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Linearization of acousto-optic modulator transmission function

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Abstract. The procedure of linearization of nonlinear transmission function of the optical transparency in the form of an acousto-optic modulator by the methods of nonlinear functional analysis is described. The transmission function of a pair of acousto-optic modulators is linearized in the context of generalized superposition principle.

1. Introduction

Acousto-optic modulator is an input device of dynamic signals in optical information processing techniques by acousto-optics methods. Here the basic operations of the processed radio signals are spectrum analysis and calculation of correlation functions and convolutions. The spectrum analysis assumes a linear regime of radio signal processing, and the calculation of correlation functions and convolution is non-linear operation performed with two radio signals [1], that is fully corresponds to the concepts of functional analysis [2]. In both cases it is assumed linear mapping of input to an optic computing device being processed signals.

The most important characteristic of acousto-optic modulator is its transmission function, which is non-linear mapping of input radio signals in optical systems of signal processing. Salvation requires transmission function acousto-optic modulator linearization. Usually this problem is solved in the context of acousto-optic modulator operation in the Raman – Nath regime at weak sound approach. Really this acousto-optic modulator operation is the exception rather than the rule. This requires problem solving of linearization of acousto-optic modulator transmission function at the most common form. This solution can be obtained on the basis of theoretical optics position [3] and nonlinear operators theory [2], and this determines the linear processing of radio signal by acousto-optical spectrum analyzer [4], which contains one acousto-optic modulator. In this case we are talking about the possibility of superposition principal applying.

Acousto-optic device of correlation processing of radio signals and calculations of their convolution consist in the pair of acousto-optic modulator, in this case the most problem is the solving of problem of establish of transmission function of the pair of acousto-optic modulators at the condition of the linearization of transmission function of each of them. In this case we are talking about the processing of radio signals appearing in the product. This problem can be solved in the context of generalized superposition principle [5].

2. Transmission function of acousto-optic modulator

In general, the transmission function of a transparent or translucent object is expressed in the form [3]

$$F(\xi,\eta)=\frac{V(\xi,\eta)}{V_0(\xi,\eta)} \tag{1}$$

where $V(\xi,\eta)$ - light field passing through the object; $V_0(\xi,\eta)=A\exp\left(ik\left(l_0\xi+m_0\eta\right)\right)\exp(-i\omega t)$, here l_0 and m_0 - the direction cosines.

Transmission function $F(\cdot)$ is the basis for describing of the action of the diffraction grating structure and lattice like structure shaped acousto-optic modulator [3]. In the latter case, the transmission function is described by spatial - temporal dependence

$$F(\xi,\eta,t)=\frac{V(\xi,\eta,t)}{V_0(\xi,\eta)} \tag{2}$$

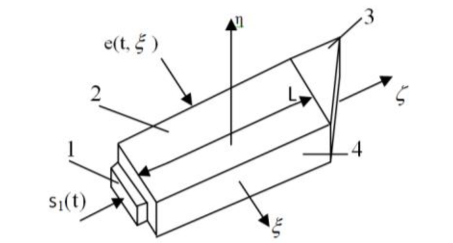

Figure 1. Acousto-optic modulator.

From formula (1) it is follows that the optical field at the output side of acousto-optic modulator, i.e. at the corresponding input stage to the optical information processing system, expressed in the form

$$e(\xi,\eta,t,\zeta)=\ F(\xi,\eta,\zeta,t)A\exp(-i\omega t)\cdot\exp\left(ik\left(l_0\xi+m_0\eta\right)\right) \tag{3}$$

where further relies $l_0 = 1$, $m_0 = 1$.

