On the modelling of strength differential and anisotropy exhibited by titanium

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Abstract. Modelling the response of titanium alloys under plastic deformation is challenging. In addition to strong anisotropy, these materials exhibit tension-compression asymmetry as well as anisotropic hardening even under monotonous loading conditions. The present contribution aims to propose a new modelling approach for titanium materials, by a homogeneous introduction of asymmetry into existing symmetric yield functions. Furthermore, anisotropic hardening effects are tracked describing the strain dependant yield surface evolution. The results are validated using the earing profile of deep drawn cups.

1. Introduction

Given its excellent strength to weight ratio as well as its corrosion resistance, commercially pure titanium is often the material of choice, especially in aerospace and medical industries. The manufacturing of titanium parts using forming operations has, significant advantages in terms of material waste and speed over alternative methods such as machining. Forming of titanium is however far from being a trivial task. Apart from strong anisotropy, titanium alloys exhibit pronounced strength differential in tension and compression. Furthermore, given the directional dependency of twinning formation, strong anisotropic hardening is observed.

Different yield locus models have been proposed in the recent literature in order to properly capture asymmetric yielding. The CPB06 model, proposed by Cazacu et al. [1], features a homogeneous combination of the principal values of the linearly transformed stress deviator with its scaled absolute value, introducing thus an asymmetry in the positive and negative stress values. This idea has been later extended to feature more linear transformations by Plunkett et al.[2]. Nixon et al. proposed in 2010 a similar model featuring also the third invariant of the stress tensor [3]. All these models are able of capturing the yield stress asymmetry as well as R-values in tension with varying quality.

The present contribution aims to propose a generic method to construct homogeneous asymmetric yield loci based on well-established symmetric yield locus formulations. The approach is based on the recently proposed Homogeneous Anisotropic Hardening (HAH) model [1], which is primarily designed to capture Bauschinger effect. The basic framework of the HAH function has been extended to define asymmetric distortions in a finite number of predefined loading directions. The results are compared to the CPB06 model based on a cup drawing test in terms of force-displacement behavior and earing profile.

2. Proposed Model

The main form of the proposed yield function reads:

$$\Phi^q = \phi^q + \sum_{k}^{n} f_k^+ \big| \mathbf{d}_k : \mathbf{s} - |\mathbf{d}_k : \mathbf{s}| \big|^q + f_k^- \big| -\mathbf{d}_k : \mathbf{s} - |\mathbf{d}_k : \mathbf{s}| \big|^q$$

where ϕ is any homogeneous symmetric yield function. The summation over k applies asymmetric distortions to the yield locus in n different deviatoric stress directions ($\mathbf{d}_k$). The parameters f_k^+ and f_k^- must be identified together with the parameters of ϕ. The exponent q is used to blend the distortional component with the base function preserving homogeneity.

3. Material Characterization

To fully characterize the material properties standard tensile tests as well as compression tests in rolling (RD) and transverse (TD) directions have been carried out. The compression experiments have been conducted in a dilatometer using prismatic samples of quadratic base (3mm) and 5mm height. Both tensile and compression tests were conducted in room temperature and with quasi-static strain-rates.
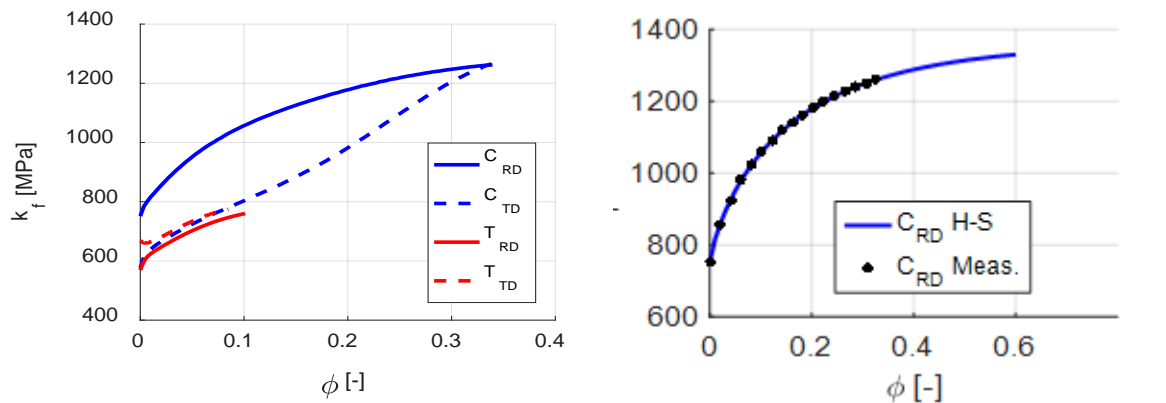

