

# Cnoidal waves, solitons and vortices in the flow of polaritons

**I V Dzedolik and V Pereskokov**

V. I. Vernadsky Crimean Federal University,  
Physics and Technology Institute, Simferopol, Russian Federation

E-mail: dzedolik@crimea.edu

**Abstract.** We theoretically have investigated the properties of the phonon-polariton waves propagating in Kerr-type dielectric medium, and examined the conditions of appearance of the spatial solitons, cnoidal waves and polariton vortices. It is shown that the polariton flow can propagate in the form of spatial soliton, or can be split into several flows in the form of filaments depending on the field density and parameters of the nonlinear medium. In the polariton flow the vortices may appear at the presence of local inhomogeneities of medium refractive index causing the wavefront rotation.

## 1. Introduction

The properties of electromagnetic waves in a medium are mainly described in the literature by regarding the local and non-local linear and nonlinear response of the medium, or by the analysis of one-photon and multiphoton processes [1 - 6]. However, the electromagnetic wave while propagating in the medium always generates the transverse and longitudinal phonons. Thus, the hybridization of the electromagnetic wave and wave of optical phonons takes place, and the quasiparticles (named polaritons) are generated as a result of coupling of the photons and phonons according to the quantum approximation [7 - 20]. The hybridization of the electromagnetic wave with the wave of dipole excitations of the medium (phonons, magnons, plasmons, excitons) leads to the generation of the corresponding type of polaritons. According to the classical approximation, the hybrid phonon-polariton wave propagates in the medium in the form of a polariton flow. The polariton spectrum has a gap, and an electromagnetic wave with the frequency falling within this spectral gap, does not propagate in the dielectric medium. The nonlinear polariton wave having initially the flat wavefront due to transverse instability is transformed into a spatial soliton or cnoidal (non-linear periodic) wave [13 - 20].

The properties and dynamics of nonlinear waves in dielectric media attract attention of researchers in connection with the problems of the propagation and control of power laser beams in a medium, of the spectral analysis at crystalline and amorphous media, of the design and development of new optical devices and logic gates for optical communications, etc. [16, 19, 20]. The properties of vector solitons and cnoidal polariton waves, nonlinear vortex solitons, nonlinear wave splitting into separate flows named filaments are actively investigated now both theoretically and experimentally [6, 16, 17, 19 - 30].

In our paper we theoretically investigate the properties of the phonon-polariton waves propagating in the Kerr-type dielectric medium on the basis of the classical approximation. We have examined the



conditions of occurrence of the spatial solitons, cnoidal waves and polariton vortices, and have shown that the polariton flow can be propagated in the form of spatial solitons, or it can split into several flows in the form of filaments depending on the field density and the nonlinear medium parameters. We have shown that a lattice of nonlinear vortices can appear in the presence of local inhomogeneities of medium refractive index that causes the rotation of the wave front of the polariton wave.

## 2. Vector equation for the phonon-polaritons

The polaritons are generated in the medium as the bound states of the transverse electromagnetic fields and optical phonons (high transverse oscillations of the ions and the electron shells), and they are propagated in the medium in the form of hybrid photon-phonon waves [10]. We can describe [16 - 20] this process using the system of equations consisting of the equation of motion of the ions in a unit cell of the crystal lattice, and the equation of motion of the outer electron shell of the ion,

$$\left\{ \frac{m_{eff}}{m} \right\} \frac{d^2}{dt^2} \left\{ \frac{\mathbf{R}}{\mathbf{r}} \right\} + \left\{ \frac{\Gamma_I}{\Gamma_e} \right\} \left\{ \frac{m_{eff}}{m} \right\} \frac{d}{dt} \left\{ \frac{\mathbf{R}}{\mathbf{r}} \right\} + \nabla_R \left\{ \frac{U_R}{U_r} \right\} = \left\{ \frac{e_{eff}}{-e} \right\} \left( \mathbf{E} + \frac{1}{c} \frac{d}{dt} \left\{ \frac{\mathbf{R}}{\mathbf{r}} \right\} \times \mathbf{B} \right), \quad (1)$$

