Journal of Physics: Conference Series **735** (2016) 012026 doi:10.1088/1742-6596/735/1/012026

Dynamics for atoms successively passing a cavity in the presence of the initial atomic entanglement

E K Bashkirov[1] and M S Mastyugin
[1]Physics Department, Samara State University, 1 Academician Pavlov Str. 443011, Russia

E-mail: bash@samsu.ru

Abstract. We investigated the dynamics of entanglement between atoms successively passing a cavity and interact with a thermal cavity field. We considered the situation when atoms are initially prepared in a pure entangled state. Using the exact solution of density matrix evolution we calculated the negativity for different values of cavity mean photon numbers. We shown the possibility to save the initial atomic entanglement even for a thermal cavity field with relatively high temperature.

1. Introduction

Entanglement is not only one of the most surprising features of quantum theory, but also provides an important resource for various quantum information processes such as quantum information, quantum communication, and quantum cryptography. Therefore, great efforts have been made to investigate entanglement characterization, entanglement control, and entanglement production in different systems. One of the particular interest schemes in which entanglement can be created is a system containing two two-level atoms, since they can represent two qubits, the building blocks of the quantum gates that are essential to implement quantum protocols in quantum information processing. Two-atom entangled states have been demonstrated experimentally using ultra cold trap ions, impurity spins in solids, superconducting circuits and cavity quantum electrodynamics schemes (QED) [1]. Cavity QED has been used to generate the atom-atom and atom-photon entanglement. The entanglement between two initially independent atoms successively passing the vacuum cavity has been demonstrated by S. Haroche et al. [2]. The entanglement procedure involves the resonant coupling, one by one, of the atoms to a high Q microwave superconducting cavity. The atoms, prepared in circular Rydberg states, exchange a single photon in the cavity and become entangled by this indirect interaction. The effect has been demonstrated with pairs of atoms separated by centimetric distances. Liao et al. [3] investigated the entanglement properties of two atoms passing through a cavity one after another when the field is initially in thermal field. They found that the phenomenon of sudden birth of entanglement occurs in some certain conditions and the entanglement between two atoms can be created even if the two atoms are initially in excited states. Yan [4] also investigated the entanglement properties of two atoms successive passing a cavity with Fock or thermal field but she especially focused on the case that when two atoms are initially in an entangled state. In this paper we examined entanglement properties of two atoms successive passing a thermal field when two atoms are initially in another type of Bell entangled atomic state.

[1] To whom any correspondence should be addressed.

Journal of Physics: Conference Series **735** (2016) 012026 doi:10.1088/1742-6596/735/1/012026

2. Model and it exact solution

Our system consists of two separate identical two-level atoms passing through a cavity one after another with different velocities. We suppose that atoms interact resonantly with cavity field via one-photon transitions. The interaction Hamiltonian of this system can be written as

$$H = \hbar g(a^{+}\sigma^{-} + \sigma^{+}a) \tag{1}$$

where, $\sigma^{+} = |+\rangle\langle-|$ and $\sigma^{-} = |-\rangle\langle+|$ are the transition operators between the excited $|+\rangle$ and the ground $|-\rangle$ states in the two-level atom, a^{+} and a are the creation and the annihilation operators of photons of the cavity mode, g is the coupling constant between atom and the cavity. The two-atom wave function can be expressed as a combination of state vectors of the form $|v_1, v_2\rangle = |v_1\|v_2\rangle$, where $v_1, v_2 = +, -$.

We first examine that the cavity is initially in a Fock state $|n\rangle$ containing *n* photons and two spatially separated two-level atoms are initially prepared in a pure entangled state of the following form

$$|\Psi(0)\rangle_A = \cos\theta|+,+\rangle + \sin\theta|-,-\rangle, \tag{2}$$

which pass through the cavity one after the other. Here θ is the amplitude.

