Features of diffraction and collimation of few-cycle optical beams

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Abstract. The paper reports theoretical spectral and spatial-temporal features of diffraction and collimation of few-cycle optical beams in transparent isotropic linear media. Equation describing asymmetric distribution in temporal structure of electric field in far field region of diffraction, which with increasing distance becomes symmetric is given. It is shown the frequency of maximum spectral density increases with distance from source of radiation along the axis of propagation. The frequency shift is inversely proportional to the initial duration of the wave packet and decreases with increasing of oscillations number.

1. Introduction

The development of methods of generation terahertz radiation made it possible to obtain few-cycle (less than 10 field oscillations) electromagnetic waves [1]. Such radiation is widely applicable in spectroscopy, in drugs and explosive materials detection systems and for medical diagnostics [2]. Changes in electro-magnetic spatial-temporal structures in diffracted radiation lead to task of optimization optical systems of emission, collimation and focusing initially few-cycle waves [3,4].

In this paper some spectral and spatial-temporal features of diffraction and collimation of few-cycle beams are reviewed. We got expression, which describes the dynamics of redistribution of electric field in temporal structure of diffracting wave packet in far field region of diffraction. Also we got analytic formulas describing spatial-temporal structure of collimated few-cycle beams.

2. Diffraction of few-cycle beams

Spectral profile of few-cycle beams is defined by the expression

$$G_0\left(\omega\right) = \frac{\sqrt{\pi}}{2} i^{n+1} \tau_p^n \omega^{n-1} E_0 \exp\left(-\left(\frac{\omega\tau_p}{2}\right)^2\right). \tag{1}$$

If we use Fourier transform $E\left(t\right) = \frac{1}{2\pi} \int\limits_{-\infty}^{+\infty} G\left(\omega\right) \exp\left(i\omega t\right) d\omega$, we get the spatial-temporal profile of electric field on radiation emitter [5]

$$E_0(t) = i^{2n} 2^{n-2} E_0 \sum_{k=2l, l\in\mathbb{N}_0}^{n-1} \left(\frac{(n-1)! i^{-k}}{2^k \left(n-1-k\right)! \left(\frac{k}{2}\right)!} \left(\frac{t}{\tau_p}\right)^{n-1-k} \right) \exp\left(-\left(\frac{t}{\tau_p}\right)^2\right), \tag{2}$$

where ω is the temporal cyclic frequency of radiation, n is the number of half-cycle electric field oscillation in wave packet, τ_p is the duration on emitter of radiation.

Paraxial dynamics of spatial-temporal spectrum of initial Gaussian beams can be described by the expression [4]

$$G(\omega, x, y, z) = z_R(\omega) \frac{z_R(\omega)+iz}{z_R^2(\omega)+z^2} \exp\left(-\frac{x^2+y^2}{\rho^2} z_R(\omega) \frac{z_R(\omega)+iz}{z_R^2(\omega)+z^2} - ik(\omega) z\right) G_0(\omega), \tag{3}$$

where x,y,z is the Cartesian axes coordinates (z axis coincides with wave propagation direction) $z_R(\omega) = \frac{\rho^2 n(\omega)\omega}{2c}$ is the Rayleigh range, ρ is the transverse beam width, $k(\omega) = \frac{\omega}{c} n(\omega)$ is the wave number, $n(\omega)$ is the refractive index of the medium.

In non-dispersive media, like atmospheric air [6], we can assume dispersive index in form $n(\omega) = N_0$.

Along the z-axis frequency of maximum of temporal spectrum can be received by time derivative of the function (3):

$$\omega_{\max}^{x=0,y=0}(z) = \frac{\sqrt{L^2(n-1) - \frac{z^2}{2} + \sqrt{\left(L^2(n-1) - \frac{z^2}{2}\right)^2 + 2nz^2L^2}}}{\tau_p L} \tag{4}$$

where $L = \frac{\rho^2 N_0}{2c\tau_p}$.

The function (4) is limited on the source of radiation by value

$$\omega_{\max}^{z=0} = \frac{\sqrt{2(n-1)}}{\tau_p}. \tag{5}$$

In far field region of diffraction:

$$\omega_{\max}^{z\to\infty} = \frac{\sqrt{2\,n}}{\tau_p}. \tag{6}$$

Fig.1 shows the dynamics of frequency temporal spectrum maximum along z-axis for different values of n.

