

Recent results on lattice QCD thermodynamics

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Abstract. I review recent results on QCD thermodynamics from lattice simulations. In particular, I will focus on the QCD equation of state at zero and finite chemical potential, the curvature of the phase diagram and fluctuations of conserved charges. The latter are compared to experimental data, to the purpose of extracting the chemical freeze-out temperature and chemical potential from first principles.

1. Introduction

The deconfined phase of Quantum Chromodynamics (QCD), the Quark-Gluon Plasma (QGP) can be created in the laboratory in heavy ion collision experiments currently running at RHIC (Brookhaven National Laboratory) and at the LHC (CERN). These experiments allow to explore the QCD phase diagram and to extract the properties of this new phase of matter. The QGP turns out to be the most ideal fluid ever observed, which led to the idea that the system created in the collisions is strongly interacting and cannot be studied by means of perturbative techniques.

Simulations on a discretized grid in space-time (lattice) represent the most reliable tool to solve the theory of strong interactions in its non-perturbative regime. The precision achieved by recent lattice QCD simulations allows a quantitative comparison between theory and experiment for the first time in our field. This is due to a steady improvement in both numerical algorithm and computational resources, as well as in our physical understanding which manifests itself in physical techniques (e.g. the Wilson-flow scale setting introduced in Ref. [1]).

The low temperature phase of strongly interacting matter can be understood in terms of a non-interacting gas of hadrons and resonances: the Hadron Resonance Gas (HRG) model provides a very good description of most thermodynamic observables. In the infinite temperature limit, matter behaves again as a non-interacting gas, but in this case the degrees of freedom are quarks and gluons. Reducing the temperature, it is possible to treat the theory as weakly coupled and to calculate thermodynamic quantities perturbatively. Resummation techniques improve the convergence of the perturbative series and bring the agreement with lattice QCD results down to $T \simeq 2.5T_c$. The temperature range between these two limiting situations corresponds to the non-perturbative regime of strongly interacting matter: the coupling constant becomes large and it is not possible to treat the system perturbatively. This is the region which is accessible experimentally, and in which QCD can only be solved numerically by means of lattice simulations.



2. QCD equation of state

The QCD equation of state at zero chemical potential is now known with high accuracy: continuum extrapolated results are available for the pressure, energy density, entropy density, interaction measure and speed of sound of a system of 2+1 quark flavors with physical quark masses [2, 3, 4]. A selection of these results is shown in Fig. 1, in which curves from the WB collaboration obtained with the 2stout action (gray) are compared to those from the HotQCD collaboration obtained with the HISQ action (colored). The agreement between the two sets of curves is a fundamental test of the validity of the lattice approach to solve QCD.

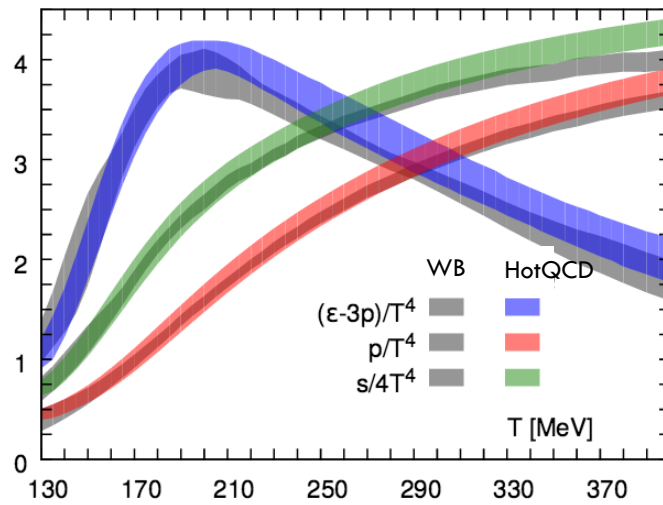


Figure 1. Scaled pressure, entropy density and interaction measure as functions of the temperature. The gray curves have been obtained by the WB collaboration with the 2stout action [2, 3], the colored ones by the HotQCD collaboration with the HISQ action [4].

Simulations on the lattice at finite density are not possible at the moment, due to the sign problem. Nevertheless, it is possible to explore a low-density region of the phase diagram by means of alternative methods. Here I will focus on the Taylor expansion of thermodynamic observables around $\mu_B = 0$ [5, 6] (which can be considered as a truncated version of the multiparameter reweighting technique [7]) and on the analytical continuation from imaginary chemical potentials [8, 9, 10].