and the electromagnetic field equations

$$\nabla \times \mathbf{B} = \frac{1}{c} \frac{\partial}{\partial t} (\mathbf{E} + 4\pi \mathbf{P}), \quad \nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial}{\partial t} \mathbf{B}, \quad (2)$$

where  $e_{eff}, m_{eff}$  are the effective charge and mass of the ions,  $\mathbf{R} = \mathbf{r}_+ - \mathbf{r}_-$  is the displacement vector of positive and negative ions,  $U_R = (Q_{1R}/2)R^2 + (Q_{2R}/3)R^3 + (Q_{3R}/4)R^4$  is the potential energy of the ion,  $U_r = (Q_{1r}/2)r^2 + (Q_{2r}/3)r^3 + (Q_{3r}/4)r^4$  is the potential energy of the electron,  $\Gamma_{I,e}$  are the attenuation coefficients,  $\mathbf{P} = e_{eff}N_C\mathbf{R} - eN_e\mathbf{r}$  is the polarization vector of the medium,  $N_C$  is the number of cells per unit volume,  $N_e$  is the number of electrons per unit volume,  $Q_{jR}$  and  $Q_{jr}$  are the phenomenological elastic lattice parameters. We take into account the bonding of charges through the electromagnetic field in the system of equations (1) - (2).

Then we can neglect the charges response on the magnetic component of electromagnetic field in the medium in the equations of motion of the ions and electrons  $|\mathbf{E}| \gg |c^{-1}(d\mathbf{R}/dt) \times \mathbf{B}|$ ,  $|\mathbf{E}| \gg |c^{-1}(d\mathbf{r}/dt) \times \mathbf{B}|$  at an optical frequency. If the incident electromagnetic wave is a harmonic one  $E \sim \exp(-i\omega t)$ , then the polarization vector of medium has the form

$$\begin{aligned} \mathbf{P} = & \chi_1 \mathbf{E}_a \exp(-i\omega t) + \chi_{20} E_a \mathbf{E}_a + \chi_{22} E_a \mathbf{E}_a \exp(-i2\omega t) \\ & + \chi_3 E_a^2 \mathbf{E}_a \exp(-i\omega t) + \chi_{33} E_a^2 \mathbf{E}_a \exp(-i3\omega t) + \dots, \end{aligned} \quad (3)$$

where  $\chi_\ell$  are the linear and nonlinear susceptibilities of the medium. We neglect the generation of harmonics in the medium considering their nonsynchronous with the fundamental (first) harmonic, i. e. we neglect the energy exchange between the harmonics. Then the polarization vector of the medium (3) can be written as

$$\mathbf{P} = \chi_1 \mathbf{E}_a \exp(-i\omega t) + \chi_3 E_a^2 \mathbf{E}_a \exp(-i\omega t), \quad (4)$$

where the susceptibilities of the medium  $\chi_1 = \frac{1}{4\pi} \left( \frac{\omega_e^2}{\tilde{\omega}_1^2} + \frac{\omega_I^2}{\tilde{\Omega}_1^2} \right)$  and

$$\chi_3 = -\frac{1}{4\pi} \left( \frac{e^2 \alpha_{3r} \omega_e^2}{m^2 (\tilde{\omega}_1^2)^3 (\tilde{\omega}_1^2)^*} + \frac{e_{eff}^2 \alpha_{3R} \omega_I^2}{m_{eff}^2 (\tilde{\Omega}_1^2)^3 (\tilde{\Omega}_1^2)^*} \right)$$