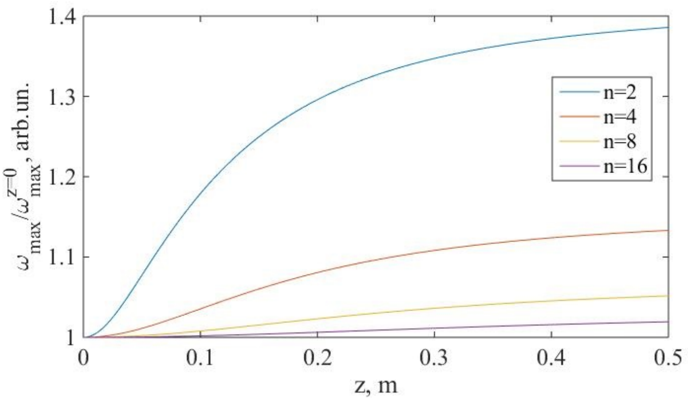

Figure 1. Dynamics of frequency temporal spectrum maximum along z-axis with initial duration $\tau_p = 0.2ps$

Spatial-temporal structure of electric field in far-field region ($z^2 \gg \{z_R(\omega)\}^2_{\max}$ is the spherics condition of the wave front curvature. $\{z_R(\omega)\}^2_{\max} = \frac{\rho^2 N_0 \omega_{\max}}{2c}$, where ω_{max} is maximum of frequency from frequency range where the majority of initial radiation energy is trapped) is defined by Furier transform of the function (3) instead of paper [3], where only initial one cycle beams was reviewed:

$$E\left(t',x,y,z\right)=i^{2n+2}E_0 2^{n-1}\frac{L}{z}\left(\frac{\tau}{\tau}\right)^{n+1}\left(\begin{array}{l}\frac{2L}{z}\frac{\tau_p}{\tau}\sum\limits_{k=2l,l\in\mathbb{N}_0}^{n+1}\left(\frac{(n+1)!i^{-k}}{2^k\left(n+1-k\right)!\left(\frac{k}{2}\right)!}\left(\frac{t'}{\tau}\right)^{n+1-k}\right)+\\+\sum\limits_{k=2l,l\in\mathbb{N}_0}^{n}\left(\frac{n!i^{-k}}{2^k\left(n-k\right)!\left(\frac{k}{2}\right)!}\left(\frac{t'}{\tau}\right)^{n-k}\right)\end{array}\right)\times\exp\left(-\left(\frac{t'}{\tau}\right)^2\right),\tag{7}$$

where "delay time"

$$t'=t-\frac{N}{c}\left(z+\frac{x^2+y^2}{2z}\right)\tag{8}$$

describes curvature of the wave front. The wave packet duration τ

$$\tau=\tau_p\sqrt{1+\frac{x^2+y^2}{\rho^2}\left(\frac{2L}{z}\right)^2}\tag{9}$$

is directly proportional to distance from z-axis.

Analitic formula (7) matches well with numerical calculations of few-cycle wave diffraction. Fig.2 shows it on example of one-cycle wave diffraction.

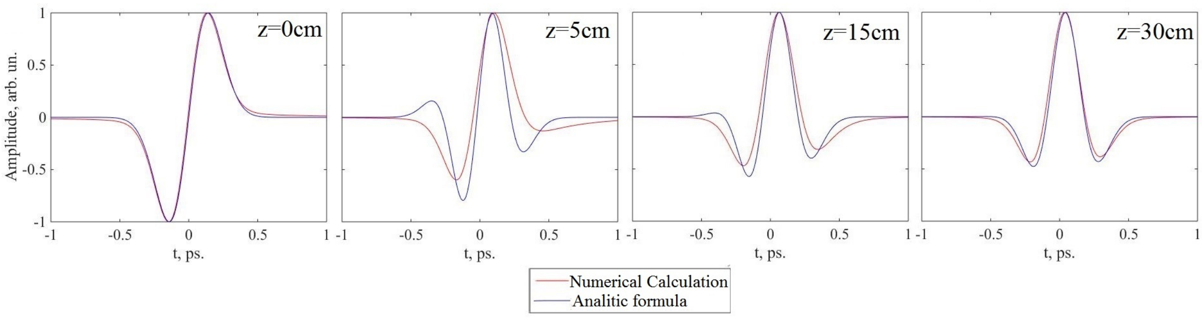

Figure 2. Comparison of numerical calculation with analytic formula (7) on example of along z-axis one-cycle wave diffraction $n = 2; \tau_p = 0.2ps$.