Acousto-optic modulator transmission function $F(\xi,\eta,\zeta,t)$ is non-linear mapping of acoustic wave $u(\xi,\eta,\zeta,t)$, propagating in the acousto-optic interaction medium. This wave is initiated by electric oscillations $s(t)$, which is input to the piezoelectric transducer. Acousto-optic modulator transmission function can generally be expressed by

$$F(\xi,\eta,\zeta,t)=\hat{\mathrm{N}}u(\xi,\eta,\zeta,t) \tag{4}$$

where $\hat{\mathrm{N}}-$ the nonlinear operator defined by diffraction regime.
The nonlinear operator $\hat{\mathrm{N}}$ is represented by the formula of finite increments i.e. in the form of sum of linear term and a term of order higher than the first concerning $\|h\|$ [2]:

$$\widehat{\mathrm{N}}(x+h) = \widehat{\mathrm{N}}(x) + \widehat{\mathrm{L}}(x)h + \omega(h) \tag{5}$$

where $\widehat{\mathrm{N}}' = \widehat{\mathrm{L}}$ - the first Frechet derivative, i.e, linear operator.

3. The linearization function passing acousto-optic modulator

With regard to the problem of determining the transmission function acousto-optic modulator naturally to assume $x = 0$, there is no signal at the input of the piezoelectric transducer and, $h = u(\xi,\eta,\zeta,t)$ then transmission function acousto-optic modulator written as:

$$F(\xi,\eta,\zeta,t) = \widehat{\mathrm{N}}(0) + \widehat{\mathrm{L}}(0)\beta u(\xi,\eta,\zeta,t) + \omega(u(\xi,\eta,\zeta,t)) \tag{6}$$

where $\omega(u(\xi,\eta,\zeta,t))$ defines nonlinear distortion, β - dimensional coefficient, $\widehat{\mathrm{N}}(0) = F_0 = \text{const.}$

Equation (6) is the basis of linear description of the transmission function of acousto-optic modulator at the solving of various problems of information processing by acousto-optics methods.
To use the formula of finite increments (6), it is necessary to find the first derivative of Frechet. This is done as follows. Initially, the Gateaux derivative is established [6]

$$\widehat{\mathrm{N}}'(x+th) = \lim \frac{\widehat{\mathrm{N}}(x+th) - \widehat{\mathrm{N}}(x)}{t}, t \to 0 \tag{7}$$

and its continuity near *x* is established. If Gateaux derivative is continuous, then Frechet derivative exists and coincides with the Gateaux derivative [6].
At Raman - Nath regime at approximation of weak sound transmission function of acousto-optic modulator is given by the well-known expression

$$F(\xi,\eta,\zeta,t) = \widehat{\mathrm{N}}u(\xi,\eta,\zeta,t) = A\exp(i\alpha u(\xi,\eta,\zeta,t)) \tag{8}$$

where A, α - dimensional coefficients.

Substituting into expression (7) $h = A\exp(i\alpha u(\xi,\eta,\zeta,t))$ and $x = 0$ gives the relation

$$\widehat{\mathrm{N}}'(t\exp(i\alpha u(\xi,\eta,\zeta,t))) = \lim \frac{A\widehat{\mathrm{N}}(t\exp(i\alpha u(\xi,\eta,\zeta,t)) - \widehat{\mathrm{N}}(0)}{t} = Ai\alpha u(\xi,\eta,\zeta,t), t \to 0 \tag{9}$$

where Gateaux derivative is continuous with respect to $i\beta u(\xi,\eta,\zeta,t)$ and therefore it coincide with the first Frechet derivative.

In expression (8) nonlinear operator $\widehat{\mathrm{N}}$ is analytical [6], it is represented in the form:

$$\widehat{\mathrm{N}}(x+h) = \sum_{k-0}^{\infty} \widehat{\mathrm{N}}^{(k)}(0)h^k \tag{10}$$

which implies the ratio (9).

In the context of accepted idealized description of the acousto-optic modulator

$$F(\xi,\eta,\zeta,t) = F_0 + \hat{\mathrm{I}}\beta u(\xi,\eta,\zeta,t) = F_0 + \beta u(\xi,\eta,\zeta,t) \tag{11}$$

where $\hat{\mathrm{I}}-$ the unit operator.

Using the general relation (11) different kinds of acousto-optic modulator transmission function determined by various acoustic beams $u(\xi,\eta,\zeta,t)$ with a base in the form of a rectangular piezoelectric transducer can be expressed. For example, in paper [7], acoustic beam is based at attenuation of the acoustic wave in the acousto-optic interaction medium, and in paper [8] the effect of the diffraction divergence of the acoustic beam is considered.

