

# Plane stress yield function described by 3rd-degree spline curve and its application

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**Abstract.** In this study, a plane stress yield function which is described by 3rd-degree spline curve is proposed. This yield function can predict a material anisotropy with flexibility and consider evolution of anisotropy in terms of both  $r$  values and stresses. As an application, hole expanding simulation results are shown to discuss accuracy of the proposed yield function.

## 1. Introduction

Precise prediction of a material anisotropy is an important factor of accurate sheet metal forming simulation. To describe a material anisotropy, many yield functions have been proposed. For example, Hill'48 [1] quadratic model is widely used because of its simplicity. Barlat *et al.* [2] introduced yield function Yld2000-2d, which can consider biaxial stress. Vegter *et al.* [3] proposed a plane stress yield function based on second order Bézier curve. Even though this yield function also requires some experiment data of biaxial stresses to determine anisotropic material parameters, flexible expression of the yield surface enables accurate simulation. These yield functions assume the initial anisotropy is kept during plastic deformation. However, it is known that some kinds of sheet metal show evolution of anisotropy. For example, Hu [4] provided an experiment data of an aluminum sheet, which show changes of normalized flow stress and  $r$  value with increasing plastic strain.

In this paper, a 3rd-degree spline based plane stress yield function is proposed. The idea of connecting yield locus by a kind of spline curves is identical to Vegter's work. On this proposed plane stress yield function, 3rd-order Bézier curves are employed for flexibility of yield locus expression. Moreover, by using plastic strain dependent material properties, the proposed yield function can consider anisotropic hardening.

## 2. Yield function description

In the two-dimensional principal stress space, a stress point is represented by the vector  $\{\sigma_1, \sigma_2\}^T$ . Let  $\theta$  ( $0 \leq \theta \leq \pi/2$ ) be the angle between the 1st principle axis and the rolling direction. In this study, only orthotropic materials are considered. Divide the angle from 0 to  $\pi/2$  into  $n$  sections, and set



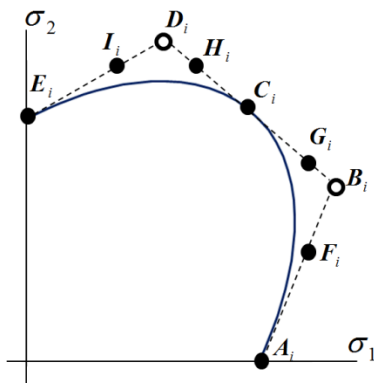
$0 \leq i \leq n$ . In the area  $\sigma_1 \geq 0$  and  $\sigma_2 \geq 0$ , a plane stress normalized yield locus is described by function  $R_i$ . If  $\sigma_1 \geq \sigma_2 \geq 0$  then yield locus is described by function  $R_i$ . If  $\sigma_1 \geq \sigma_2 \geq 0$  then

$$R_i(t) = (1-t)^3 A_i + 3(1-t)^2 t F_i + 3(1-t) t^2 G_i + t^3 C_i \quad (1)$$

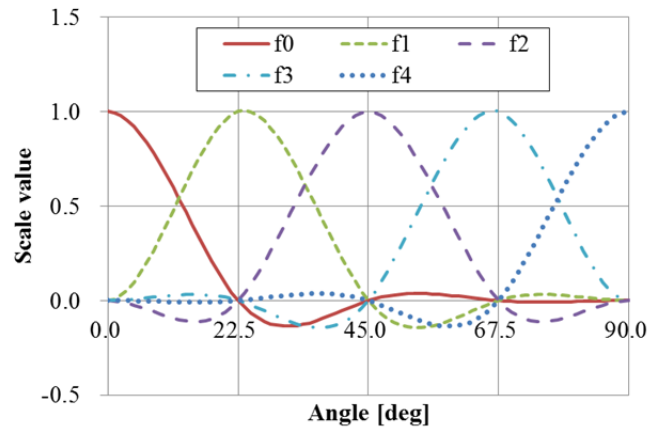
and if  $\sigma_2 > \sigma_1 \geq 0$  then