Visualization of formula (7) is shown at fig.3 on example of diffraction of half and one cycle beam.

3. Collimation of few-cycle beams

Collimation mirror with the reflection function $F_{col}\left(\omega,x,y\right)=\exp\left(ik\left(\omega\right)\left(x^2+y^2\right)/2f_{col}\right)$ is placed at the distance $z_0 = f_{col}$ from radiation source. Temporal spectrum of radiation at that distance is transformed to form

$$G_{col}\left(\omega,x,y,z\right)=\frac{iz_0\left(z_{col}\left(\omega\right)+i\left(z-z_0\right)\right)}{\left(z_{col}\left(\omega\right)\right)^2+\left(z-z_0\right)^2}\exp\left(\begin{array}{l}-\frac{x^2+y^2}{\rho^2}\frac{z_R(\omega)(z_{col}(\omega)+i(z-z_0))}{(z_{col}(\omega))^2+(z-z_0)^2}\\-ik\left(\omega\right)\left(z_0+z\right)\end{array}\right)G_0\left(\omega\right),\tag{10}$$

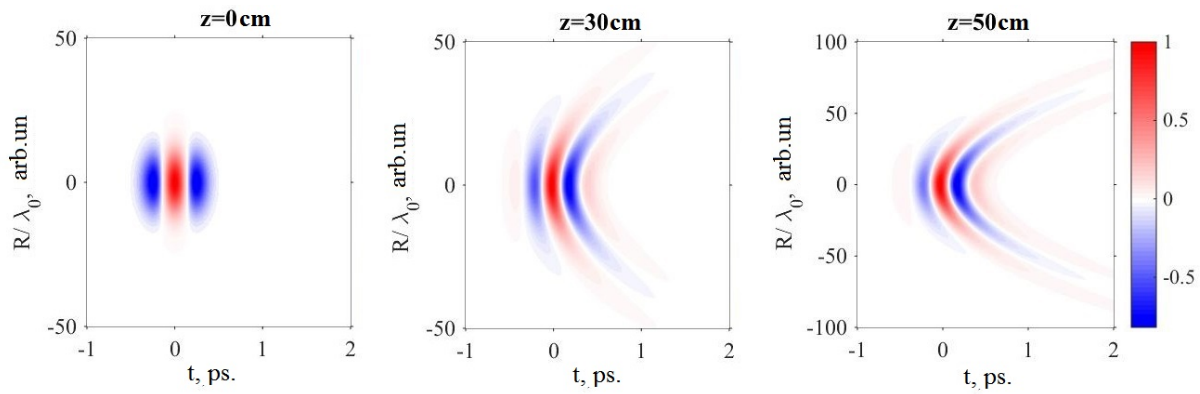

Figure 3. Spatial-temporal structure of electric field at different distance from source of radiation on example of half and one cycle beam. $n = 3; \tau_p = 0.2ps, R = \sqrt{x^2 + y^2}$.

where $z_{col}(\omega) = \frac{z_0^2}{z_R(\omega)}$.
If we know the expression of dynamics of spatial-temporal spectrum in linear isotropic media [4] and Fourier transform

$$G(\omega, x, y, z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} g(\omega, k_x, k_y, z) \exp(i(k_x x + k_y y)) dk_x dk_y,$$

we can produce expressions, which describe temporal dynamics of collimated beam's spectrum:

$$G_{col}(\omega, x, y, z) = \frac{i z_0 (z_{col}(\omega) + i(z - z_0))}{(z_{col}(\omega))^2 + (z - z_0)^2} \exp\left(\begin{array}{l} -\frac{x^2+y^2}{\rho^2} \frac{z_R(\omega)(z_{col}(\omega)+i(z-z_0))}{(z_{col}(\omega))^2+(z-z_0)^2} - \\ -ik(\omega)(z_0 + z) \end{array} \right) G_0(\omega), \quad (11)$$

