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# Entanglement between two qubits induced by thermal field

**E K Bashkirov[1] and M S Mastyugin**
Physics Department, Samara State University, 1 Academician Pavlov Str. 443011, Russia

E-mail: bash@samsu.ru

**Abstract.** We have investigated the influence of dipole-dipole interaction and initial atomic coherence on atomic entanglement dynamics of two qubits. We have considered a model, in which only one atom couples to a quantum electromagnetic field in a cavity, since one of them can move around the cavity. We have shown that the entanglement arises for all pure atomic state even when both atoms are initially in the excited states. We have also derived that degree of entanglement is enhanced in the presence of the atomic coherence.

## 1. Introduction

Entanglement is a major resource for many fundamental applications in quantum information science. It is important to implement reliable methods to generate entangled states between systems. In order to function optimally, these applications require maximally entangled states. Several methods of creating entanglement have been proposed involving trapped and cooled ions or neutral atoms in cavities, superconducting circuits, spins in solids etc. [1]. In order to function optimally various applications require maximally entangled states. Because of decoherence, which is generally related to noise, there is great difficulty in generating and keeping the integrity of a pure entangled states. Although the interaction between the environment and quantum systems can lead to decoherence, it may also be associated with the formation of non-classical effects such as entanglement. Thus, understanding and investigating entanglement of mixed states becomes one of the actual problems of quantum information. Kim et al. [2] have investigated the atom-atom entanglement in a system of two identical two-level atoms with one-photon transitions induced by a single-mode thermal field. They showed that a chaotic field with minimal information can entangle atoms which were prepared initially in a separable state. Zhang directly generalizes Kim's study to the case when the atoms are slightly detuned from the thermal field, and study how the detuning would affect atom-atom entanglement [3]. The entanglement between two identical two-level atoms through nonlinear two-photon interaction with one-mode thermal field has been studied by Zhou et al. [4]. They showed that atom-atom entanglement induced by nonlinear interaction is larger than that induced by linear interaction. In [5] has discovered that two atoms can be entangled also through nonlinear nondegenerate two-photon interaction with two-mode thermal field. The influence of dipole-dipole interaction on entanglement between two cubits induced by one-mode and two-mode thermal field has been investigated in [6],[7].

The problem of creating or controlling the atomic entanglement is greatly related to the atomic coherence of population between different levels. Hu et al [8] have shown that the entanglement between two atoms induced by one-mode or two-mode thermal field can be manipulated by changing

[1] To whom any correspondence should be addressed.

the initial parameters of the atoms, such as the superposition coefficients and the relative phases of the initial atomic coherent state and the mean photon number of the cavity field. They have also discovered that entanglement may be greatly enhanced due to dipole-dipole interaction in the presence of the atomic coherence.
The practical applications in quantum information processing require engineering entangled atoms so this expects operable atoms which can be moved to distance without losing of information. Recently many schemes have been proposed to realize the engineering entangled atoms [9]. Guo and co-authors [10] have proposed a simple scheme to realize an easily engineered two-atom entangled state. The advantage of this scheme is only one atom is trapped in a cavity and the other one can be spatially moved freely outside the cavity. But the authors have investigated the atom-atom entanglement only for vacuum initial cavity field. In this paper we study the entanglement dynamics for model supposed in [10] for thermal cavity field taking into account the initial atomic coherence. Let's note that the model under consideration can be most simply realised for system of two superconducting qubits, one of which interacts with the LC superconducting circuit or microwave coplanar cavity. For superconducting qubits the effective dipole-dipole interaction may be much greater than for natural atoms [1].

## 2. Model and it exact solution

We consider two identical two-level atoms and one-mode quantum electromagnetic cavity field. The first atom is trapped in a lossless microcavity and resonantly interacts with the cavity field of the frequency $\omega$. The second atom lies beside the first atom out of the cavity. We assume that the distance between atoms can compare with a wavelength on working transition. In this case the dipole-dipole interaction should be included. The interaction Hamiltonian of this system can be written as

$$( H = \hbar g(\sigma_1^+ a + a^+\sigma_1^-) + \hbar J(\sigma_1^+\sigma_2^- + \sigma_1^-\sigma_2^+), \tag{1}$$

