Calculation of eigenfunctions of bounded waveguide with quadratic refractive index

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Abstract. In this work, the one-dimensional finite fractional Fourier transform has been considered in application to gradient optical waveguides. The eigenfunctions of it has been discussed. The relation between obtained functions, Hermite-Gaussian modes and spheroidal functions has been shown. The dependence of input domain width and number of nonzero eigenfunctions of transformation has been demonstrated. The functions which recover its form at given distance from the front plane have been calculated.

1. Introduction

Fractional Fourier transform (FrFT) is a set of linear transformations, which generalizes Fourier transform. The typical interpretation of Fourier transform is conversion of time domain of signal to its frequency domain.

The canonical FrFT was considered [1] as Fourier transform of α-order, where α is real value. We can define the FrFT as operation of frequency-time distribution rotation by some angle [2].

FrFT is used in differential equation solving, in quantum mechanics and quantum optics, in optical theory of diffraction, for description of optical systems and optical signal processing, including the applying of frequency filters, time filtration and multiplexing, as well as in pattern recognition, in wavelet-transformations, in operations with chirp-functions, in encryption, for neural networks creating and other applications. More detailed review of FrFT can be found in work [3] of T. Alieva et al.

One of application of FrFT is the description of laser beam propagation in gradient index media [4, 5]. In this work, we use one-dimensional FrFT for modeling of optical signal propagation in optical waveguide with parabolic dependence of the refractive index. In case of unlimited medium, an analytical solution for eigenfunctions problem of transformation is known. They are Hermite-Gaussian modes [1]. Taking bounding of gradient waveguide into account, we need to solve numerically the eigenvalues and eigenfunctions problem. In this work, we calculate eigenfunctions for various parameters of waveguide with quadratic refractive index.

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2. General theory

The passing of light beam through gradient index media with finite input domain (Figure 1) is a particular case of paraxial propagation through ABCD-system [6]. It can be described by finite fractional Fourier transform [1]:

$$F_{\alpha,l}(u)=\sqrt{\frac{k}{2\pi i a\sin\alpha}}\int_{-l}^{l} f(x)\exp\left[\frac{ik}{2a\sin\alpha}\left(\cos\alpha\, x^2-2xu+\cos\alpha\, u^2\right)\right]dx\,, \tag{1}$$

where $f(x)$ – input plane optical distribution, $F_{\alpha,l}(u)$ – distribution at a given distance z from if, k – wave number, a – the parameter of refractive index $n(x)=n_0\left(1-\left(x^2/2a^2\right)\right)$, $\alpha=z/a$, $2l$ – width of input domain.

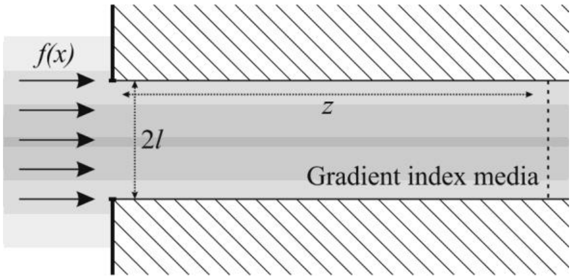

Figure 1. Scheme of optical system with bounded gradient waveguide.

The light beam propagation through the gradient index medium has property of periodicity. It is valid for both unlimited and limited (in input plane) FrFTs. The period of self-recovering is $z_T=2\pi a$. Moreover, the result of FrFT at a distance $z_{T/4}=\pi a/2$ equals to result distribution of typical Fourier transform, which is valid for lens system. The source signal “inverts” at a point $z_{T/2}=\pi a$. The applying of inverse Fourier transform to the source distribution equals to using FrFT at a distance $z_{3T/4}=3\pi a/2$.

If we consider the case of unlimited input plane domain, its eigenfunctions recovery at any distance (accurate to phase shift). Such functions are well-known as Hermite-Gaussian modes [7]:

$$h_n=\exp\left[-\frac{x^2}{2\sigma_0^2}\right]H_n\left(\frac{x}{\sigma_0}\right). \tag{2}$$

3. Simulation

The bounding of input domain of the system affects the eigenfunctions. If width l is sufficiently large that eigenfunction turns to zero on its borders, then its propagation will be without distortions and its form will be similar to Hermite-Gaussian mode. Otherwise, it will be eigenfunction only for current parameter α, wherein it was calculated. The examples of such functions propagation is shown in Figures 2 and 3. The parameters of computations are: $\lambda=1\mu\text{m}$, $a=25\lambda$, $\alpha=\pi/5$.

Besides eigenfunctions, we can consider the pair of sets of orthogonal “shifted” eigenfunctions $\left\{\varphi_n\left(x;\alpha,l\right)\right\}$ and $\left\{\chi_n\left(u;\alpha,l\right)\right\}$, which transfers to each other at distances $z_1=\alpha a$ and $z_2=(2\pi-\alpha)a$, correspondingly. The definitions of the functions are:

$$\varphi_n\left(x;\alpha,l\right)=\psi_n\left(x;\alpha,l\right)\exp\left\{-ik\,x^2\,\mathrm{ctg}\,\alpha/2a\right\}, \tag{3}$$

$$\chi_n\left(u;\alpha,l\right)=\psi_n\left(u;\alpha,l\right)\exp\left\{ik\,u^2\,\mathrm{ctg}\,\alpha/2a\right\}, \tag{4}$$

where $\psi_n\left(u;\alpha,l\right)$ – spheroidal functions [8]. The examples of its propagation through the system are shown in Figures 4 and 5.