Figure 1. Flow curves in tension (T) and in compression (C) respectively in rolling (RD) and transverse (TD) directions (left). Approximation of the compression curve in rolling direction with the H-S model (right)

Figure 1 depicts the measured flow curves. Significant asymmetry can be observed in RD even for the initial yield stress, with a slightly divergent trend with increasing strain. In TD the response is nearly symmetric except for the first few percent strain. The compression test in transverse direction shows an atypical s-like hardening behavior. This is considered to be caused by the twinning of the hexagonal lattice, which hinder dislocation movement and thus increase the macroscopic yield stress.

3.1. Hardening model
The Hockett-Sherby function has been used to capture the hardening behavior:

$$k_f = B - (B - A)\exp(-m\phi^p)$$

where k_f represents flow stress, φ plastic strain, A the initial flow stress, B the saturated flow stress and m and p are hardening parameters. The function is able of accurately fitting the experimental data as depicted in Figure 1 (right).

3.2. Identification of the Yield Function

The yield function proposed in Section 2 has been identified to capture the experimental behaviour delineated in Section 3. The yld2004-18p model [1] has been used as the symmetric base function, which has been distorted in the following three deviatoric stress directions to enable asymmetry:

$$\mathbf{d_1} = \begin{bmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{bmatrix}, \mathbf{d_2} = \begin{bmatrix} -1/3 & 0 & 0 \\ 0 & 2/3 & 0 \\ 0 & 0 & -1/3 \end{bmatrix}, \mathbf{d_3} = \begin{bmatrix} 1/3 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & -2/3 \end{bmatrix}$$

The first two operate respectively in RD and TD whereas the third component acts over the equibiaxial stress direction. The identified yield locus, in comparison to the CPB06 yield locus can be seen in Figure 2 (left), the corresponding Lankford coefficients are depicted in Figure 2 (right). The functions have been identified by minimizing a cost-function related to yield stresses and Lankford coefficients in both RD and TD. Clearly the MRH approach provides more flexibility than required to exactly match these values. The following additional heuristic constraints have been used:

- The equibiaxial yield stress in both tension and compression has been considered to the corresponding stress in TD
- R-values in compression are assumed to be equal to the ones in tension
- R-value in 70 degrees has been assumed to be equal to the average of 45 and 90 degrees (to avoid oscillation in between

Isotropic hardening model was used to capture the evolution of the yield locus, the compression flow-curve in RD has been chosen as reference for all considered yield loci as higher equivalent plastic strain values have been reached in this case.
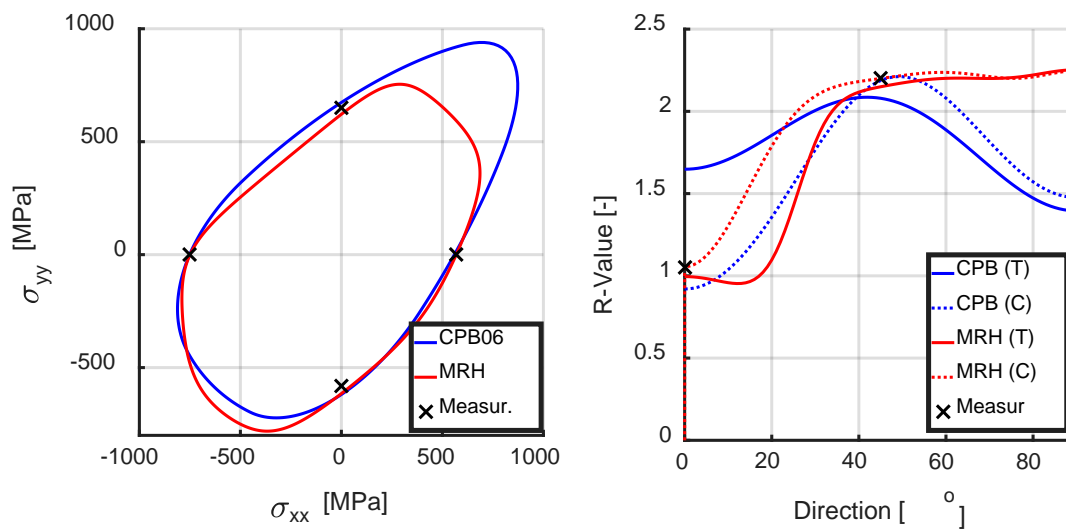