where, $\sigma_i^+ = |+\rangle_{ii}\langle -|$, and $\sigma_i^- = |-\rangle_{ii}\langle +|$ are the transition operators between the excited $|+\rangle_i$ and the ground $|-\rangle_i$ states in the $i$ th atom, $a^+$ and $a$ are the creation and the annihilation operators of photons of the cavity mode, $g$ is the coupling constant between atom and the cavity field and $J$ is the coupling constant of the dipole interaction between the atoms and $|+\rangle$ and $|-\rangle$ are the excited and the ground states of a single two-level atom. The two-atom wave function can be expressed as a combination of state vectors of the form $|v_1, v_2\rangle = |v_1\|v_2\rangle$, where $v_1, v_2 = +,-$.

We consider that the initial state of each atoms is a coherent superposition of the two levels, that is,

$$|\Psi_1(0)\rangle = \cos\theta_1 |+\rangle_1 + e^{i\varphi_1}\sin\theta_1 |-\rangle_1,$$

$$|\Psi_2(0)\rangle = \cos\theta_2 |+\rangle_2 + e^{i\varphi_2}\sin\theta_2 |-\rangle_2.$$

Here $\theta_1$ and $\theta_2$ denote the amplitudes of the polarized atoms, and $\varphi_1$ and $\varphi_2$ are relative phases of two atoms, respectively.

The initial cavity mode state are assumed to be the thermal one-mode state

$$\rho_F(0) = \sum_n p_n |n\rangle\langle n|.$$

The weight functions are

$$p_n = \frac{\bar{n}^n}{(1+\bar{n})^{n+1}},$$

where $\bar{n}$ is the mean photon number in the $i$-th cavity mode

$$\bar{n} = (\exp[\hbar\omega_i / k_B T] - 1]^{-1},$$

And $k_B$ is the Boltzmann constant and $T$ is the equilibrium cavity temperature.

The initial density matrix of the whole system is

$$\rho(0) = \rho_A(0)\rho_F(0) = \sum_n p_n \,|\,\Psi_1(0)\rangle\,|\,\Psi_2(0)\rangle\langle\Psi_1(0)\,|\,\langle\Psi_2(0)\,\|\,n\rangle\langle n\,|. \tag{2}$$

Before considering the dynamics of the system for thermal initial cavity field, it is straightforward to first study the case when the trapped two-level atom interacts with Fock state. Suppose that the excitation number of the atom-field system is $n$ ($n \geq 0$) the evolution of the system is confined in the subspace $|-,-,n+2\rangle, |+,-,n+1\rangle, |-,+,n+1\rangle, |+,+,n\rangle$. on this basis, the Eigen functions of Hamiltonian of the system (1) can be written as [10]

$$|\Phi_{in}\rangle = \xi_{in}(X_{i1n}\,|-,-,n+2\rangle + X_{i2n}\,|+,-,n+1\rangle +$$

$$+X_{i3n}\,|-,+,n+1\rangle + X_{i4n}\,|+,+,n\rangle) \quad (i = 1,2,3,4),$$

where

$$\xi_{in} = 1/\sqrt{|X_{i1n}|^2 + |X_{i2n}|^2 + |X_{i3n}|^2 + |X_{i4n}|^2}$$

and

$$X_{i1n} = 1, \quad X_{i2n} = (-1)^{i+1}\frac{S_n}{\sqrt{n+2}},$$

$$X_{i3n} = \frac{S_n^2 - (n+2)}{(\sqrt{n+2})\alpha}, \quad X_{i4n} = (-1)^{i+1}\frac{S_n(S_n^2 - (n+2) - \alpha^2)}{(\sqrt{n+1}\sqrt{n+2})\alpha}.$$

Here $S_n = A_n$ for odd $i$ and $S_n = B_n$ for even $i$. The corresponding Eigen value is

$$E_{1n}/\hbar = A_n, \qquad E_{2n}/\hbar = -A_n, \qquad E_{3n}/\hbar = B_n, \qquad E_{4n}/\hbar = -B_n,$$

where

$$A_n = \sqrt{W_n + V_n}\,/\,2, \quad B_n = \sqrt{W_n - V_n}\,/\,2$$

and

$$W_n = 4n + 6 + 2\alpha^2, \quad V_n = 2\sqrt{4(n+1)\alpha^2 + (\alpha^2+1)}, \quad \alpha = J/g.$$