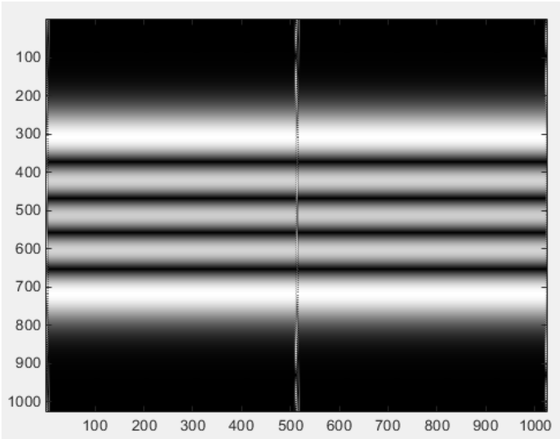

Figure 2. The propagation of eigenfunction: function turns to zero on the boundaries ($l=12\lambda$).

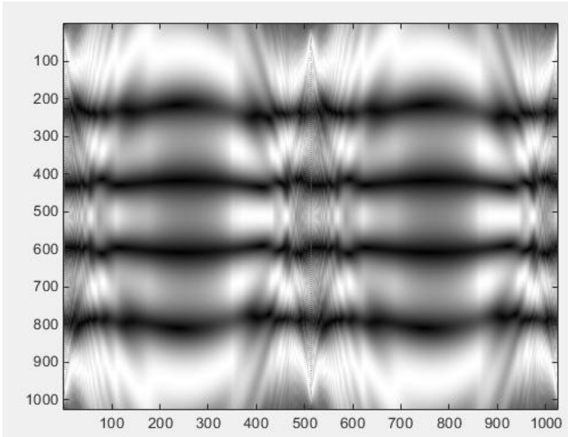

Figure 3. The propagation of eigenfunction: function doesn't turn to zero on the boundaries ($l=6\lambda$).

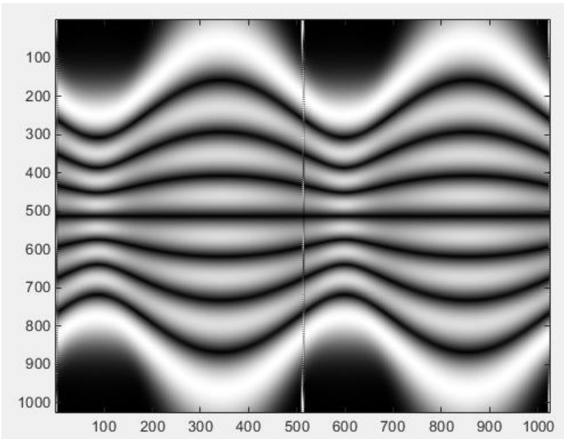

Figure 4. The propagation of shifted eigenfunction $\varphi_n\left(x;\alpha,l\right)$.

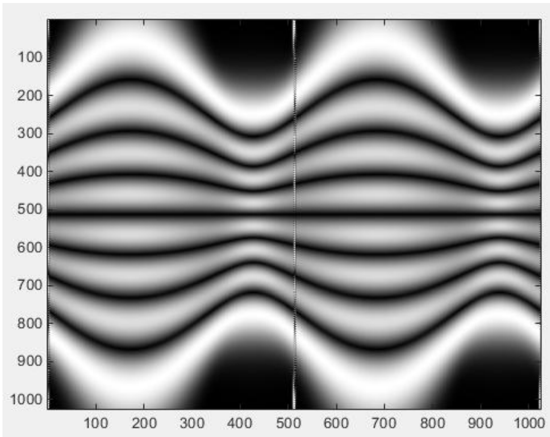

Figure 5. The propagation of shifted eigenfunction $\chi_n\left(u;\alpha,l\right)$.

The eigenvalues corresponding to transformations with different width of input domain is shown in Figures 6 and 7. The lesser input domain width is, the lesser is the number of significant eigenvalues. Therefore, the number of eigenmodes transferring through waveguide without energy lost is decreases.

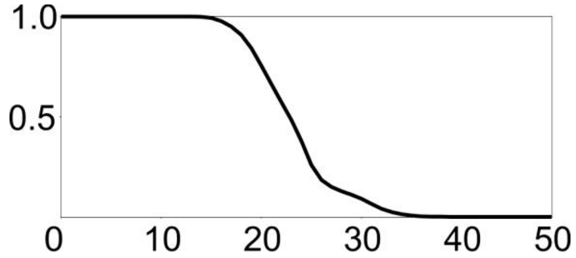

Figure 6. Moduli of eigenvalues, $l = 12\lambda$.

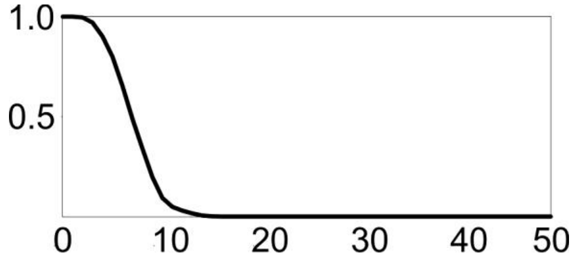

Figure 7. Moduli of eigenvalues, $l = 6\lambda$.

4. Conclusion

The one-dimensional FrFT, corresponding to gradient optical medium, is considered in the paper. We examined space-limited operator of propagation in gradient index medium, based on FrFT. The calculation of eigenfunctions at various parameters of optical waveguide with quadratic refractive index was performed.

We also considered "shifted" eigenfunctions corresponding to "input" and "output" of propagation operator.

The results of numerical simulation shown, that eigenfunctions with eigenvalues, which approximately equals to one, almost completely include in finite size of waveguide and are similar to Hermite-Gaussian modes and spheroidal functions.

Acknowledgments

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