$$R_i(t) = (1-t)^3 C_i + 3(1-t)^2 t H_i + 3(1-t) t^2 I_i + t^3 E_i \quad (2)$$

where  $0 \leq t \leq 1$ . Note that  $A_i, C_i, E_i, F_i, G_i, H_i$  and  $I_i$  are vectors of control points of 3rd-order Bézier curves as shown in Fig. 1.  $B_i$  and  $D_i$  are vectors which provide the control points of Bézier curves. Point  $A_i$  implies uniaxial tension stress, and point  $E_i$  corresponds to uniaxial tension stress that is angled at  $\pi/2$  from the direction of  $A_i$ . Point  $C_i$  corresponds to equi-biaxial stress that is  $\sigma_1 = \sigma_2$ . The tangent at  $A_i$  corresponds to  $r$  value that is determined by uniaxial tension test, and the tangent at  $C_i$  corresponds to  $r$  value of equi-biaxial stress state that is  $r_b = d\varepsilon_2^p / d\varepsilon_1^p$ . Point  $B_i$  is defined as a point of intersection of two tangents. Point  $D_i$  is determined similarly. If  $\sigma_1 \geq \sigma_2$ , point  $F_i$  is the point of interior division ratio  $(1-p:p)$  of line  $A_i B_i$ , where  $p$  is a material parameter, and point  $G_i$  is the point of interior division ratio  $(1-p:p)$  of line  $C_i B_i$ . If  $\sigma_1 < \sigma_2$ , points  $H_i, I_i$  are determined similarly.



**Figure 1.** Yield locus and its control points



**Figure 2.** Spline functions to interpolate (Case  $n=4$ )

For planar anisotropic sheet metal, the yield locus and the reference strain ratio depend on the angle between the 1st principle axis and the rolling direction. The angular dependency of the reference points and strain ratio can be described by using cubic spline basis function  $f_i$ . If  $n=4$ , spline basis functions  $f_i$  are determined as in Fig. 2. By using spline function  $f_i$ , the function which describes yield locus is written as following. If  $\sigma_1 \geq \sigma_2 \geq 0$  then

$$R_i(\theta, t) = \sum_{i=0}^n f_i(\theta) \{ (1-t)^3 A_i + 3(1-t)^2 t F_i + 3(1-t) t^2 G_i + t^3 C_i \} \quad (3)$$

and if  $\sigma_2 > \sigma_1 \geq 0$  then

$$R_i(\theta, t) = \sum_{i=0}^n f_i(\theta) \{ (1-t)^3 C_i + 3(1-t)^2 t H_i + 3(1-t) t^2 I_i + t^3 E_i \} \quad (4)$$

The yield locus of uniaxial compression is assumed that it is symmetric with respect to the origin. To describe the evolution of anisotropy in terms of both stresses and  $r$  values, plastic strain dependency of the normalized yield locus needs to be considered. In this model, anisotropic hardening

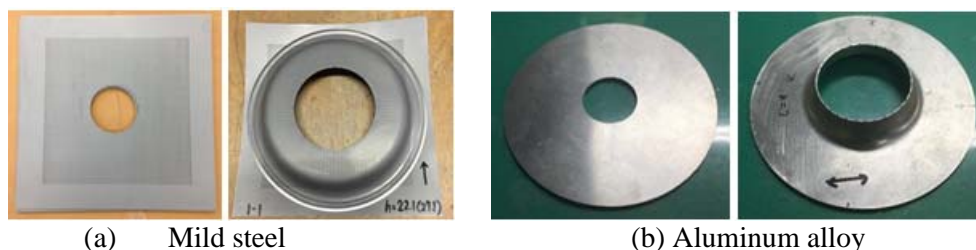
is described by considering plastic strain dependent control points which imply yield locus, tangents of the control points and parameter  $p$ .

### 3. Application

To demonstrate the applicability of the proposed yield function, hole expanding tests with a mild steel sheet and a 6000 series aluminum alloy sheet were investigated. The thickness of the mild steel sheet was 0.8mm and that of the aluminum alloy sheet was 1.0 mm. The specifications of hole expanding tests are summarized as in Table 1 and shape of blank sheets are shown in Fig.3. The mild steel specimen was 240mm  $\times$  240mm, and had the hole of 50mm diameter. Moreover, the diameter of the aluminum alloy blank was 100mm, and the initial hole diameter was 25mm. The sampling points of thickness were 1mm from the hole edge.

