# Influence of resonance radiation transfer on ionization balance in a positive column plasma

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**Abstract.** A method of self-consistent solution of charged particles balance equation, which is described by a differential equation of ambipolar diffusion, and an equation of resonance atom balance, which is described by an integral equation of radiation transfer, is proposed. The method is related to a replacement of an integral operator and a differential operator by a system of linear algebraic equations. The difference between a precise solution and a solution in the approximation of the effective resonance transition probability is shown. The influence of highest diffusion and radiation modes becomes apparent during transition to a contracted state.

## 1. Introduction

This article describes a method of precise self-consistent solution of an ambipolar diffusion equation and a resonance radiation transfer equation. A role of resonance radiation trapping on positive column plasma parameters discussed as well. It is significantly important to take into consideration the processes of transfer precisely when the excitation sources radically differ from the fundamental modes of diffusion and radiation problems (contracted discharge, skin effect, localization of excitation sources in a limited volume etc.). At present time there are only a few works [1,2] where diffusion and resonance transfer equation were solved along. Current state of problem is described in article [3]. A precise solution of a problem when resonance transfer is taken into consideration and a solution of an approximation of the effective resonance transition probability according to Biberman [4], Holstein [5] are compared. The difference between the solutions obtained for simple model concentration dependences of ionization and recombination terms are discussed. Symmetric cylindrical positive column and large absorption coefficients according to the Lorentz contour of a spectral line considered during the calculations.

## 2. Self-consistent solution of an ambipolar diffusion equation and a resonance radiation transfer equation considering a three-level ionization balance model

The difference between the diffusion of particles and photons is evident when a simple three-level scheme of ionization balance, which takes into account an electron excitation of a resonant state from the ground state at rate $Z$, a stepwise ionization with a probability $W_i$,a two-particle recombination on

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a resonance state $\Gamma$, an ambipolar diffusion of charged particles $D_a\Delta n$ and an output of resonance radiation with the effect of trapping, is considered.

This scheme can be described by two initial balance equations for resonance atoms (1) and charged particles (2).

$$\begin{cases} Z + \Gamma = N_r(\vec{r})W_i + G(\vec{r}) & (1) \\ N_r(\vec{r})W_i = \Gamma - D_a\Delta n & (2) \end{cases},$$

where an integral operator of resonance radiation transfer $G(\vec{r})$ equals to

$$G(\vec{r}) = AN_r(\vec{r}) - \int_V AN_r(\vec{r}')K(|\vec{r} - \vec{r}'|)d^3r', \quad (3)$$

$N_r(\vec{r})$ - resonance atom concentration at a position $\vec{r}$,
$A$ - spontaneous decay probability of resonance atoms,
$K(|\vec{r} - \vec{r}'|)$ - kernel of an integral operator.

Thus, initial system requires a self-consistent solution of an integral equation of resonance radiation transport (1) and a differential equation of an ambipolar diffusion (2). It should be also noticed that both of them have non-linear dependences on electron concentration. An approximate solution of this problem can be obtained in a common approximation of the effective transition probability according to Biberman [4] which assumes that the kernel $K(|\vec{r} - \vec{r}'|)$, as a function of the distance $|\vec{r} - \vec{r}'|$, decreases much faster than the atom density $N_r(\vec{r})$. This allows to take the density $N_r(\vec{r})$ at $\vec{r}$ outside the integral in equation (3) and replace the integral operator $G(\vec{r})$ by $N_r(\vec{r})A_{eff}$, where $A_{eff}$ is an effective resonance transition probability.