where distance z is counted from collimation mirror. Spatial-temporal structure of collimated wave packet at the distance z_0 from mirror, where the waist of beam is located, can be expressed in the form

$$E_{col}^{z=z_0}(t', x, y, z_0) = i^{2n+2} E_0 \frac{2^{n-1} L}{z_0} \left(\frac{\tau_p}{\tau}\right)^{n+2} \sum_{k=2l, l\in\mathbb{N}_0}^{n} \left(\frac{n!(2i)^{-k}}{(n-k)!\left(\frac{k}{2}\right)!} \left(\frac{t'}{\tau}\right)^{n-k} \right) \exp\left(-\left(\frac{t'}{\tau}\right)^2\right), \quad (12)$$

where duration of the wave packet τ is described by expression

$$\tau = \tau_p \sqrt{1 + \frac{x^2 + y^2}{\rho^2} \left(\frac{2L}{z_0}\right)^2}, \quad (13)$$

which is equivalent to beam's duration at the distance $z0$ from the radiation source. Delay time is

$$t' = t - \frac{N_0}{c}(z_0 + z). \quad (14)$$

shows the flat phase shift at the focus distance from collimation mirror.

In far-field region spatial-temporal structure simplifies to form

$$E_{col}^{far}(t'', x, y, z_0) = -i^{2n} E_0 \frac{L}{z_0} 2^{n-1} \sum_{k=2l, l\in\mathbb{N}_0}^{n-1} \left(\frac{n-1!(2i)^{-k}}{(n-1-k)!\left(\frac{k}{2}\right)!} \left(\frac{t''}{\tau_p}\right)^{n-1-k} \right) \exp\left(-\left(\frac{t''}{\tau_p}\right)^2\right) \quad (15)$$

where curvative of the wave front becomes to be spherical and can be described by expression:

$$t'' = t - \frac{N_0}{c}\left(z_0 + z + \frac{x^2+y^2}{2(z-z_0)}\right). \quad (16)$$

The example of collimation of few-cycle waves are shown at fig.4

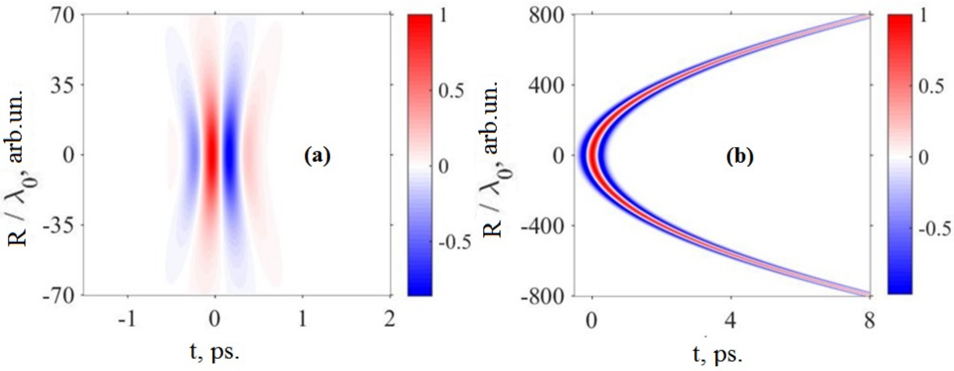

Figure 4. Spatial-temporal structure of electric field at distance $z_0 = f_{col}$ (a) and $z_0 = 100 f_{col}$ (b) from collimation mirror on example of initial half and one cycle beam. $n = 3; \tau_p = 0.2ps; R = \sqrt{x^2+y^2}$.

4. Conclusion

The spectral approach was used for the task of few-cycle wave propagation in linear isotropic media. In far-field region diffracted n-half-cycle beam is described by superposition of (n+1)-half-cycle and (n+2)-half-cycle wave packets. It produce asymmetric temporal structure, which with distance becomes symmetric, because (n+2)-half-cycle component decreases faster than (n+1)-half-cycle. The frequency of maximum spectral density increases with distance from source of radiation along the axis of propagation (4). The frequency shift is inversely proportional to the initial duration of the wave packet and decreases with increasing of oscillations number. For collimated few-cycle beams equations, which describes electric field spatial-temporal structure at focus distance from collimation mirror and in far-field region of diffraction are produced.

5. Acknowledgments

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