Thermodynamic quantities can be expanded in Taylor series around $\mu_B = 0$; for example for the pressure such an expansion reads:

$$\frac{p(\mu_B)}{T^4} = c_0(T) + c_2(T) \left(\frac{\mu_B}{T}\right)^2 + c_4(T) \left(\frac{\mu_B}{T}\right)^4 + c_6(T) \left(\frac{\mu_B}{T}\right)^6 + \mathcal{O}(\mu_B^8). \quad (1)$$

The coefficients $c_i(T)$ in the above expansion can be calculated on the lattice. Several results exist in the literature: $c_2 \dots c_6$ have been calculated long ago on coarse lattices and with heavier than physical quark masses [11]; continuum extrapolated results for c_2 at the physical quark mass were published for the first time in Ref. [12]; results for c_4 at finite lattice spacing were shown in Ref. [13]. In Fig. 2 I show preliminary results for c_2 , c_4 and c_6 in the continuum limit and for physical quark masses. The systematic error is not included in the plot. The results have been obtained with the method of analytical continuation from imaginary chemical potential [14, 15]. It is important to notice that the chemical potentials μ_B , μ_S and μ_Q are not

independent of each other: μ_S and μ_Q are both functions of T and μ_B , such that the following experimental conditions are satisfied:

$$\rho_S = 0, \quad \rho_Q = 0.4\rho_B, \quad (2)$$

where ρ_S , ρ_Q and ρ_B are the densities of strangeness, charge and baryonic number, respectively. The coefficients $c_2 \dots c_6$ in Eq. (1) are full derivatives with respect to μ_B/T , namely they take into account the dependence of μ_S and μ_Q on μ_B and T .

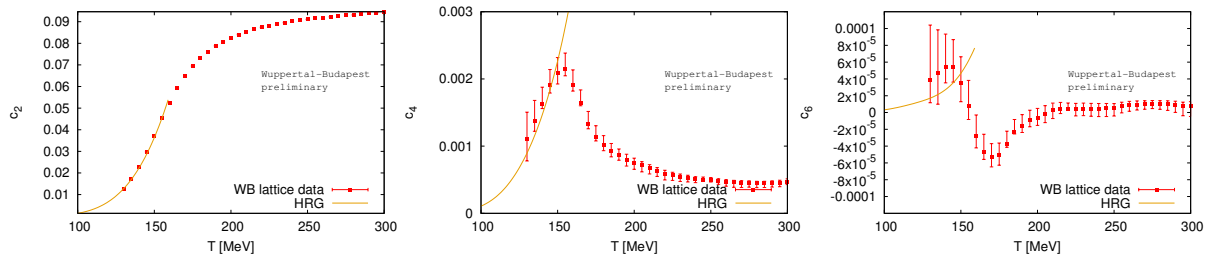


Figure 2. Preliminary results for the Taylor coefficients c_2 , c_4 and c_6 as functions of the temperature from the WB collaboration, obtained from imaginary μ_B simulations. The data are continuum extrapolated; the error-bars are only statistical: the systematics of the μ_B fitting are not included [14, 15].

3. The QCD phase diagram

Due to the sign problem, only a small portion of the QCD phase diagram can be investigated by means of lattice QCD calculations at the moment. It was unambiguously shown [16] that at $\mu_B = 0$ the phase transition is an analytical crossover, which takes place over a broad range of temperatures around $T_c \simeq 155$ MeV [17, 18, 19, 20]. The transition temperature can be defined by locating the inflection point of relevant observables. By following the change in the position of this inflection point as the chemical potential is increased, it is possible to express the μ_B -dependence of T_c in the following way:

$$\frac{T_c(\mu_B)}{T_c(\mu_B = 0)} = 1 - \kappa \left(\frac{\mu_B}{T_c(\mu_B)} \right)^2 + \lambda \left(\frac{\mu_B}{T_c(\mu_B)} \right)^4 + \dots \quad (3)$$

κ is the curvature of the phase diagram, and it was recently investigated by several groups. By looking at three different observables (chiral condensate, chiral susceptibility and strange quark susceptibility) the WB collaboration recently published a value of $\kappa = 0.0149 \pm 0.0021$ [21]; this has been obtained by fixing the strange quark chemical potential to impose strangeness neutrality. The phase diagram corresponding to this value of κ is showed in the left panel of Fig. 3, together with a compilation of freeze-out parameters obtained with different methods. The shaded band indicates the broadness of the QCD transition, while the dark blue band shows that it is possible to find a value for T_c with great accuracy when looking at a single observable (in this case the chiral condensate). Similar results have been obtained recently by two other groups: P. Cea *et al.* obtain a value of $\kappa = 0.020(4)$ by fixing $\mu_s = \mu_l$ [22], while Bonati *et al.* find $\kappa = 0.0135(20)$ both with $\mu_s = 0$ and $\mu_s = \mu_l$ [23]. In the right panel of Fig. 3, the phase diagram with the curvature from Ref. [22] is shown.