But if the resonance atom density strongly changes in space on the scale of a decrease of the kernel $K(|\vec{r} - \vec{r}'|)$, the effective transition probability approximation is not appropriate. A promising and precise method for the solution of initial equation is a replacement of the integral operator by a system of linear algebraic equations. The direct method for the solution of equation (1) uses discretization of the entire plasma volume $V$ into small cells $\Delta V_i$ inside which the resonance atom density $N_r(r_i)$ can be considered as a constant. Then, it is possible to remove it from the integral from equation (3) by substitution of the corresponding value in the cell center $r_i$. In case of large absorption coefficients the calculation of matrix elements can be carried out using asymptotes of the spectral line wings [3]. After the described substitution equation (3) has the form:

$$G(r) = \sum_j N_r(r_j)A\left[\delta_{jm} - \int_{\Delta v_j} K(|r_k - r'|)d^3r'\right],$$

where $\delta_{jm}$ - is the Kronecker symbol. The term in square brackets is taken as a matrix element $b_{jm} = \delta_{jm} - \int_{\Delta v_j} K(|r_k - r'|)d^3r'$. In matrix $B$ positive diagonal elements $b_{jj}$ describe a decay of resonance atoms and the negative non-diagonal elements $b_{jm}$ describe a redistribution of the resonance atoms over the plasma volume.

The diffusion operator, describing the ambipolar diffusion, is reduced to a tridiagonal matrix $A$ by finite differences. A dimension $N$ of both matrices is determined by a number of fragmentations of an interval $[0; R]$, where $R$ is a tube radius.

After a replace of the operators by the described matrices $A$, $B$ in the initial equations (1), (2) and introducing new variables: $x = r/R$, $y = \frac{n(x)}{n_0}$, $z = \frac{N_r(x)}{n_0}$, $\tilde{I}(y) = \frac{I(n)}{I_0}$, $a = \frac{I_0}{\Gamma_0}$, where index zero signifies the value at the axis of a discharge, the initial system has a form which is convenient for solving:

$$\begin{cases} \sum_j (b_{jm} z_j) + z_m \left( \frac{W_{i_m}}{A} \right) = \frac{\Gamma_0}{An_0} (a\tilde{I}_m - \tilde{\Gamma}_m) \\ \sum_j (a_{jm} y_j) + z_m \left( \frac{W_{i_m}}{D_a} \right) = \frac{\Gamma_0}{D_a n_0} \tilde{\Gamma}_m \end{cases}.$$

In this system a relative electron concentration $y_j = n_j / n_0$ and a relative resonance atom concentration $z_j = N_{r_j} / n_0$ are unknown variables. Matrix $A$ contains boundary conditions for an electron concentration. The resonance atoms integral equation does not require any of boundary conditions. To apply a method of iterations, the matrix of diffusion and the matrix of resonance radiation transfer should be combined into a single matrix

$$M = \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix}.$$

Model rates of ionization and recombination, which provide the switch from diffuse state to a contracted one with an increase of current, were introduced ($\tilde{I} = y^5$, $\tilde{\Gamma} = y^2$).

Considered approach allows to present dependences of plasma discharge parameters on current

$$i = 2\pi R^2 \int_0^R j(r) r dr,$$

where $j(r)$ - current density, $j(r) \sim n$. In relative variables this equation has a form

$$i = 2\pi R^2 j_0 \int_0^1 y(x) x dx.$$

It gives an opportunity to determine a dimensionless effective cross section of current flow

$$S_{eff} = \int_0^1 y(x) x dx,$$

where $j_0$ - current density at the axis. $S_{eff}$ =0,215 for the Bessel radial concentration distribution and $S_{eff}$ =0,5 for the square-shaped profile.

**3. Comparison of model problem solutions**

Solutions of model problem, which were obtained using a common approximation of the effective transition probability by Biberman [4] and the matrix method described above, are compared in figure 1. It can be seen that the resonance radiation trapping leads to a radial broadening of the electron and resonance atoms densities with a growth of an electron concentration value at the axis in case of precise solution with taking into account the transfer of resonance radiation transfer. Oppositely, in case of Biberman approximation those profiles are shrinking.