are obtained by solving the equations of motion of electrons and ions. Here  $\tilde{\omega}_1^2 = \omega_0^2 - \omega^2 - i\Gamma_e \omega$ ,  $\tilde{\Omega}_1^2 = \Omega_\perp^2 - \omega^2 - i\Gamma_I \omega$ ,  $\omega_e^2 = 4\pi e^2 N_e m^{-1}$ ,  $\omega_I^2 = 4\pi e_{eff}^2 N_C m_{eff}^{-1}$ ,  $\omega_0^2 = Q_{1r} m^{-1}$ ,  $\Omega_\perp^2 = Q_{1R} m_{eff}^{-1}$ ,  $\omega_e$  and  $\omega_I$  are the electron and ion plasma frequencies,  $\omega_0$  is the electron resonance frequency,  $\Omega_\perp$  is the resonance frequency of the lattice,  $\alpha_{3r} = Q_{3r} m^{-1}$ ,  $\alpha_{3R} = Q_{3R} m_{eff}^{-1}$  are the phenomenological parameters that depend on the nonlinear properties of the medium,  $\Gamma_e$  and  $\Gamma_I$  are the attenuation coefficients of electron and ion subsystems. The third-order susceptibility of the medium  $\chi_{31} > 0$  and  $\chi_{33} > 0$  are greater than zero, because the phenomenological parameters  $\alpha_{3R} = -Q_{3R} m_{eff}^{-1} < 0$  и  $\alpha_{3r} = -Q_{3r} m^{-1} < 0$  are negative. The phenomenological parameters are expressed in the terms of measurable quantities such as resonance frequency, etc.

The dynamics of polaritons are described in reviewed case by the vector equation obtained from equations of motion of ions, electrons and electromagnetic fields. We represent the equation for the electric vector of the polariton waves in the form of

$$-\nabla^2 \mathbf{E} + \nabla(\nabla \mathbf{E}) + \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \mathbf{E} = -\frac{4\pi}{c^2} \frac{\partial^2}{\partial t^2} \mathbf{P}, \quad (5)$$

where  $\mathbf{P}$  is the polarization vector (4) of the medium. As it follows from the Maxwell equation  $\nabla(\varepsilon \mathbf{E}) = 0$ , the gradient of divergence of the electric field is non-zero in the Eq. (5)  $\nabla(\nabla \mathbf{E}) \neq 0$ , because  $\nabla \mathbf{E} = -\varepsilon^{-1}(\mathbf{E} \nabla \varepsilon) \neq 0$ . Generally, we cannot neglect the mixed derivatives in Eq. (5), because the permittivity of the medium depends on the coordinates in the presence of inhomogeneities.

### 3. Equations for the envelopes of polariton waves

Suppose that the electric field is polarized in the transverse plane  $\mathbf{E} = \mathbf{1}_x E_x(x, y, z) + \mathbf{1}_y E_y(x, y, z)$ , then from Eq. (5) we obtain the system of equations

$$\begin{aligned} \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} - \frac{\partial^2 E_y}{\partial x \partial y} + \frac{\omega^2}{c^2} (1 + 4\pi\chi_1) E_x + \frac{4\pi\omega^2 \chi_3}{c^2} (|E_x|^2 + |E_y|^2) E_x &= 0, \\ \frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial z^2} - \frac{\partial^2 E_x}{\partial x \partial y} + \frac{\omega^2}{c^2} (1 + 4\pi\chi_1) E_y + \frac{4\pi\omega^2 \chi_3}{c^2} (|E_x|^2 + |E_y|^2) E_y &= 0. \end{aligned} \quad (6)$$

In this case, the fundamental harmonic of polariton wave can be represented as  $E_{x,y} = \tilde{E}_{x,y}(x, y, z) \exp(ikz)$ , where  $\tilde{E}_{x,y}(x, y, z)$  are the slowly varying amplitudes of the transverse components of the electric field,  $k$  is the wave vector. In case when the amplitude of the electric vector of the polariton wave varies slowly along the axis  $z$ , it is possible to determine its dependence on the longitudinal coordinate as  $\tilde{E}_j = e_j(x, y) \exp(iqz)$  by entering a constant phase shift  $q$ ,  $j = x, y$ . Then from Eq. (6) we obtain the system of equations for the envelopes of the electric field  $e_j(x, y)$  of the polariton wave, depending only on the transverse coordinates

$$\begin{aligned}\frac{\partial^2 e_x}{\partial y^2} - \frac{\partial^2 e_y}{\partial x \partial y} + \alpha_1 e_x + \alpha_3 (e_x^2 + e_y^2) e_x &= 0, \\ \frac{\partial^2 e_y}{\partial x^2} - \frac{\partial^2 e_x}{\partial x \partial y} + \alpha_1 e_y + \alpha_3 (e_x^2 + e_y^2) e_y &= 0,\end{aligned}\quad (7)$$

where  $\alpha_1 = c^{-2} \omega^2 (1 + 4\pi\chi_1) - k^2 - 2kq$ ,  $\alpha_3 = 4\pi c^{-2} \omega^2 \chi_3$ ;  $\alpha_3 > 0$  at  $\chi_3 > 0$  in self-focusing medium, and  $\alpha_3 < 0$  at  $\chi_3 < 0$  in self-defocusing medium.