To derive the full dynamics of our model one can consider also the basis states $|-,-,1\rangle, |+,-,0\rangle, |-,+,0\rangle$. In this basis the Eigen functions and Eigen values of the Hamiltonian (1) are

$$|\varphi_1\rangle = (\alpha^2/\Omega)[|-,-,1\rangle - (1/\alpha)\,|-,+,0\rangle], \qquad E_1 = 0;$$

$$|\varphi_2\rangle = (1/\sqrt{2})[(1/\Omega)\,|-,-,1\rangle + |+,-,0\rangle + (\alpha/\Omega)\,|-,+,0\rangle], \qquad E_2/\hbar = \Omega;$$

$$|\varphi_3\rangle = (1/\sqrt{2})[-(1/\Omega)\,|-,-,1\rangle + |+,-,0\rangle - (\alpha/\Omega)\,|-,+,0\rangle], \qquad E_2/\hbar = -\Omega,$$

where $\Omega = \sqrt{1+\alpha^2}$ .

At last, the Hamiltonian has one more Eigen function

$$\varphi_0 = |-,-,0\rangle.$$

This corresponds to energy $E_0 = 0$ .
Using the

$$|\Psi(t)\rangle = e^{-iHt/\hbar}\,|\Psi(0)\rangle = \sum_i C_i(0)e^{-iE_i t/\hbar}\,|\Phi_i\rangle,$$

where the coefficients $C_i(0)$ are determined by initial conditions, one can derive the time evolution of the initial states $|-,-,n\rangle$, $|+,-,n\rangle$, $|-,+,n\rangle$ and $|+,+,n\rangle$. As a result, collecting the gained expressions, we can find a temporary density matrix for an initial state (2). Taking a partial trace over the heat bath variable one can obtain that the reduced atomic density operator in the two-atom basis $|+,+\rangle$, $|+,-\rangle$, $|-,+\rangle$, $|-,-\rangle$ evolves to

$$\rho_A(t) = \begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13} & \rho_{14} \\ \rho_{12}^* & \rho_{22} & \rho_{23} & \rho_{24} \\ \rho_{13}^* & \rho_{23}^* & \rho_{33} & \rho_{34} \\ \rho_{14}^* & \rho_{24}^* & \rho_{34}^* & \rho_{44} \end{pmatrix}. \quad (3)$$

The elements of matrix (3) are not shown because they have too cumbersome form.

For two-qubit system described by the density operator (3), a measure of entanglement or negativity can be defined in terms of the negative eigenvalues $\mu_i^-$ of partial transpose of the reduced atomic density matrix $\rho_A^{T_1}$ [11], [12]. The partial transpose of the reduced atomic density matrix (3) can be written in the form

$$\rho_A^{T_1}(t) = \begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13}^* & \rho_{23}^* \\ \rho_{12}^* & \rho_{22} & \rho_{14}^* & \rho_{24}^* \\ \rho_{13} & \rho_{14} & \rho_{33} & \rho_{34} \\ \rho_{23} & \rho_{24} & \rho_{34}^* & \rho_{44} \end{pmatrix}.$$

and the negativity is

$$\varepsilon = -2\sum \mu_i^- . \quad (4)$$

When $\varepsilon = 0$ two qubits are separable and $\varepsilon > 0$ means the atom-atom entanglement. The case $\varepsilon = 1$ indicates maximum entanglement.

## 3. Results and discussion

Using the numerical of calculations of entanglement parameter (4) for initial atomic states (2) we have obtained, firstly, that entanglement can be induced by thermal field for all initial atomic state. The more significant result is that the atomic entanglement may be induced by thermal field when both the atoms are prepared in their excited states. By contrast, the atomic entanglement does not induced by thermal field when both atoms are trapped in cavity and prepared in their excited states [2-8]. Secondly, the maximum degree of entanglement is enhanced owing to the atomic coherence. With increasing of the mean photon number the value of atom–atom negativity decreases. But for coherent states this decreasing is much sharper. Such entanglement behaviour essentially differs from that for model when both atoms are trapped in cavity and interact with a thermal field [8]. In the last case the entanglement is greatly enhanced due to the initial atomic coherence for thermal cavity with large mean photon numbers. The increasing of the strength of the dipole-dipole interaction leads to increasing of the degree of entanglement. In Fig 1 we show time behaviour of the negativity for incoherent initial atomic state $|+,+\rangle$ (a) and coherent initial atomic state $|\Psi_1(0)\rangle = 1/\sqrt{2}(|+\rangle_1 + |-\rangle_1)$, $|\Psi_2(0)\rangle = 1/\sqrt{2}(|+\rangle_2 + |-\rangle_2)$ for the model with mean photon number $\bar{n} = 0.1$ and the strengh of the dipole-dipole interaction $\alpha = 0.5$ .