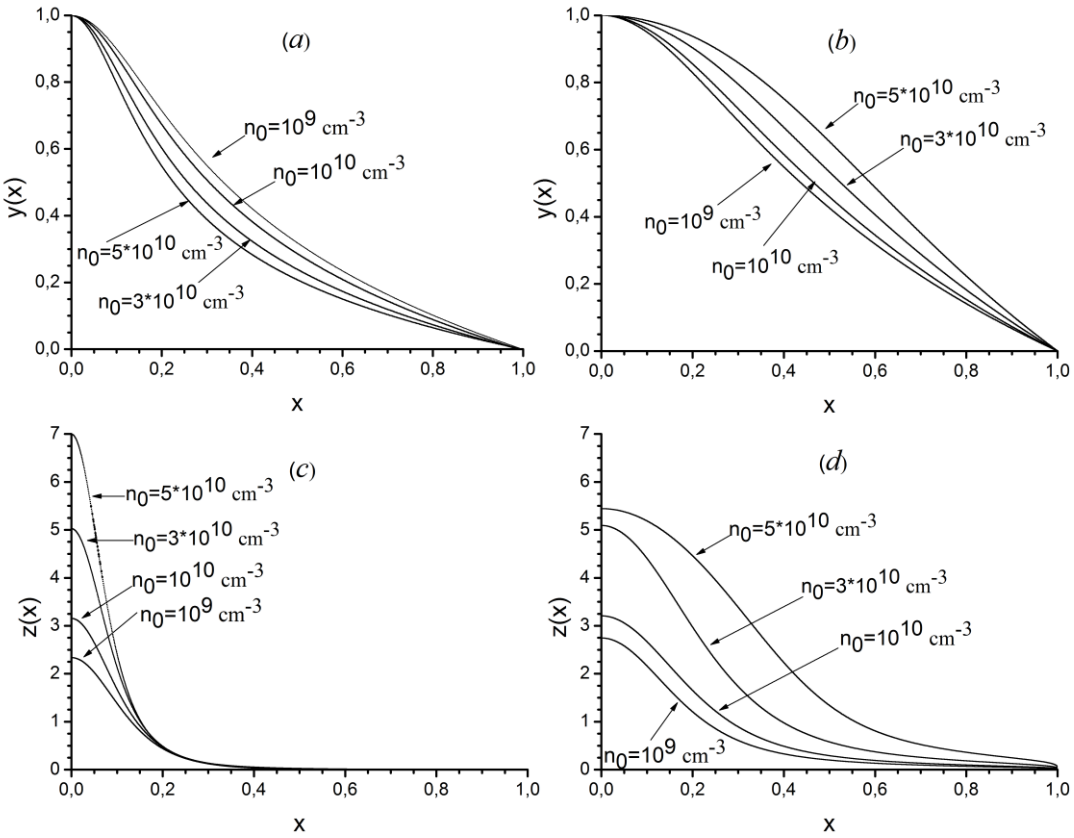


**Figure 1.** Radial relative electron concentration distribution (above) in the approximation of the effective transition probability (*a*) and with taking into account the transfer of resonance radiation (b). Radial relative resonance atom concentration distribution (below) in the approximation of the effective transition probability (*c*) and with taking into account the transfer of resonance radiation (d).

Dependences of an effective cross section of current flow $S_{eff}$, electron and resonance atom concentrations at the axis on current are presented in figure 2. It illustrates that in case of the local approximation an increase of current leads to a contraction, whereas the effect of trapping causes spreading of the current cord (figure 2 (a)).

The influence of resonance radiation trapping on electron and resonance atom concentration formation can be seen in figure 2 (b), (c). While current is relatively small (when radial profiles of electron concentration decrease smoothly), two solutions give a good agreement with each other. With a growth of current the difference in radial decreases starts to appear and leads to a significant distinguish in the values of plasma parameters. This fact demonstrates an influence of highest modes in a formation of plasma properties. At low current radial distributions are close to fundamental modes of diffusion and radiation problems and a solution, obtained in the approximation of the effective transition probability, is precise enough. But when the radial distributions differ from fundamental modes, the disadvantages of local approximation appear. The extent of accuracy of common approximation can be seen on presented results.

