

An isogeometric Reissner-Mindlin shell element for dynamic analysis considering geometric and material nonlinearities

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Abstract. The present approach deals with the dynamical analysis of thin structures using an isogeometric Reissner-Mindlin shell formulation. Here, a consistent and a lumped mass matrix are employed for the implicit time integration method. The formulation allows for large displacements and finite rotations. The Rodrigues formula, which incorporates the axial vector is used for the rotational description. It necessitates an interpolation of the director vector in the current configuration. Two concept for the interpolation of the director vector are presented. They are denoted as continuous interpolation method and discrete interpolation method. The shell formulation is based on the assumption of zero stress in thickness direction. In the present formulation an interface to 3D nonlinear material laws is used. It leads to an iterative procedure at each integration point. Here, a J2 plasticity material law is implemented. The suitability of the developed shell formulation for natural frequency analysis is demonstrated in numerical examples. Transient problems undergoing large deformations in combination with nonlinear material behavior are analyzed. The effectiveness, robustness and superior accuracy of the two interpolation methods of the shell director vector are investigated and are compared to numerical reference solutions.

1. Introduction

In this contribution an isogeometric Reissner-Mindlin shell formulation for transient problems including large deformations and nonlinear material behavior is presented. Isogeometric analysis for structural dynamics was proposed in [1]. It leads to an excellent performance in the optical branch, see [2]. In the frame of plate formulations a Kirchhoff formulation is presented in [3] and a Reissner-Mindlin formulation is derived in [4]. In the context of shells, isogeometric Kirchhoff-Love shell analysis is studied in [5]. The good performance was achieved at the cost of higher computational effort. Benson et al. [6, 7] proposed isogeometric shell formulations for transient problems using explicit time integration, which are also able to deal with large deformations and nonlinear material behavior. The present approach employs an implicit time integration. The purpose of the present approach is to analyze the impact of the director interpolation. The continuous interpolation method of the shell director vector, proposed in [8], is compared to a discrete interpolation method as used e.g. in [6, 9, 10].



2. Governing equations for the Reissner-Mindlin shell

Let \mathbf{x} be a point on the shell mid surface of the reference configuration and $\bar{\mathbf{x}}$ be a point of the mid surface of the current configuration. The shell continuum is defined by

$$\begin{aligned}\Phi(\xi^1, \xi^2, \xi^3) &= \mathbf{x}(\xi^1, \xi^2) + \xi^3 \mathbf{d}(\xi^1, \xi^2) \\ \bar{\Phi}(\xi^1, \xi^2, \xi^3) &= \bar{\mathbf{x}}(\xi^1, \xi^2) + \xi^3 \bar{\mathbf{d}}(\xi^1, \xi^2)\end{aligned}\quad (1)$$

Here, the quantities with an overset bar denote the values in the current configuration. Φ represents an arbitrary point in the shell and \mathbf{d} describes the director vector, see Fig.1. ξ^1, ξ^2 are the in-plane coordinates and ξ^3 denotes the thickness coordinate with $-h/2 \leq \xi^3 \leq h/2$, where h is the thickness of the shell. The membrane strains $\varepsilon_{\alpha\beta}$, the change of curvature $\kappa_{\alpha\beta}$