## 4. Conclusions

We have investigated the effect of the atomic coherence on the entanglement of two dipole coupled two-level atoms when only one atom is trapped in a lossless cavity and interacts with one-mode thermal field, and the other one can be spatially moved freely outside the cavity. We have shown that thermal field can produce atom-atom entanglement for all pure initial atomic states. The results also show that the atom-atom entanglement can be controlled by changing the system parameters, such as the amplitudes of the polarized atoms, the mean photon numbers of thermal field, and the strength of the dipole interaction (or distance between atoms).

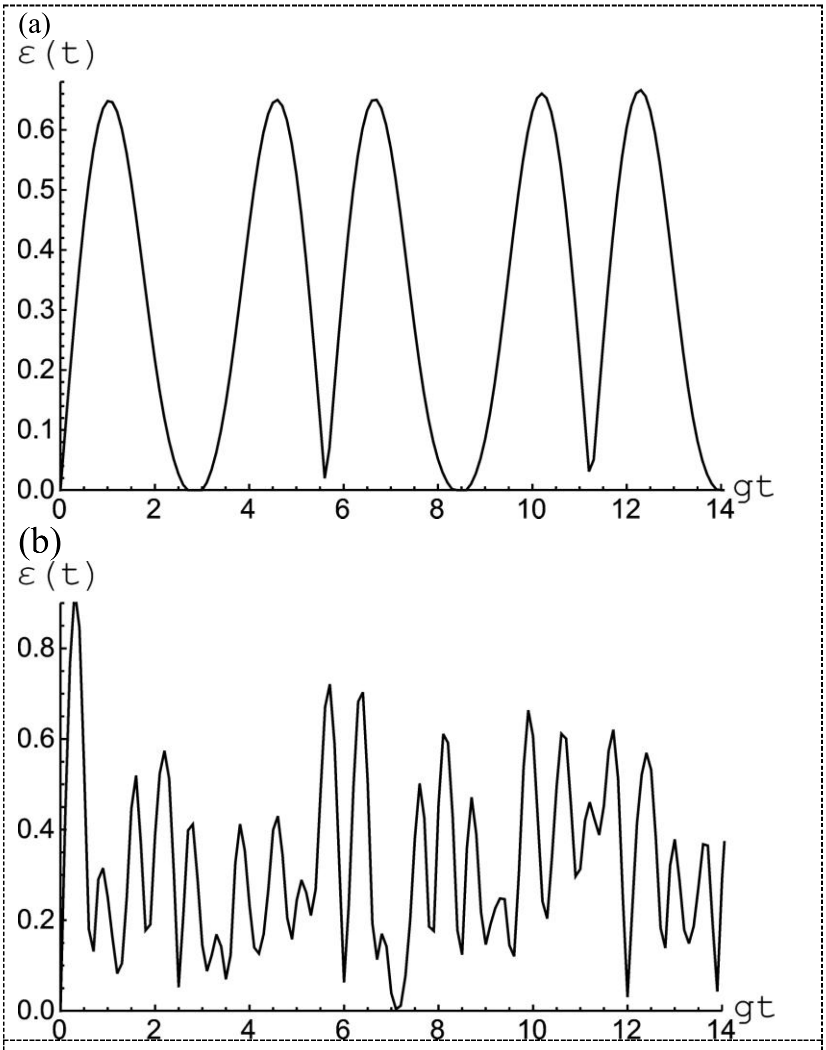


**Figure 1.** The negativity vs. gt for the incoherent $\theta_1 = \theta_1 = 0$ (a) and coherent $\theta_1 = \theta_1 = \pi / 4$ (b). The model parameters are $\bar{n} = 0.1$ and $\alpha = 0.5$.

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