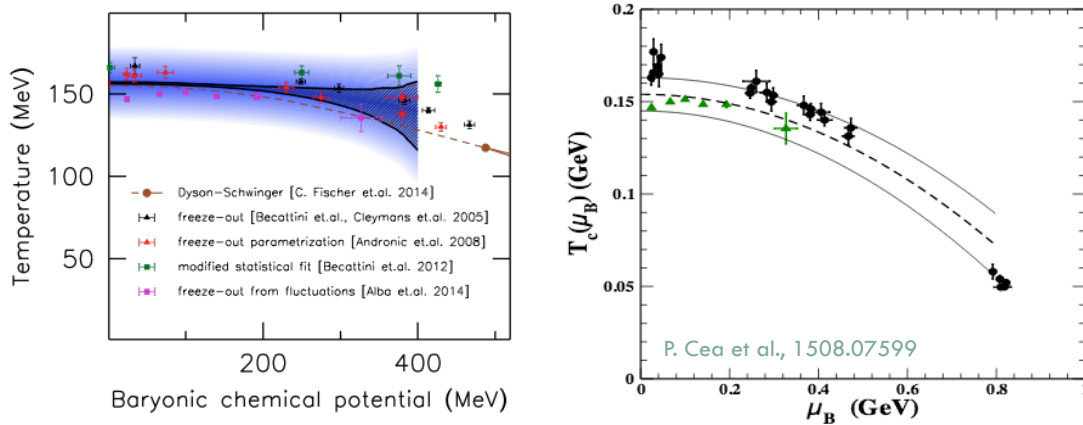


Figure 3. Left: The phase diagram based on the μ_B -dependent T_c from the chiral condensate, analytically continued from imaginary chemical potential [21]. The blue band indicates the width of the transition. The shaded black region shows the transition line obtained from the chiral condensate. The widening around 300 MeV is coming from the uncertainty of the curvature and from the contribution of higher order terms, thus the application range of the results is restricted to smaller values. We also show some selected non-lattice results: the Dyson-Schwinger result [24], and the freeze-out data of Refs. [25]–[31]. Right: analogous plot from Ref. [22].

4. Fluctuations of conserved charges and chemical freeze-out

Fluctuations of conserved charges have received increasing interest in recent years. The reason is that they can be measured in heavy ion collision experiments, and they can be simulated on the lattice. Thus, by comparing the experimental observables with the theoretical predictions, it is possible to gain information on the evolution of a heavy ion collision from first principles. Fluctuations have been proposed long ago as a signature for the QCD critical point [32, 6]. Besides, since they are fixed at the chemical freeze-out, they can be used to extract the freeze-out parameters [33, 35]. They are defined as derivatives of the pressure with respect to the chemical potential of the corresponding conserved charge:

$$\chi_{lmn}^{BQS}(T, \mu_B) = \frac{\partial^{l+m+n} p / T^4}{\partial(\mu_B/T)^l \partial(\mu_Q/T)^m \partial(\mu_S/T)^n} \quad (4)$$

and they can be related to the cumulants of the event by event distribution of the corresponding net conserved charge measured in experiments. In particular, it is possible to define volume-independent ratios which only depend on T and μ_B :

$$\begin{aligned} M/\sigma^2 &= \chi_1/\chi_2 & S\sigma^3/M &= \chi_3/\chi_1 \\ S\sigma &= \chi_3/\chi_2 & \kappa\sigma^2 &= \chi_4/\chi_2 \end{aligned} \quad (5)$$

where M is the mean of the experimental distribution, σ the variance, S the skewness and κ the kurtosis. The lattice QCD results for the above ratios will depend on T and μ_B : by comparing them to the experimental value it is therefore possible to extract the freeze-out temperature and chemical potential.

One has to keep in mind, that the outcome of lattice QCD simulations are thermal fluctuations. In order to compare them to experiment, one has to make sure that any source of non-thermal fluctuations which might affect the experimental data is under control. The system created in a heavy-ion collision is a canonical ensemble, in which net-charges such as electric charge, baryon number and strangeness are strictly conserved. However, due to