Using the Hamiltonian (1) and the initial conditions (2) one can obtain the time dependent wave function for considered system at the moment τ when the first atom leaves the cavity

$$|\Psi_{1n}(\tau)\rangle = Y_{1n}(\tau)|+,+,n\rangle + Y_{2n}(\tau)|+,-,n-1\rangle + Y_{3n}(\tau)|-,+,n+1\rangle + Y_{4n}(\tau)|-,-,n\rangle, \tag{3}$$

where

$$Y_{1n}(\tau) = \cos\theta\cos(\sqrt{n+1}g\tau),$$
$$Y_{2n}(\tau) = -i\sin\theta\sin(\sqrt{n}g\tau),$$
$$Y_{3n}(\tau) = -i\cos\theta\sin(\sqrt{n+1}g\tau),$$
$$Y_{4n}(\tau) = \sin\theta\cos(\sqrt{n}g\tau).$$

The wave function (3) on the other hand is the initial state of the system prior to entering into the cavity of the second atom. At the moment *t* when the second atom leaves the cavity the time-dependent wave function takes the form (for $n \geq 1$)

$$\begin{aligned}|\Psi_{2n}(t)\rangle = {} & X_{1n}(t)|+,+,n\rangle + X_{2n}(t)|+,+,n-2\rangle + X_{3n}(t)|+,-,n-1\rangle + X_{4n}(t)|+,-,n+1\rangle + \\ & + X_{5n}(t)|-,+,n+1\rangle + X_{6n}(t)|-,+,n-1\rangle + X_{7n}(t)|-,-,n\rangle + X_{8n}(t)|-,-,n+2\rangle,\end{aligned} \tag{4}$$

where

$$X_{1n}(t) = \cos\theta\cos(\sqrt{n+1}g\tau)\cos(\sqrt{n+1}gt),$$
$$X_{2n}(t) = \sin\theta\sin(\sqrt{n}g\tau)\cos(\sqrt{n-1}gt),$$
$$X_{4n}(t) = -i\cos\theta\cos(\sqrt{n+1}g\tau)\sin(\sqrt{n+1}gt),$$
$$X_{5n}(t) = -i\cos\theta\sin(\sqrt{n+1}g\tau)\cos(\sqrt{n+2}gt),$$
$$X_{6n}(t) = -i\sin\theta\cos(\sqrt{n}g\tau)\cos(\sqrt{n}gt),$$
$$X_{7n}(t) = \sin\theta\cos(\sqrt{n}g\tau)\cos(\sqrt{n}gt),$$
$$X_{8n}(t) = \cos\theta\sin(\sqrt{n+1}g\tau)\sin(\sqrt{n+2}gt).$$

For the case $n = 0$ the wave-function can be derived as

$$|\Psi_2(t)\rangle = Z_1(t)|+,+,0\rangle + Z_2(t)|+,-,1\rangle + Z_3(t)|-,+,1\rangle + Z_4(t)|-,-,0\rangle + Z_5(t)|-,-,2\rangle, \quad (5)$$

where

$$Z_1(t) = \cos\theta\cos(g\tau)\cos(gt), \quad Z_2(t) = -i\cos\theta\sin(g\tau)\sin(gt),$$

$$Z_3(t) = -i\cos\theta\sin(g\tau)\cos(\sqrt{2}gt), \quad Z_4(t) = \sin\theta, \quad Z_5(t) = \cos\theta\sin(g\tau)\sin(\sqrt{2}gt).$$

To investigate the atom-atom entanglement we calculate negativity. The reduced atomic density matrix can be derived by tracing the density matrix of the whole system on the field variables. As a result one can obtain for atomic density matrix

$$\rho_{at}(\tau,t) = \begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13} & \rho_{14} \\ \rho_{12}^* & \rho_{22} & \rho_{23} & \rho_{24} \\ \rho_{13}^* & \rho_{23}^* & \rho_{33} & \rho_{34} \\ \rho_{14}^* & \rho_{24}^* & \rho_{34}^* & \rho_{44} \end{pmatrix}, \quad (6)$$

where for the case $n \geq 1$

$$\rho_{11} = |X_{1n}(t)|^2 + |X_{2n}(t)|^2, \quad \rho_{12} = 0, \quad \rho_{13} = 0, \quad \rho_{14} = X_{1n}(t)X_{7n}(t)^*,$$

$$\rho_{22} = |X_{3n}(t)|^2 + |X_{4n}(t)|^2, \quad \rho_{23} = X_{3n}(t)X_{6n}(t)^* + X_{4n}(t)X_{5n}(t)^*,$$

$$\rho_{24} = 0, \quad \rho_{33} = |X_{5n}(t)|^2 + |X_{6n}(t)|^2, \quad \rho_{34} = 0, \quad \rho_{44} = |X_{7n}(t)|^2 + |X_{8n}(t)|^2 .$$