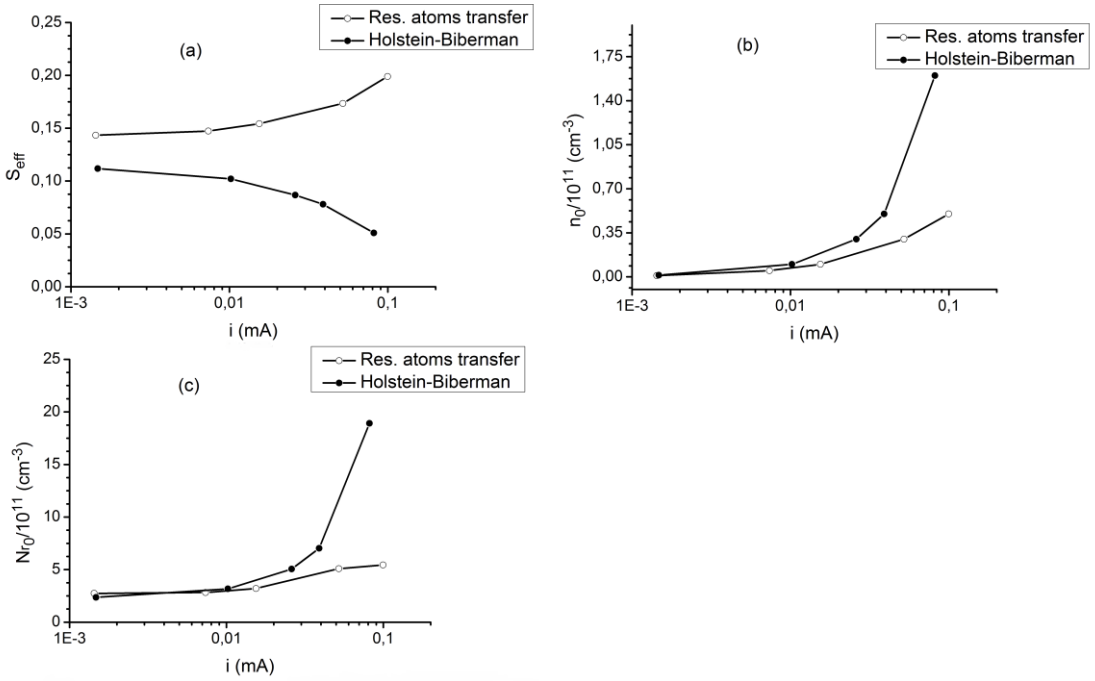


**Figure 2.** Dependences of an effective cross section current flow (a), an electron concentration (b) and a resonance atom concentration (c) on current.

## 4. Conclusion

The main result of this work is a development of a new approach for solving the balance equations, which is related to an accurate consideration of a resonance radiation transfer, since at present time almost all works are made in the approximation of the effective transition probability by Biberman [4]. Proposed technique of obtaining a solution of the integral radiation transfer equation has number of advantages. It is possible to obtain accurate numerical solutions by dividing the interval of integration into a greater number of steps and then subsequently compare a new solution with the previous. Thus one can obtain the spatial distribution of the resonance atoms at the same level of accuracy as the distribution of the electrons, which can be calculated by precise methods of numerical integration of the differential equation of an ambipolar diffusion. Eigenvalues and eigenmodes of the radiation transport operator can be easily calculated by using the discussed matrix method. It makes it possible to solve various time-dependent problems accurately by approximating the excitation sources and the expected solution by a system of the radiation modes alike in case of the diffusion modes. In addition it is possible to calculate the Green functions as an inverse matrix of the radiation problem.

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