#### 4. Circularly polarized polariton waves

The system of equations (7) represented in the Cartesian coordinates can be transformed into a system of equations for the polariton waves of circular polarization in the rotating coordinate system [17, 19, 20]. We introduce the vector  $\mathbf{e}_+ = \mathbf{e}_x + i\mathbf{e}_y = e(\mathbf{1}_x + i\mathbf{1}_y)/\sqrt{2} = e\mathbf{1}_+$  for the polaritons with the right-handed helicity at the left-handed circular polarization wave, and the vector  $\mathbf{e}_- = \mathbf{e}_x - i\mathbf{e}_y = e(\mathbf{1}_x - i\mathbf{1}_y)/\sqrt{2} = e\mathbf{1}_-$  for polaritons with the left-handed helicity at the right-handed circular polarization wave, and rotating two-dimensional coordinates  $\xi = x + iy$  and  $\eta = x - iy$ . Then from Eqs. (7) we obtain the system of complex equations for the envelopes of polariton waves with the right  $\mathbf{e}_+$  and left  $\mathbf{e}_-$  helicity in the rotating coordinates  $(\xi, \eta)$ ,

$$\begin{aligned}\frac{\partial^2 \mathbf{e}_+}{\partial \xi^2} + \frac{\partial^2 \mathbf{e}_+}{\partial \eta^2} + i \frac{\partial^2 \mathbf{e}_-}{\partial \xi^2} - i \frac{\partial^2 \mathbf{e}_-}{\partial \eta^2} - 2 \frac{\partial^2 \mathbf{e}_-}{\partial \xi \partial \eta} - (\alpha_1 + \alpha_3 e^2) \mathbf{e}_- &= 0, \\ \frac{\partial^2 \mathbf{e}_-}{\partial \xi^2} + \frac{\partial^2 \mathbf{e}_-}{\partial \eta^2} + i \frac{\partial^2 \mathbf{e}_+}{\partial \xi^2} - i \frac{\partial^2 \mathbf{e}_+}{\partial \eta^2} - 2 \frac{\partial^2 \mathbf{e}_+}{\partial \xi \partial \eta} - (\alpha_1 + \alpha_3 e^2) \mathbf{e}_+ &= 0.\end{aligned}\quad (8)$$

Equating to zero the determinant  $\text{Det}(\mathbf{e}_+, \mathbf{e}_-)$  of the system of vector equations (8), we obtain the equation for the envelope of the polariton wave  $e(\xi, \eta)$ ,

$$(1-i) \frac{\partial^2 e}{\partial \xi^2} + (1+i) \frac{\partial^2 e}{\partial \eta^2} + 2 \frac{\partial^2 e}{\partial \xi \partial \eta} + (\alpha_1 + \alpha_3 e^2) e = 0. \quad (9)$$

The Eq. (9) in the rotating coordinates  $(\xi, \eta)$  describes the polariton flow both with the right and left polariton helicity. Obtaining the solution of Eq. (9) we can return to the Cartesian coordinates for describing the polariton wave with linear polarization.

We can obtain from Eq. (9) an equation for the envelope of “scalar” polariton flow with the right  $e_+ = e(\xi)$  helicity, or with the left  $e_- = e(\eta)$  helicity

$$\frac{d^2 e_{\pm}}{d\varsigma^2} + \alpha_1^{(\pm)} e_{\pm} + \alpha_3^{(\pm)} e_{\pm}^3 = 0, \quad (10)$$

where  $\alpha_1^{(\pm)} = (1 \pm i)\alpha_1/2$ ,  $\alpha_3^{(\pm)} = (1 \pm i)\alpha_3/2$ ,  $\varsigma = \begin{cases} \xi \\ \eta \end{cases}$ .

