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Monitoring of quantum mode correlation functions in the presence of pointer state phase relaxation

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Abstract. In present work we investigate the process of interaction between two-level atom-pointer and cavity mode to obtain correlation functions of mode quantum state. In particular, we analyze the protocol of indirect photodetection which allow estimations for average values of photon number operator in any order using statistics of atom state detector clicks.

1. Introduction

Weak and indirect measurements are widely used in quantum physics for extracting information about the quantum system with minimum perturbation of its state [1]. The indirect photodetection process of quantum resonator mode implies organization of interaction between this mode and two level atom, which is passed through the cavity. This process is followed by atom state detection. Repetition of this process several times and collection of detector statistical sampling allow to estimate the statistical properties of cavity field quantum state [2].

Actually each finite quantum system is open. The atom and the electromagnetic field interact with the environment and it leads to relaxation of their common state. In connection to the cavity, in our case, it influence the evolution of monitored object (cavity mode). However state relaxation of the atom-pointer leads to distortions of the detection results [3, 4]. Finally it gives an erroneous statistical properties estimation of the mode. Therefore it is highly useful to study detector's nonideality, originated from environment influence on atomic state, and to take this into account in the statistics processing of the atomic state detectors counts.

In this work we present a method for describing the evolution of cavity mode, conditioned by atomic state measurement result in the case of indirect photodetection protocol. Assuming weak excited state relaxation of the atom compared to decoherence of its state in resonance conditions we obtain analytical expressions for superoperators of cavity mode conditional evolution. These allows us to show that averaged values of photon number operator and it grades are proportional to time derivatives of probability to detect atom in its ground state.

2. The model

Let us describe the common evolution of two level atom and cavity mode of electromagnetic field in the presence of environment which influences on the atom and follows to decoherention. In this case the master equation for density matrix $\rho(t)$ has the following (Lindblad) form:

$$i\frac{d}{dt}\rho = \left[H_{\text{int}}, \rho\right] + iD_A\rho, \tag{1}$$

where H_{int} is interaction Jaynes - Cummings Hamiltonian in interaction picture:

$$H_{\text{int}} = \Omega\sigma_+ a\exp(i\Delta t) + \Omega^*\sigma_- a^\dagger \exp(-i\Delta t), \tag{2}$$

D_A is the atomic dissipater characterized the interaction of the atom with the external modes:

$$D_A\rho = \frac{\gamma_{ge}}{2}\left(2\sigma_-\rho\sigma_+ - \sigma_+\sigma_-\rho - \rho\sigma_+\sigma_-\right) + \frac{\gamma_{ge}}{2}\left(c.c.\right), \tag{3}$$

Ω is the Raby frequency of atomic transition; $\sigma_+ = |e\rangle\langle g|$, $\sigma_- = |g\rangle\langle e|$ are atomic operators, a is photon annihilation operator in the cavity, Δ is atom-field detuning, γ_{ge}, γ_{eg} are coefficients of spontaneous decay and excitation, respectively. For convenience we rewrite Eq. (3) to the from:

$$\begin{aligned} \left(D_A\rho\right)_{gg} &= \gamma_{eg}\rho_{ee} - \gamma_{ge}\rho_{gg}, \qquad & \left(D_A\rho\right)_{ge} &= -\rho_{ge}\Gamma, \\ \left(D_A\rho\right)_{ee} &= \gamma_{ge}\rho_{gg} - \gamma_{eg}\rho_{ee}, \qquad & \left(D_A\rho\right)_{eg} &= -\rho_{eg}\Gamma, \end{aligned} \tag{5}$$

where Γ has the meaning of atomic phase relaxation (decoherence) rate.

To solve Eq.(1) we represent the density matrix as a decomposition:

$$\rho(t) = \sum_{\mu,\nu\in\{g,e\}} |\mu\rangle\langle\nu| \otimes \rho_{\mu\nu}(t) \tag{4}$$

and assume $\Gamma \gg \Omega$, which is equivalent to condition that coherence relaxation time is much shorter than the typical evolution time of the system [4]. From here it is easy to show that the following approach is applicable:

$$\frac{d}{dt}\left(\rho_{ge}e^{i\Delta t}\right) \approx \frac{d}{dt}\left(\rho_{eg}e^{-i\Delta t}\right) \approx 0, \tag{6}$$

and system (1) reduces to the closed form for diagonal elements of density matrix in the basis of atomic states:

$$\begin{aligned} \dot{\rho}_{gg} &= \kappa\Gamma\left(2a^\dagger\rho_{ee}a - \rho_{gg}a^\dagger a - a^\dagger a\rho_{gg}\right) - i\kappa\Delta\left[a^\dagger a, \rho_{gg}\right] + \gamma_{eg}\rho_{ee} - \gamma_{ge}\rho_{gg}, \\ \dot{\rho}_{ee} &= \kappa\Gamma\left(2a\rho_{ee}a^\dagger - \rho_{gg}aa^\dagger - aa^\dagger\rho_{gg}\right) - i\kappa\Delta\left[a^\dagger a, \rho_{ee}\right] + \gamma_{ge}\rho_{gg} - \gamma_{eg}\rho_{ee}. \end{aligned} \tag{7}$$