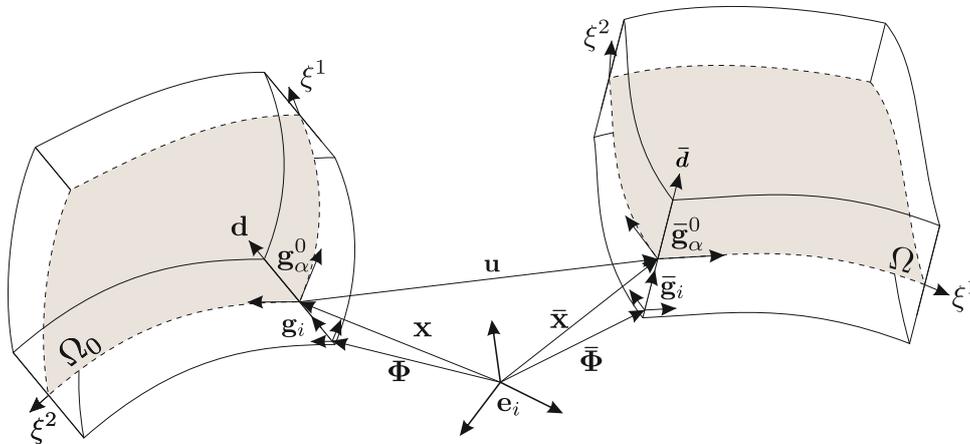


Figure 1. The reference- and current configuration of the shell continuum

and the transversal shear strains γ_α read

$$\begin{aligned}\varepsilon_{\alpha\beta} &= \frac{1}{2} (\bar{\mathbf{x}}_{,\alpha} \cdot \bar{\mathbf{x}}_{,\beta} - \mathbf{x}_{,\alpha} \cdot \mathbf{x}_{,\beta}) \\ \kappa_{\alpha\beta} &= \frac{1}{2} (\bar{\mathbf{x}}_{,\alpha} \cdot \bar{\mathbf{d}}_{,\beta} - \mathbf{x}_{,\alpha} \cdot \mathbf{d}_{,\beta}) \\ \gamma_\alpha &= \bar{\mathbf{x}}_{,\alpha} \cdot \bar{\mathbf{d}} - \mathbf{x}_{,\alpha} \cdot \mathbf{d}.\end{aligned}\quad (2)$$

The Greek letters indicate the local coordinate direction, it holds $\alpha, \beta \in \{1, 2\}$. The vector containing all shell strains reads $\boldsymbol{\varepsilon}^T = [\varepsilon_{11}, \varepsilon_{22}, 2\varepsilon_{12}, \kappa_{11}, \kappa_{22}, 2\kappa_{12}, \gamma_1, \gamma_2]$.

3. The weak form

Let $\delta \mathbf{v} \in H^1$ be an admissible test function and \mathbf{v} the displacement vector, the weak form of the equation of motion reads

$$G(\mathbf{v}, \delta \mathbf{v}) = \int_{B_0} \delta \mathbf{v}^T \rho_0 \ddot{\mathbf{v}} dV + \int_{\Omega_0} \delta \boldsymbol{\varepsilon}^T \boldsymbol{\sigma} dA - \int_{\Omega_0} \delta \mathbf{v}^T \mathbf{p} dA - \int_{\Gamma_\sigma} \delta \mathbf{v}^T \mathbf{f} ds = 0. \quad (3)$$

Here, $\boldsymbol{\sigma}$ represents the stress resultants which are work conjugate to the shell strains. ρ_0 is the initial mass density and $\ddot{\mathbf{v}}$ the acceleration. The surface load \mathbf{p} is defined per area reference surface and the traction \mathbf{f} is given on the boundary.

4. Interpolation methods of the director vector

4.1. Discrete interpolation method - Rotation before interpolation

Within the discrete interpolation method (DIM), see e.g. [11], the nodal basis systems in the current configuration

$$\bar{\mathbf{a}}_{iI} = \mathbf{R}\mathbf{a}_{iI} \quad (4)$$

are calculated by the rotation of each nodal basis system in the reference configuration at each control point I . The current director vector $\bar{\mathbf{d}}_I$ in each control point I equals $\bar{\mathbf{a}}_{3I}$. This *discrete* rotation of nodal basis systems is eponymous for the interpolation method at hand. The rotational matrix

$$\mathbf{R} = \mathbf{1} + \frac{\sin \omega}{\omega} \boldsymbol{\Omega} + \frac{1 - \cos \omega}{\omega^2} \boldsymbol{\Omega}^2$$

$$\text{with } \boldsymbol{\Omega} = \varepsilon_{ijk} \omega_i = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \quad \text{and } \omega = |\boldsymbol{\omega}| \quad (5)$$

is calculated via the Rodrigues formula, where ε_{ijk} is the Levi-Civita tensor. Nodal values ω_I have to be inserted into Eq. (5). These approximation of teh director vector through the element area is given as

$$\bar{\mathbf{d}}^h = \sum_{I=1}^{n_{en}} N_I \bar{\mathbf{d}}_I \quad \bar{\mathbf{d}}_{,\alpha}^h = \sum_{I=1}^{n_{en}} N_{I,\alpha} \bar{\mathbf{d}}_I. \quad (6)$$

4.2. Continuous interpolation method - Interpolation after Rotation

The idea of the continuous interpolation method (CIM), which is described in [11], is to interpolate the rotational parameter $\boldsymbol{\omega}$ instead of the director vector itself. Instead of the interpolation of discretely rotated values as is done in Eq. (6), here the current director vector

$$\bar{\mathbf{d}}^h = \mathbf{R}\mathbf{d}^h = \mathbf{R} \sum_{I=1}^{n_{en}} N_I \mathbf{d}_I \quad (7)$$

is computed by first interpolating the director vector of the reference configuration and then applying the rotation \mathbf{R} in each integration point. Thus, the condition $|\bar{\mathbf{d}}| = |\mathbf{d}|$ holds exactly. Within this approach interpolated values ω^h and $\bar{\mathbf{d}}^h$ have to be used for the computation of \mathbf{R} .