For two-qubit system described by the density operator (6), a measure of entanglement or negativity can be defined in terms of the negative eigenvalues μ_i^- of partial transpose of the reduced atomic density matrix $\rho_A^{T_1}$ [5], [6]. The partial transpose of the reduced atomic density matrix (6) can be written in the form

$$\rho_{at}^{T_1}(\tau,t) = \begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13}^* & \rho_{23}^* \\ \rho_{12}^* & \rho_{22} & \rho_{14}^* & \rho_{24}^* \\ \rho_{13} & \rho_{14} & \rho_{33} & \rho_{34} \\ \rho_{23} & \rho_{24} & \rho_{34}^* & \rho_{44} \end{pmatrix}. \quad (7)$$

and the negativity is

$$\varepsilon = -2\sum \mu_i^- . \quad (8)$$

When $\varepsilon = 0$ two qubits are separable, and $\varepsilon > 0$ means the atom-atom entanglement. The case $\varepsilon = 1$ indicates maximum entanglement.

The partial transpose of the reduced atomic density matrix (7) has only two eigenvalues that can be negative

$$\mu_1 = (1/2)\Big[|X_{1n}|^2 + |X_{2n}|^2 + |X_{7n}|^2 + |X_{8n}|^2 -$$
$$-\sqrt{(|X_{1n}|^2 + |X_{2n}|^2 - |X_{7n}|^2 - |X_{8n}|^2)^2 + 4|X_{6n}X_{3n}^* + X_{5n}X_{4n}^*|^2}\,\Big], \quad (9)$$
$$\mu_2 = (1/2)\Big[|X_{3n}|^2 + |X_{4n}|^2 + |X_{5n}|^2 + |X_{6n}|^2 -$$
$$-\sqrt{(|X_{3n}|^2 + |X_{4n}|^2 - |X_{5n}|^2 - |X_{6n}|^2)^2 + 4|X_{1n}|^2|X_{7n}|^2}\,\Big].$$

Using formulae (9) one can calculate the negativity (9). Analogously we can derive negativity for the case when $n = 0$.

Now we consider the case in which two initially entangled atoms successively pass a thermal field, which is the most easily available radiation field. This initial one-mode thermal field can be written as

$$\rho_F(0) = \sum_n p_n |n\rangle\langle n|.$$

The weight functions are

$$p_n = \frac{\bar{n}^n}{(1+\bar{n})^{n+1}},$$

where $\bar{n}$ is the mean photon number in the cavity mode $\bar{n} = (\exp[\hbar\omega_i / k_B T]-1]^{-1}$, k_B is the Boltzmann constant and T is the equilibrium cavity temperature.

For the thermal field the matrix element of (7) have the form

$$\rho_{11} = \sum_{n=1} p_n (|X_{1n}(t)|^2 + |X_{2n}(t)|^2 + p_0 |Z_1|^2, \quad \rho_{12} = 0, \quad \rho_{13} = 0,$$

$$\rho_{14} = \sum_{n=1} p_n X_{1n}(t) X_{7n}(t)^* + p_0 Z_1(t) Z_4(t)^*, \ \rho_{22} = \sum_{n=1} p_n (|X_{3n}(t)|^2 + |X_{4n}(t)|^2) + p_0 |Z_1(t)|^2,$$

$$\rho_{23} = \sum_{n=1} p_n (X_{3n}(t) X_{6n}(t)^* + X_{4n}(t) X_{5n}(t)^* + p_0 Z_2(t) Z_3(t)^*$$

$$\rho_{24} = 0, \quad \rho_{33} = \sum_{n=1} p_n (|X_{5n}(t)|^2 + |X_{6n}(t)|^2 + p_0 |Z_3(t)|^2,$$

$$\rho_{34} = 0, \quad \rho_{44} = \sum_{n=1} p_n |X_{7n}(t)|^2 + |X_{8n}(t)|^2 + p_0 (|Z_4|^2 + |Z_5|^2).$$

For the thermal field the partial transpose of the reduced atomic density matrix (7) also has only two eigenvalues that can be negative. But these have too cumbersome form to present in the paper.