At the decision of principle, the general problems of the theory of optical information processing systems by acousto-optics methods acoustic beam in the shape of a cuboid is assumed, i.e. at distortion-free acousto-optic interaction medium

$$u(\xi,\eta,\zeta,t) = \chi(\xi)\chi(\eta)\chi(\zeta)u(\zeta - vt) = B\chi(\xi)\chi(\eta)\chi(\zeta)s(\zeta - vt) \tag{12}$$

where *B* - the dimensional coefficient, $\chi(\cdot)-$ characteristic functions of corresponding intervals determined by the size of the rectangular piezoelectric transducer and an aperture diaphragm; *v* - the velocity of acoustic waves.

Equation (12) defines acousto-optic modulator transmission function

$$F(\xi,\eta,\zeta,t) = F_0 + \chi(\xi)\chi(\eta)\chi(\zeta)\beta u(\zeta - vt) = F_0 + B\chi(\xi)\chi(\eta)\chi(\zeta)s(\zeta - vt) \tag{13}$$

Relation (13) is the most special case of the general relation (6), that form acousto-optic modulator transmission function usually is used for solving variety common, fundamental problems of optical information processing by acousto-optics methods.

Equations (9) and (13) allow us to write acousto-optic modulator transmission function at Raman - Nath regime at weak sound relation

$$F(\xi,\eta,\zeta,t) = \chi(\xi)\chi(\eta)\chi(\zeta)(F_0 + i\gamma u(\zeta - vt)) = F_0\chi(\xi)\chi(\eta)\chi(\zeta)(1 + i\kappa s(\zeta - vt)) \tag{14}$$

where $\kappa = \dfrac{\mu}{F_0} << 1$

Equations (13) and (14) set fulfilment of superposition principle at input of the electrical signal acousto-optic information processing system, in particular in acousto-optic spectrum analyzer.

4. The function of sequentially arranged pair of acousto-optic modulators

Figure 2 shows a pair of acousto-optic modulators, placed close. It is assumed that the transmission function of each of them is given by the form (8) .With regard relations (2) their joint transmission function defined as

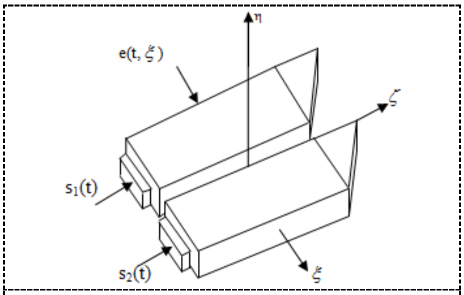

Figure 2. Acousto-optic modulator.

$$F_{1,2}(\xi,\eta,\zeta,t) = A_1 \exp(i\alpha_1 u(\xi,\eta,\zeta,t)) \cdot A_2 \exp(i\alpha_2 u(\xi,\eta,\zeta,t)) = \\ = A_1 A_2 \exp(i(\alpha_1 u(\xi,\eta,\zeta,t) + \alpha_2 u(\xi,\eta,\zeta,t)). \tag{15}$$

The Raman-Nath in the weak sound relation (15) takes the form:

$$F_{1,2}(\xi,\eta,\zeta,t) = A_1 A_2 (2 + (i(\alpha_1 u(\xi,\eta,\zeta,t) + \alpha_2 u(\xi,\eta,\zeta,t)) \tag{16}$$

Relation (16) can be regarded as a form of generalized superposition principle [5].

5. Conclusion

The aim set out in this paper researches is the application general methodology of theory of nonlinear operators to the problem of linear approximation of nonlinear operator describing acousto-optic modulator transmission function. Namely, this approach allows us to solve the problem of acousto-optic signal processing in the context of linear theory. Since these researches are based on common methodology of theory of non-linear operators, the proposed approach is suitable for solving of problems of linearization of any dynamic systems.

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