#### 5. Solitons and cnoidal waves in the flow of polaritons

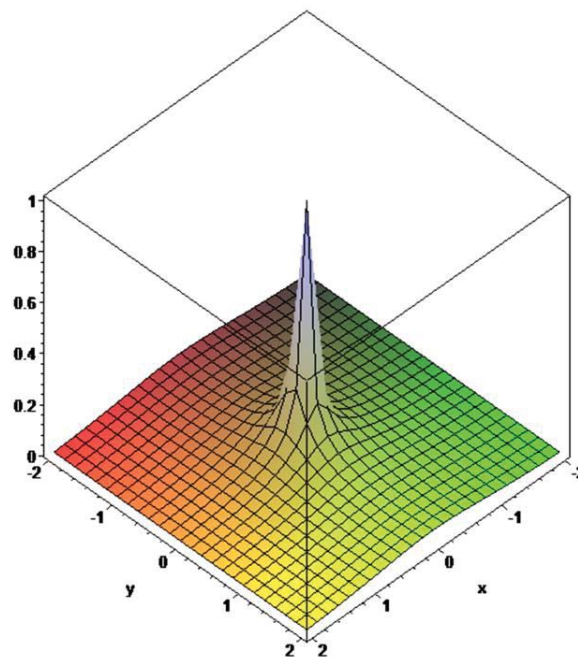
It is well known that a harmonic plane wave is unstable in a nonlinear medium [6]. Instability of the plane wave depends on the parameters of field and medium, and it leads to the transverse and / or longitudinal

modulation and further transformation of the plane wave to the spatial soliton or cnoidal wave [16, 17, 19 - 25, 27 - 30].

Consider the formation of spatial soliton or transverse cnoidal wave in nonlinear infinite dielectric medium based on the polariton concept. The polariton flow takes the form of the spatial soliton due to the plane wave modulation in the transverse plane when the boundary conditions have the form  $e \rightarrow 0$  and  $de/d\zeta = 0$  at  $|\xi| \rightarrow \infty$ , or  $|\eta| \rightarrow \infty$  [6]. The boundary conditions for the soliton center  $\zeta = 0$  at the input of medium  $e(0) = \text{const}$  and  $de(0)/d\zeta = 0$  enable us to determine the phase shift  $q$ . We can get the soliton solutions of Eq. (10) for polaritons with the right and with the left helicity [17, 19, 20] as

$$e_{\pm} = \left| \sqrt{\frac{2\bar{\alpha}_1^{(\pm)}}{\alpha_3^{(\pm)}}} \right| \text{sch} \left[ \sqrt{|\bar{\alpha}_1^{(\pm)}|} \zeta - \text{sch}^{-1} \left| e_{\pm}(0) \sqrt{\frac{\alpha_3^{(\pm)}}{2\bar{\alpha}_1^{(\pm)}}} \right| \right] \exp(iq_{\pm}z) \quad (11)$$

in the self-focusing medium at  $\alpha_1 = -\bar{\alpha}_1$ ,  $\alpha_3 > 0$ , or in the self-defocusing medium at  $\alpha_1 = -\bar{\alpha}_1$ ,  $\alpha_3 < 0$ , where  $q_{\pm} = [\alpha_3 e_{\pm}^2(0)/2 + c^{-2}\omega^2(1 + 4\pi\chi_1) - k^2]/2k$  is the phase shift. The distribution of polaritons  $A_{\pm} = (Re^2 e_{\pm} + Im^2 e_{\pm})^{1/2}$  at the cross-section of the flow in the form of spatial soliton is shown in figure 1; here the flow propagates along the axis  $z$ .