3. Results and discussion

The results of the numerical of calculations of entanglement parameter (8) for initial atomic states (2) ant thermal fields with different mean photons numbers are shown in the Fig.1. The curves were obtained under the assumption that $t = 2\tau$ as in [2]. One can see from the Fig.1 that for vacuum cavity state the initial maximum degree of atomic entanglement restore for the moments $t_k = 2\pi k \ (k = 1, 2, ...)$. Thermal field destroys the initial atomic quantum correlations only for certain time of the atom-field interaction. For other moments the partial degree of atom-atom entanglement is restored, even for relatively high cavity temperatures. to increasing of the degree of entanglement.

4. Conclusions

Thus, we investigated the dynamics of entanglement of two two-level atoms successively passing the thermal cavity and resonantly interacting with one-mode quantum cavity electromagnetic field through one-photon transitions. The atoms are assumed to be initially prepared in the entangled state. We derived the exact expression for the reduced atomic density matrix and calculated the analytical expression for negativity. Our numerical results reveal that the initial atomic entanglement can partially restore its original value for a finite interval of time, even for relatively high cavity temperatures.

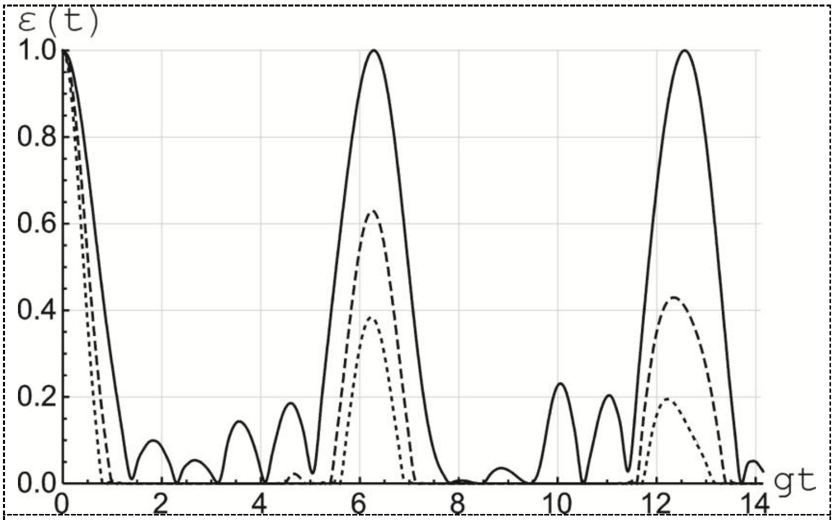

Figure 1. The negativity vs. *gt* for entangled initial atomic state (2) with $\theta = \pi/4$. The thermal mean photon numbers are $\bar{n} = 0 (T = 0)$ (solid), $(T = 2.4\ K)$ (dashed) and $\bar{n} = 1 (T = 3.8\ K)$ (dotted).

5. References

[1] Buluta I, Ashhab S and Nori F 2011 Rep.Prog. Phys. **74** 104401
[2] Hagley E, Maitre X, Nogues G, Wunderlich C, Brune M, Raimond J M and Haroche S. 1997 Phys. Rev. Lett. **79** 1
[3] Liao Q, Fang G, Ahmad M A and Liu S 2011 Optics Commun. **384** 201
[4] Yan X Q 2009 Chaos, Solitons and Fractals **41** 1645
[5] Peres A 1996 Phys. Rev. Lett. **77** 1413
[6] Horodecki R, Horodecki M and Horodecki P 1996 Phys. Lett. A **223** 333