5. Numerical Example - Impulsively loaded square plate

The simply supported square plate is taken from the *shell obstacle course* proposed in [12]. The material properties, loadings and dimensions are depicted in Fig. 2. The load is applied impulsively within the first time step $t_0 = 0$. Due to double-symmetry only one quarter of the system is modeled. The reference solution of the maximum deflection of the midpoint has been calculated with 25x25 elements with quartic basis functions with the CIM concept. The purpose of thi example is to investigate the effect of the different interpolation methods for the director vector, see Fig. 3. With the DIM concept, NUBRS with $p = 6$ lead to a decreasing quality of the solution (cf. Fig. 3b). Here, two different aspects are involved. The approximation of the deformed geometry increases with higher order basis functions. However, also the area of control points that influence the director vector increases. If the director vector is rotated before it is interpolated on curved geometries, the length of the director vector will in general be smaller than one and this effect increases with an increasing number of involved control points. This effect decreases with mesh refinement, because the distance and therefore the difference of the rotation angle of the involved control points becomes smaller. Note that the continuous and discrete interpolation method of the shell director vector are identical for configuration without curvature.

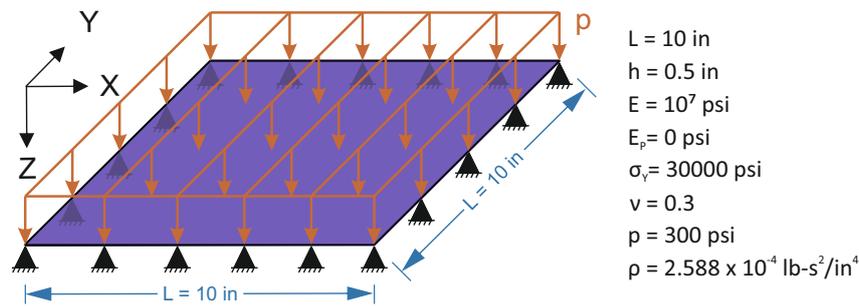


Figure 2. Impulsively loaded square plate

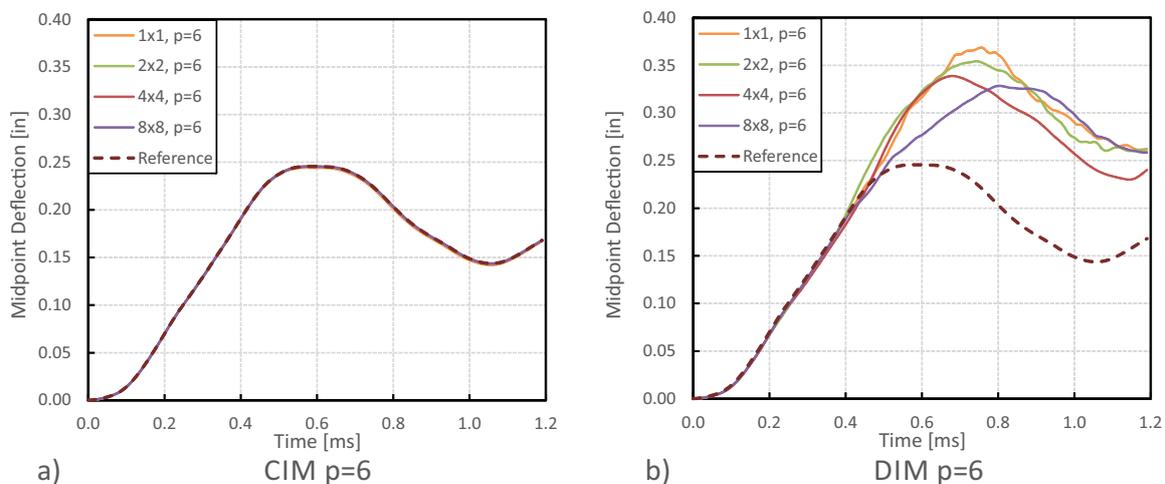


Figure 3. Comparison between interpolation methods of the shell director vector: a) Continuous Interpolation Method (CIM) for h- and k-refinement. For $p = 6$ the results for all meshes are almost indistinguishable from the reference solution; b) Discrete Interpolation Method (DIM) for h- and k-refinement.

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