice leads to emerging of a flat sheet (flat band) between conventional zones. Similar optical lattices can be realized by means of waveguides as nodes of the lattice. Some kinds of the optical lattices that demonstrate the photonic spectrum with flat band have been discussed in [1,2,3]. In [1] the waveguide array consisting from three parallel linear chain of waveguides was considered. The central waveguide chain is shifted according to either chains at half lattice period. Resulting configuration seams as linear chain of the rhombus. This array of waveguides was named as the quasi-one-dimensional rhombic array [1,4,5].

In this paper, we consider the electromagnetic field distribution in the quasi-one-dimensional rhombic array of the waveguides. All waveguides are supposed as linear waveguides. The exact solution of the coupled mode equations is found. These solutions demonstrate both the flat band mode and the discrete diffraction phenomenon.

2. Base equations

System of equations describing coupled waves in this quasi-one-dimensional rhombic array of waveguides has the following form [1,4,8]

$$i\frac{\partial A_n}{\partial \zeta} = (B_n + B_{n-1}) + (C_n + C_{n-1}),$$

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$$i\frac{\partial B_n}{\partial \zeta} = (A_{n+1} + A_n), \qquad i\frac{\partial C_n}{\partial \zeta} = (A_{n+1} + A_n). \tag{1}$$

Here $\zeta = Kz$ is normalized coordinate, K is coupling constant. The fields A_n, B_n and C_n are dimensionless slowly varying amplitudes of the electric fields in *n*-th waveguide. The sub-indices *n* are markers of the elementary cells. Phase matching condition is assumed to be satisfied, and coupling constants between waveguides are equal to unit. Since system of equation (1) is linear, the dispersion relation can be found in a standard way:

$$q_{\pm}(s) = \pm 2\sqrt{2}\,|\cos(\pi s/M)|, \quad q_0(s) = 0. \tag{2}$$

Here $q(s)$ is transversal wave number of the mode with index *s*, where $s = -M,\ldots,-1,0,1,\ldots,M$, $N = 2M+1$ is number of the elementary cells of waveguide array.

3. Solution of the base equations

We can use the generation function method. Let us introduce the following functions

$$P_A(\zeta, y) = \sum_{n=-\infty}^{\infty} A_n(\zeta)e^{iny}, \quad P_B(\zeta, y) = \sum_{n=-\infty}^{\infty} B_n(\zeta)e^{iny}, \quad P_C(\zeta, y) = \sum_{n=-\infty}^{\infty} C_n(\zeta)e^{iny}.$$

From (1) the following system of equations can be obtained

$$i\frac{\partial P_A}{\partial \zeta} = \kappa^*(P_B + P_C), \quad i\frac{\partial P_B}{\partial \zeta} = \kappa P_A, \quad i\frac{\partial P_C}{\partial \zeta} = \kappa P_A, \tag{3}$$

where

$$\kappa = \kappa(y) = (1 + e^{-iy}) = 2\cos(y/2)\exp(-iy/2).$$

With taking into account the boundary conditions at $\zeta = 0$

$$P_{A0} = \sum_{n=-\infty}^{\infty} A_n(0)e^{iny}, \quad P_{B0} = \sum_{n=-\infty}^{\infty} B_n(0)e^{iny}, \quad P_{C0} = \sum_{n=-\infty}^{\infty} C_n(0)e^{iny},$$

the solution of (3) can be written as

$$P_A(\zeta, y) = P_{A0}\cos\Omega\zeta - i\beta(P_{B0} + P_{C0})\sin\Omega\zeta, \tag{4}$$

$$P_B(\zeta, y) = \frac{1}{2}\left[P_{B0} - P_{C0} + (P_{B0} + P_{C0})\cos\Omega\zeta - \frac{i}{\beta}P_{A0}\sin\Omega\zeta\right], \tag{5}$$

$$P_C(\zeta, y) = \frac{1}{2}\left[P_{C0} - P_{B0} + (P_{B0} + P_{C0})\cos\Omega\zeta - \frac{i}{\beta}P_{A0}\sin\Omega\zeta\right], \tag{6}$$

where

$$\Omega^2 = 8\cos^2(y/2), \quad \beta = \frac{\kappa^*}{\Omega} = \frac{1}{\sqrt{2}}\exp(-iy/2).$$

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By using the orthogonality condition

$$\int_{-\pi}^{\pi} \exp[iy(n-m)]dy = 2\pi\delta_{nm}$$

the amplitudes $A_n(\zeta)$, $B_n(\zeta)$ and $C_n(\zeta)$ can be determined from 4)-(6):

$$2\pi A_n(\zeta) = \int_{-\pi}^{\pi} P_A(\zeta, y)\exp(-iyn)dy\,, \quad 2\pi B_n(\zeta) = \int_{-\pi}^{\pi} P_B(\zeta, y)\exp(-iyn)dy\,,$$

$$2\pi A_n(\zeta) = \int_{-\pi}^{\pi} P_A(\zeta, y)\exp(-iyn)dy\,.$$

4. Particular solution of the base equations

Let us consider some particular solutions of the base equations (1). The particular solutions are defined by the different boundary conditions. Supposing the conditions, which are correlated to situation where the radiation is initially input only to waveguides of one elementary cell in array.