Figure 2. Radial distance of the flange edge of the deep drawn cup (left). Force-Displacement diagram (right)

4. Cup Drawing Test

The quality of the proposed model has been tested based on a cup drawing experiment. The circular blank of 110mm diameter and 0.4mm thickness has been drawn to a depth of 30mm using punch with

diameter 50 mm. The blank holder force has been set to 30kN. A corresponding FE-Model has been generated in LS-Dyna featuring a quarter of the blank to exploit symmetry. The blank has been modelled with 3 layers of brick elements with selectively reduced integration. The mesh has been chosen to have a size of about 0.5mm. The Coulomb friction coefficient between the blank and the rigid tools was assumed to be 0.05, given the fact that the blank has been well lubricated.

4.1. Results

The results obtained using the different modelling approaches are depicted in Figure 3. It is seen that, as far as earing is concerned, the proposed model is well able of capturing the shape of the flange edge with especially accurate results at RD and TD. The prediction in 45 degrees is equivalent to that of Hill model. The CPB06 model underestimates draw-in in RD and overestimates it in TD. This is mostly due to the fact that CPB06 cannot capture R-values in tension and compression equally well. In fact looking at Figure 3 (right) it is observed that both MRH and CPB models more accurately capture force-displacement, given their ability to model asymmetry, whereas the Hill model overestimates the force.
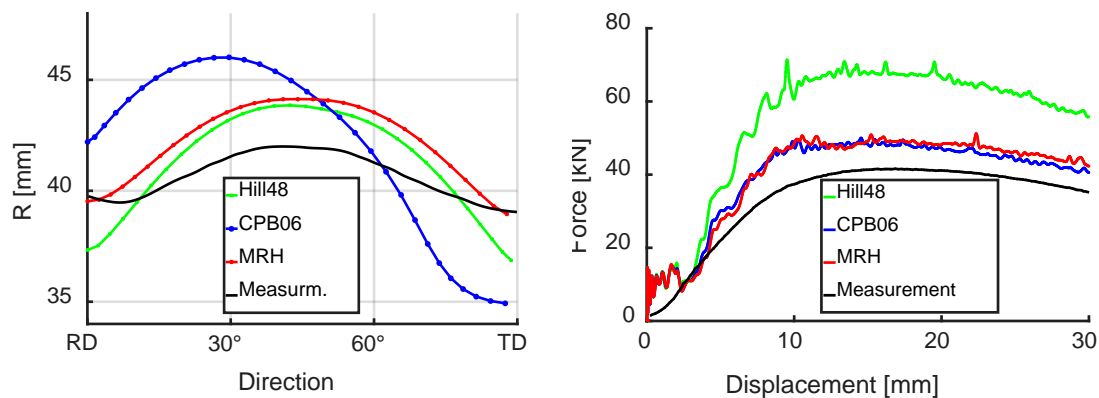

Figure 3. Radial distance of the flange edge of the deep drawn cup (left). Force-Displacement diagram (right)

5. Conclusions

A new approach has been proposed for the modelling of both asymmetry and anisotropy observed in commercially pure titanium. The model is designed to construct asymmetric functions based on well-established symmetric expressions, thus providing a generic framework in this sense. The results based on cup drawing experiments show that the proposed approach can successfully model both earing and force-displacement behaviour accurately.

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