**Table 1** Hole expanding tool specifications

	Mild Steel	Aluminum alloy
Punch diameter (mm)	100.0	40.0
Punch porofile radius (mm)	15.0	6.0
Die opening diameter (mm)	130.0	43.0
Die profile radius (mm)	15.0	6.0
Blank holding force (kN)	100.0	60.0
Punch stroke (mm)	22.1	10.0



(a) Mild steel

(b) Aluminum alloy

**Figure.3** Pre-hole expanding and post-hole expanding blank sheet

To obtain the material characteristics, uniaxial tension tests were performed in five directions. The results are shown in Table 2 and Table 3. In hole expanding tests, the stress states on the hole edge is known to be similar to uniaxial stress states. Thus, in this study, the yield locus described by a yield function is determined by only the results of uniaxial tension tests. To determine the yield locus of proposed yield function, the angle from 0 to  $\pi/2$  is divided into four sections. Thus five directions of uniaxial stresses and  $r$  value were considered as material properties. The position of equi-biaxial stress points are estimated by Hill'48 yield function. Material parameter  $p$  was determined to be the yield surface of proposed model to fit to Hill'48's yield surface.

Experiment and simulation results are shown in Fig.4. For the implementation of the proposed yield function, the user material subroutine based on the LS-DYNA's explicit time integration was employed. In the isotropic hardening case, the shape of yield locus was determined by material properties with plastic strain=20%. In the case of anisotropic hardening, evolution of the anisotropy of stresses and  $r$  values were considered by plastic strain dependent sequential curve data.

As illustrated in Fig. 4 (a), the peak of thickness decreasing direction of the mild steel is around 30 degree. The proposed model describes more accurate prediction than Hill'48. In the comparison results of the aluminum alloy in Fig.3 (b), prediction of proposed model was closer to experiment data at 90 degree. Hence the superiority of proposed model, which is based on considering uniaxial stresses and  $r$  values in five directions, is confirmed. In terms of effect of considering anisotropic hardening,

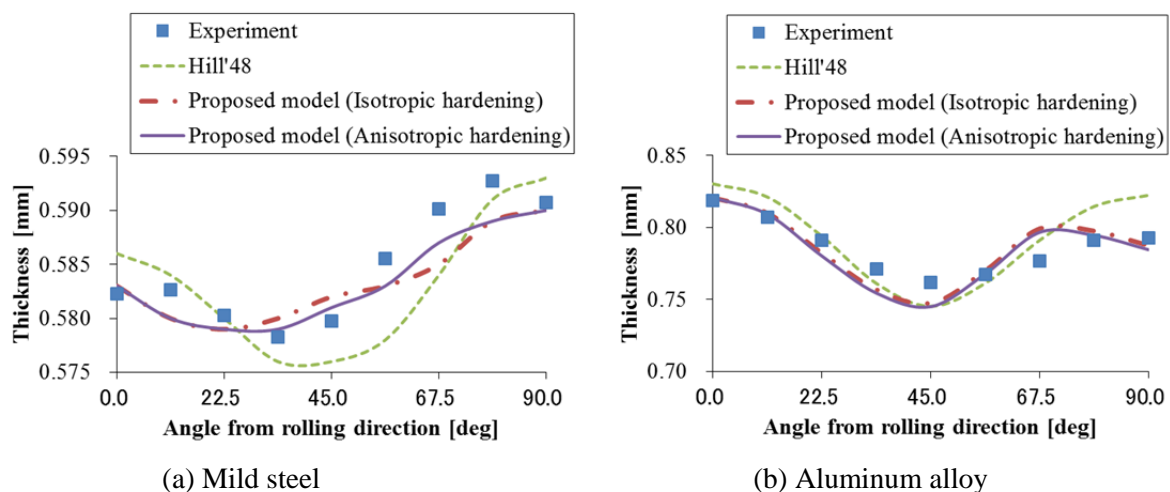
there were no major differences between the isotropic hardening and the anisotropic hardening in the results. Therefore, the effect of anisotropic hardening is minor on hole expanding tests which are mainly influenced by uniaxial stress state.

**Table 2** Material properties of the mild steel sheet

Angle (deg)	YS (MPa)	TS (MPa)	r value ( $\epsilon_p=1\%$ )	r value ( $\epsilon_p=20\%$ )
0.0	156.8	298.2	1.63	2.20
22.5	161.0	295.1	1.49	2.00
45.0	158.4	295.2	1.51	2.05
67.5	160.4	296.0	1.66	2.30
90.0	162.0	299.3	1.83	2.60

**Table 3** Material properties of the aluminum alloy sheet

Angle (deg)	YS (MPa)	TS (MPa)	r value ( $\epsilon_p=1\%$ )	r value ( $\epsilon_p=20\%$ )
0.0	98.1	247.3	0.72	0.78
22.5	108.8	248.2	0.42	0.51
45.0	82.0	246.4	0.21	0.30
67.5	83.1	238.0	0.37	0.48
90.0	92.6	246.2	0.42	0.65



**Figure 4.** Comparison of thickness along the edge of the hole after hole expansion

## References

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