$$A_n(0) = A_0\delta_{n0}\,, \quad B_n(0) = B_0\delta_{n0}\,, \quad C_n(0) = C_0\delta_{n0}\,.$$

Thus, $P_{A0} = A_0$, $P_{B0} = B_0$, $P_{C0} = C_0$. With taking into account (4)-(6) the express the amplitudes $A_n(\zeta)$, $B_n(\zeta)$ and $C_n(\zeta)$ can be written as

$$A_n(\zeta) = A_0(-1)^n J_{2n}(\eta) - \frac{iR_0}{\sqrt{2}}(-1)^n J_{2n+1}(\eta)\,, \tag{7}$$

$$B_n(\zeta) = \frac{1}{2}S_0\delta_{n0} + \frac{1}{2}R_0(-1)^n J_{2n}(\eta) - \frac{iA_0}{\sqrt{2}}(-1)^n J_{2n+1}(\eta)\,, \tag{8}$$

$$C_n(\zeta) = -\frac{1}{2}S_0\delta_{n0} + \frac{1}{2}R_0(-1)^n J_{2n}(\eta) - \frac{iA_0}{\sqrt{2}}(-1)^n J_{2n+1}(\eta)\,, \tag{9}$$

where the constants $S_0 = B_0 - C_0$ and $R_0 = B_0 + C_0$ are introduced, and $\eta = 2\sqrt{2}\zeta$. The details of the calculations are presented in [8]. Theses expressions describe the electromagnetic radiation spreading along array.

In the case of boundary conditions

$$A_n(0) = 0\,, \quad B_n(0) = -C_n(0) = B_0\delta_{n0}\,,$$

we have $R_0 = 0$, but $S_0 = 2B_0$. The distribution of the electromagnetic fields in waveguide array is

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$$A_n(\zeta) = 0\,, \quad B_n(\zeta) = B_0\delta_{n0}\,, \quad C_n(\zeta) = -B_0\delta_{n0} \tag{10}$$

In this case the discrete diffraction is absent. It corresponds for the excitation of the flat-band modes [4,5]. However, if the radiation will input into one of the waveguide of the central part of array, i.e., to use the boundary condition

$$A_n(0) = A_0\delta_{n0}\,, \quad B_n(0) = -C_n(0) = B_0\delta_{n0}\,,$$

then the distribution of the amplitudes $A_n(\zeta)$, $B_n(\zeta)$ and $C_n(\zeta)$ takes the following form

$$A_n(\zeta) = A_0(-1)^n J_{2n}(\eta)\,, \tag{11}$$

$$B_n(\zeta) = B_0\delta_{n0} - \frac{iA_0}{\sqrt{2}}(-1)^n J_{2n+1}(\eta)\,, \tag{12}$$

$$C_n(\zeta) = -B_0\delta_{n0} - \frac{iA_0}{\sqrt{2}}(-1)^n J_{2n+1}(\eta)\,, \tag{13}$$

The discrete diffraction takes place in this case of the boundary conditions.

5. Conclusion

The propagation of the electromagnetic continues wave in the quasi-one-dimensional rhombic array of the waveguides is investigated. The exact solution of the coupled mode equations (1) is found. In general case of the boundary conditions the discrete diffraction is described by these solutions. However, the without diffraction regime of the wave propagation at the particular boundary condition exists.

The obtained exact solution could be correct in the case of slowly varying envelopes of the electromagnetic solitary waves, if the group velocities for each waveguide in array are equal.

If the rhombic waveguide array is composed of the unit cells containing three waveguides: two positive refraction index waveguides and one negative refraction index-waveguide, then the electromagnetic wave spectrum contains two gap modes and one gapless mode. However, the gapless mode approximately corresponds to flat one, thus we can say about quasi-flat band.

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References

[1] Longhi S 2014 *Opt. Lett.* **39** 5892
[2] Maimistov A 2015 *J.Phys. Conference Series* **613** 012011
[3] Maimistov A I and Gabitov I R 2016 *J.Phys. Conference Series* **714** 012013
[4] Mukherjee S and Thomson R R 2015 *Opt. Lett.* **40** 5443
[5] Mukherjee S, Spracklen A, Choudhury D. Goldman N, Ohberg P, Andersson E and Thomson R.R. 2015 Phys.Rev.Lett. **114** 245504
[6] Vicencio R A, Cantillano C, Morales-Inostroza L, Real B, Mejia-Cortes C, Weimann S, Szameit A and Molina M I 2015 *Phys.Rev.Lett* **114** 245503
[7] Zong Y, Xia Sh, Tang L, Song D, Hu Yi, Pei V, Su J, Li Y and Chen Zh .2016 *Opt.Express* **24** 8877
[8] Maimistov A I, Patrikeev V A 2016 *Preprint* arXiv:1606.03697 [physics.optics]