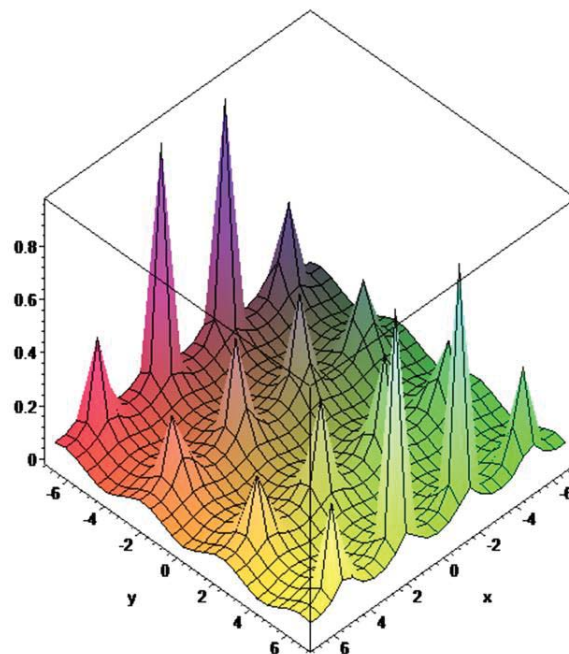
**Figure 1.** Polariton flow in the form of spatial soliton.

In addition to the spatial soliton formation, the cnoidal (non-linear periodic) wave can be formed in nonlinear medium at wavelength modulation in the transverse plane. Cnoidal polariton wave occurs when the boundary conditions are  $e = \text{const}$  and  $de/d\zeta = 0$  at  $|\xi| \rightarrow \infty$ , or  $|\eta| \rightarrow \infty$ , i. e. the cnoidal wave does not vanish at infinity. Thus, for generation of the cnoidal waves the medium should be more transparent in comparison with the medium for the soliton generation.

The envelopes of cnoidal polariton waves have the form of elliptic cosine

$$e_{\pm} = \tilde{e}_{0\pm} \operatorname{cn} \left[ \left( \alpha_{\pm}'^2 / 4 + C_{\pm}' \right)^{1/4} \sqrt{\alpha_3} \zeta - K(\tilde{k}_{\pm}), \tilde{k}_{\pm} \right] \exp(iq_{\pm}z), \quad (12)$$

where  $\tilde{e}_{0\pm} = \left[ \alpha_{\pm}' / 2 + \left( \alpha_{\pm}'^2 / 4 + C_{\pm}' \right)^{1/2} \right]^{1/2}$ ,  $\alpha_{\pm}' = 2\bar{\alpha}_1^{(\pm)} / |\alpha_3^{(\pm)}|$ ,  $C_{\pm}' = 2C_{\pm} / \alpha_3^{(\pm)}$ ,  $C_{\pm} = \alpha_3 e_{\pm}^4(\infty) / 2 - \bar{\alpha}_1^{(\pm)} e_{\pm}^2(\infty)$ ,  $q_{\pm} = \left[ \alpha_3^{(\pm)} e_{\pm}^2(0) / 2 + c^{-2} \omega^2 (1 + 4\pi\chi_1) - k^2 - C_{\pm} e^{-2}(0) \right] / 2k$ ;  $K(\tilde{k}_{\pm})$  is the complete elliptic integral;  $\tilde{k}_{\pm} = \left[ 2 + \alpha_{\pm}' \left( \alpha_{\pm}'^2 / 4 + C_{\pm}' \right)^{-1/2} \right]^{1/2} / 2$  is the elliptic integral module. The polariton flow is split into several flows in the transverse plane, and they are propagated along the axis  $z$  (figure 2), i. e. the filamentation of the polariton flow takes place [17, 19, 20]. The cnoidal wave which is described by the Eq. (12) can be transformed into the spatial soliton  $\operatorname{cn}(\zeta, 1) \rightarrow \operatorname{sch}(\zeta)$  at  $\tilde{k}_j \rightarrow 1$ , when the wave attenuation takes place  $e_{\pm}(\infty) \rightarrow 0$ , i. e.  $C_{\pm} \rightarrow 0$ .