Here $\kappa = |\Omega|^2 / \left(\Gamma^2 + \Delta^2\right)$. We will solve this system in *superoperator form.* To this end we introduce the following superoperators:

$$2K_0\rho_F = a^\dagger a\rho_F + \rho_F aa^\dagger,\ K_+\rho_F = a^\dagger\rho_F a,\ K_-\rho_F = a\rho_F a^\dagger,\ N\rho_F = \left[a^\dagger a, \rho_F\right], \tag{8}$$

which act on the mode state space and form SU(1,1) algebra. The solution of Eq.(7) may be found in formalism of evolution superoperator. To obtain this we write:

$$\rho(0) = U(t)\rho(0) = \sum_{\mu,\nu\in\{g,e\}} M_{\mu,\nu}(t) \otimes |\mu\rangle\rangle\langle\langle\nu|\rho(0), \tag{9}$$

where we use notations $|\mu\rangle\rangle\langle\langle\mu|\rho = |\mu\rangle\langle\nu|\rho|\nu\rangle\langle\mu|$ and $M_{\mu\nu}$ is conditional evolution superoperator of mode state corresponding to the case then atom prepared in state $|\mu\rangle$ was detected in state $|\nu\rangle$. Rewriting (7) with (8) and (9) in use the following system of differential equation appears:

$$\begin{cases} \dfrac{d}{dt}M_{gg} = -\left(\alpha K_0 + \beta + \gamma_{ge}\right)M_{gg} + \left(\alpha K_+ + \gamma_{eg}\right)M_{eg}, \\ \dfrac{d}{dt}M_{eg} = -\left(\alpha K_0 - \beta + \gamma_{eg}\right)M_{eg} + \left(\alpha K_- + \gamma_{ge}\right)M_{gg}. \end{cases} \tag{10}$$

Here we use the notations $\alpha = 2\kappa\Gamma$, $\beta = -\kappa\left(i\Delta N + \Gamma\right)$.

3. Analytical estimations

Assume resonance interaction between atom and cavity mode, that is $\Delta = 0$, and absence of spontaneous relaxation and excitation at the considered time scale, that means $\gamma_{ge} = \gamma_{eg} = 0$. In this case the solution of system (10) can be found in analytical form. Let write system (10) in symmetric form using the substitution $M_{gg} = \exp\left[-\left(\alpha K_0 + \beta\right)t\right]M_{gg}^{'}$, $M_{eg} = \exp\left[-\left(\alpha K_0 - \beta\right)t\right]M_{eg}^{'}$. Then

$$\begin{cases} \dfrac{d}{dt}M'_{gg} = \alpha e^{2\beta t} e^{\alpha K_0 t} K_+ e^{-\alpha K_0 t} M'_{eg}, \\ \dfrac{d}{dt}M'_{eg} = \alpha e^{-2\beta t} e^{\alpha K_0 t} K_- e^{-\alpha K_0 t} M'_{gg}. \end{cases} \tag{11}$$

Using operator equalities $e^{\alpha K_0 t} K_+ e^{-\alpha K_0 t} = e^{\alpha t} K_+$, $e^{\alpha K_0 t} K_- e^{-\alpha K_0 t} = e^{-\alpha t} K_-$ [6], and condition $\alpha + 2\beta = 0$ we obtain

$$\begin{cases} \dfrac{d}{dt}M'_{gg} = \alpha K_+ M'_{eg}, \\ \dfrac{d}{dt}M'_{eg} = \alpha K_- M'_{gg}. \end{cases} \tag{12}$$

Superoperator M'_{gg} satisfies the differential equation

$$\frac{d^2}{dt^2}M'_{gg} - \alpha^2 K_+ K_- M'_{gg} = 0, \quad M'_{gg}\left(0\right) = I,\ M'_{eg}\left(0\right) = 0. \tag{13}$$

The solution of this Cauchy problem (13) is superoperator

$$M'_{gg} = \cosh \alpha\sqrt{K_+ K_-}\, t, \tag{14}$$

wherefrom

$$M_{gg} = e^{-(\alpha K_0 + \beta)t} \cosh \alpha\sqrt{K_+ K_-}\, t. \tag{15}$$

To find the averaged value of photon number operator lets expand (15) in series in powers of t:

$$M_{gg} = 1 - \left(\alpha K_0 + \beta\right)t + \frac{1}{2}\left(\alpha^2 K_+ K_- + \alpha^2 K_0^2 + 2\alpha\beta K_0 + \beta^2\right)t^2 + \dots. \tag{16}$$