experimental limitations in the detector acceptance, the system can be treated as a Grand Canonical Ensemble, with net-charges fluctuating event-by-event [36]. Sources of non-thermal fluctuations may include fluctuations in the initial volume, protons coming from the interaction with the beam pipe, effects due to cuts in rapidity and transverse momentum, and so on. A variety of effects has been identified and studied in the literature [37]-[43]. In 2014 the WB collaboration found an upper limit for the chemical freeze-out temperature of $T_{ch} \leq 151 \pm 4$ MeV. It was also pointed out that, by analyzing the fluctuations of electric charge and baryon number independently, consistency is found between the freeze-out chemical potentials corresponding to the highest RHIC energies [44, 45, 46]. Recently, the authors of Ref. [47] were able to obtain both the freeze-out temperature and the curvature of the freeze-out line. The value of the freeze-out temperature ($T_f = (147 \pm 2)$ MeV) is in agreement with the one obtained in Ref. [44]. The left panel of Fig. 4 shows the ratio of ratios of χ_1/χ_2 for electric charge and proton number used for this fit. Since consistently between the freeze-out lines for electric charge and baryon number was found, the WB collaboration performed a combined fit of χ_1/χ_2 for electric charge and proton number and found the freeze-out temperature and chemical potential for the highest RHIC energies. These preliminary results are shown in the right panel of Fig. 4, together with the isentropic lines which match the freeze-out data, the contours for constant mean/variance of net-electric charge from the lattice, and the results of a previous analysis based on the HRG model [31]. The WB results agree with the ones of Ref. [47] and with the HRG model ones. Unfortunately, fluctuation data are not yet available at the LHC. However the authors of Ref. [50], assuming that the lower moments follow a Skellam distribution, expressed the second moments in terms of the particle yields and compared the lattice results to the ALICE experimental data, finding a slightly higher freeze-out temperature than the ones obtained at RHIC. Recently, the study of fluctuations has been extended to very large temperatures [51, 52] to extract the onset of the HTL perturbative expansion [53, 54], which is found to be $T \simeq 250$ MeV. Fluctuations of conserved charges can also be used to identify the relevant

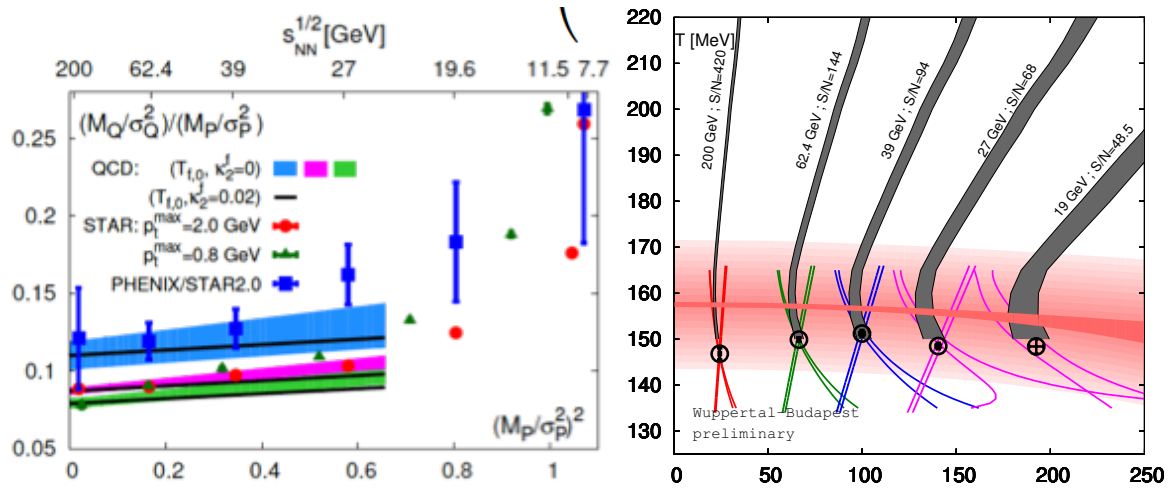


Figure 4. Left: From Ref. [47]: the ratio of ratios of χ_1/χ_2 for net electric charge and net-proton fluctuations measured by the STAR and PHENIX Collaborations [45, 46, 48, 49]. Right: Preliminary results of the WB collaboration. The colored full and dashed lines are the contours at constant mean/variance ratios of the net electric charge from lattice simulations. The contours that correspond to STAR data intersect in the freeze-out points of Ref. [31]. The red band is the QCD phase diagram shown in Fig. 3. Also shown are the isentropic contours that match the chemical freeze-out data [14, 15].

degrees of freedom in the vicinity of the QCD phase transition: by using linear combinations of charm fluctuations and correlations, in Ref. [55] it was pointed out that even if the onset of deconfinement for the charm quark takes place around $T \simeq 165$ MeV, it becomes the dominant degree of freedom in the thermodynamics of the charm sector only at $T \simeq 200$ MeV, while between these two temperatures the dominant contribution to the charmed pressure is given by open charm meson- and baryon-like excitations with integral baryonic charge.

In conclusion, the progress and the precision achieved by lattice QCD simulation in the last few years is really impressive. Precise results are available for QCD thermodynamics at zero and small chemical potentials, which allow a quantitative comparison with experimental results for the first time. This will enable us to achieve a comprehensive understanding of bulk properties of QCD matter from first principles.

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