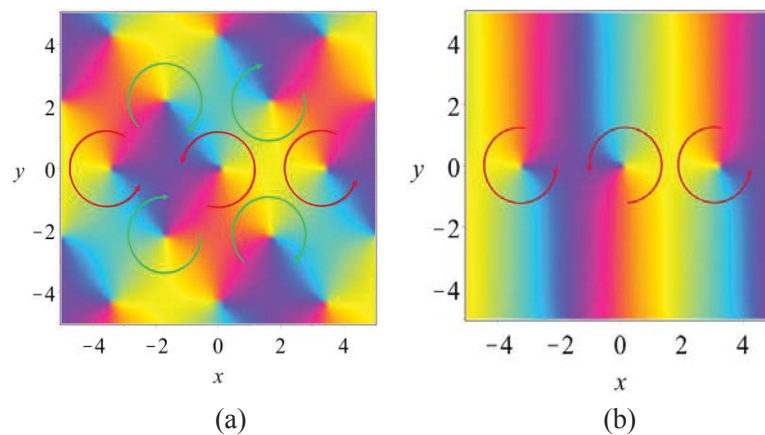
**Figure 2.** The filamentation of polariton flow at  $\tilde{k}_{\pm} = 0.5$ .

## 6. Polariton vortices

The optical vortices arise in the areas of the medium with inhomogeneities where the wavefront acquires the rotation around the singular points [6, 16, 17, 19 - 21, 23 - 26, 28]. Such areas are, for example, the dislocations of crystal lattice, the local inhomogeneities of refractive index of the medium, etc. In this case, the singular points with screw phase dislocations appear on the surface of the wavefront, and we call them the optical vortices.

The wave amplitude vanishes in the singular point  $e = 0$ , and the phase  $\phi$  is not defined. The optical vortex appears around the singular point. In the linear medium the vortex is described as

$\left[ (Ree)^2 + i(Ime)^2 \right]^{|\ell|/2} \exp(i\ell\phi)$ , where  $\phi = \arctan(Ime/Ree)$ ,  $\ell = \pm 1, \pm 2, \dots$  is the topological charge of the vortex. The phase of the wave is changed to  $\oint d\mathbf{r} \nabla \phi = 2\pi\ell$  while the path tracing around the singular point in which  $Ree = 0$  and  $Ime = 0$ . The phase distribution of the cnoidal polariton wave (12) is described by the function  $\phi(x, y) = \arctan[Ime(x, y)/Ree(x, y)]$ , that allows us to visualize the location of the vortices on the wavefront in cross-section of the polariton flow  $z = const$  in the nonlinear medium (figure 3).



**Figure 3.** Vortices in the cross-section of polariton flow at its filamentation in the nonlinear medium:  
(a)  $\tilde{k}_{\pm} = 0.5$ , (b)  $\tilde{k}_{\pm} = 0.01$ ; the arrows show the topological charge of vortex, clockwise  $\ell = -1$ , counterclockwise  $\ell = +1$ .

The phase of polariton wave is described by the ratio of the elliptic functions, whose values depend on the modulus  $\tilde{k}_{\pm}$  of the elliptic integral. In its turn the modulus  $\tilde{k}_{\pm}$  depends on the boundary conditions that determine the possibility of generation of the vortices in nonlinear polariton wave. The boundary conditions in inhomogeneous medium can also be formulated as  $e = 0$  and  $de/d\zeta = const$  at  $|\xi| \rightarrow \infty$ , or  $|\eta| \rightarrow \infty$ , for describing of the polariton vortex appearance.

## 7. Conclusion

Electromagnetic wave hybridizes with an optical phonon wave in the dielectric medium that leads to generation of the phonon-polaritons. The plane polariton wave is a nonstable wave in the nonlinear medium, and its transverse envelope can be transformed into the spatial soliton or cnoidal wave depending on the cross-modulation and the medium transparency. In the first case, the polaritons are propagated as a single flow. In the second case, the polariton wave splits into several flows and the filaments appear in the cross-section of the flow. The boundary conditions determine the value of modulus of the elliptic function describing the soliton or cnoidal polariton waves. The cnoidal wave can be transformed into the spatial soliton as a result of dissipation in the medium.

In the presence of local inhomogeneities of the refractive index in dielectric medium causing the wavefront rotation, the singular points arise on the wavefront of polariton wave, where the vortices are formed. The vortex location in the cross-section of polariton flow depends on the configuration of inhomogeneities in the medium.

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