Let us find the probability of detecting the atom in the ground state if it was prepared in ground state. From (14) we get

$$P_{gg}(t) = \left\langle M_{gg}(t)\right\rangle = \mathrm{Tr}\left(M_{gg}\rho\right) = 1 - \alpha\left\langle a^\dagger a\right\rangle t + \alpha^2 \left\langle \left(a^\dagger a\right)^2 \right\rangle t^2 + \dots. \tag{17}$$

Hence we can write the expressions for photon numbers operator and its square:

$$\left\langle a^\dagger a\right\rangle = -\frac{1}{\alpha}\frac{d}{dt}P_{gg}(t)\Big|_{t=0}, \quad \left\langle \left(a^\dagger a\right)^2 \right\rangle = \frac{1}{2\alpha^2}\frac{d^2}{dt^2}P_{gg}(t)\Big|_{t=0}. \tag{18}$$

4. Discussion

Figure 1 shows the dynamics of the average photon number for initial Fock state with $n = 2$ (a) and coherent state with amplitude equals to $\sqrt{2}$ (b). The black horizontal solid line in both cases corresponds to the analytical estimation (18) of the average photon number at the time of photodetection. Calculations were made for different values of the coefficient of spontaneous excitation (γ_{eg}, first number in the legend) and decay (γ_{ge}, second number in the legend). We can see that curves are grouped in threes according to the same value of γ_{ge}. In the case of the initial Fock state (a) the curves are grouped in pairs with the same value of the difference $\gamma_{ge} - \gamma_{eg}$.

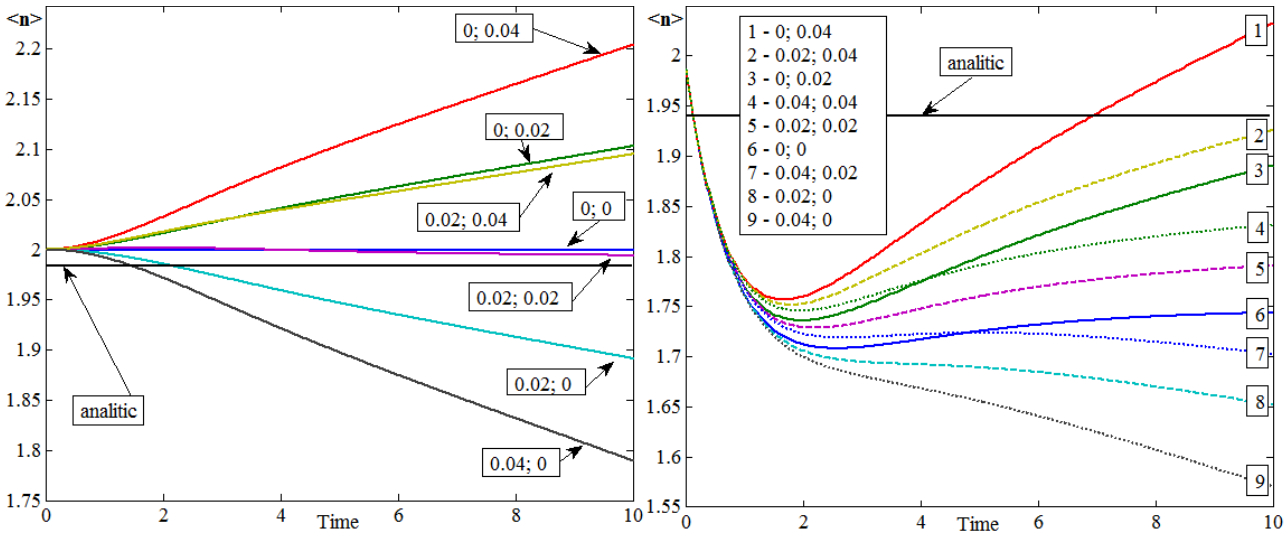

Figure 1. Average photon number for initial Fock state (a) and coherent state (b) of cavity mode. Black horizontal line coresponds to analytical estimation (18).

The conditional number of intracavity photons is cruicially depends on relation between spontaneous excitation and relaxation rates. For the case when the first process dominates we observe increasing in time number of photons while if the second process dominates the number of cavity photons is decreased. The character of dynamics is different in presented cases and this is a problem for the following research. Analytical estimation of mean photon number in the cavity is in good agreement with numerical simulations. Small difference between initial value and observed result may rise from approximate calculation of time derivative of probability and restricted basis of mode state by ten photons.

In conclusion, we suggest a protocol of mode statistical properties monitoring with atom pointer in use. Here we acount phase relaxation of the pointer and obtain conditional dynamics of intracavity mode in this case. Finaly, using exact solution for field state transformers we got analytical expressions for some statistical properties of cavity mode in the case of imperfect measurement procedure.

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