The study of undulator radiation of transversal oscillator

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Abstract. We study the radiation intensity for a charged transversal oscillator moving around a dielectric cylinder. Similar to the case of coaxial circular motion, under certain conditions for the parameters of the trajectory and dielectric cylinder, strong narrow peaks appear in the angular distribution of the radiation intensity in the exterior medium for a given harmonic. Comparing to the case of a uniform coaxial circular motion, for a transversal oscillator new peaks may appear. The oscillations serve as a mechanism for the control of the spectral-angular characteristics of the radiation intensity.

1. Introduction

The radiation processes in medium from various types of sources have been widely discussed in the literature (see, for example, [1]-[4]). The presence of medium can essentially change the radiation features and gives rise to new types of phenomena. Well-known examples are Cherenkov, transition and diffraction radiations. Moreover, the operation of a number of devices assigned to production of electromagnetic radiation is based on the interaction of high-energy particles with materials.

By taking into account extensive applications of synchrotron radiation in a wide variety of experiments and in many disciplines, it is of interest to investigate the influence of a medium on the spectral and angular distributions of the synchrotron emission. This study is also important with respect to some astrophysical problems. The synchrotron radiation from a charged particle circulating in a homogeneous medium was considered in [5]-[13], where it has been shown that the interference between the synchrotron and Cherenkov radiations leads to remarkable effects. New interesting features arise in inhomogeneous media. In particular, the investigation of the radiation from a charged particle circulating around or inside a dielectric cylinder immersed in a homogeneous medium has shown [14]-[18] that, under the Cherenkov condition for the material of the cylinder and for the velocity of the particle image on the cylinder surface, there are narrow peaks in the angular distribution of the number of quanta emitted into the exterior space. For some values of the parameters the density of the number of quanta in these peaks exceeds the corresponding quantity for the radiation in vacuum by several orders. The electromagnetic fields and the radiation intensity for a charged particle moving along an arbitrary closed orbit around a dielectric cylinder immersed into a homogeneous medium have been investigated in [19].

In the present paper, we aim to study the sensitivity of the peaks with respect to distortions of the trajectory from circular one and possible new features in the radiation process. As an

example of a charge motion, we consider transversal oscillations with respect to mean circular motion. The paper is organized as follows. In the next section we describe the charge motion under consideration and present the expression for the spectral-angular density of the radiation intensity. The features of the radiation and some numerical results are discussed in section 3. The section 4 concludes the main results.

2. Radiation intensity from a transversal oscillator

Consider a point charge q moving around a dielectric cylinder with radius ρ_c and with the dielectric permittivity ε_0. We assume that the cylinder is immersed in a homogeneous medium with the dielectric permittivity ε_1 (magnetic permeabilities for both the cylinder and surrounding medium will be taken to be unit). In cylindrical coordinates (ρ, ϕ, z), with the z-axis along the cylinder axis, the trajectory of the charge is given by the functions $(\rho_e(t), \phi_e(t), z_e(t))$, $\rho_e(t) > \rho_c$. We will assume that along the radial direction the charge makes an oscillatory motion around a circular trajectory with the radius $\rho_0 > \rho_c$ described by

$$\rho_e(t) = \rho_0 \left[1 + a \cos(n_0 \omega_0 t)\right], \tag{1}$$

where n_0 is a positive integer and a is the relative amplitude of the oscillations. Here $\omega_0 = 2\pi/T$ with T being the period of the transversal motion. Hence, n_0 gives the number of transversal oscillations of the charge during one period of the rotation. From the condition $\rho_e(t) > \rho_c$ it follows that $a < 1 - \rho_c/\rho_0$. For the motion along the cylinder axis z we will take the uniform one with the constant velocity $v_\parallel$: $z_e(t) = v_\parallel t$.

In order to fix the function $\phi_e(t)$ for the azimuthal motion, we consider a special case with the constant energy of the charge: $\mathcal{E} = \text{const}$. This is the case when the charge moves under a static magnetic field only. For the components of the charge velocity along the radial and azimuthal directions one has $v_\rho = d\rho_e(t)/dt$, $v_\phi = \rho_e(t) d\phi_e(t)/dt$. The function $\phi_e(t)$ is found from the condition $v^2 = v_\perp^2 + v_\parallel^2 = \text{const}$, where $v_\perp = \sqrt{v_\rho^2 + v_\phi^2}$ is the velocity of the transversal (with respect to the cylinder axis) motion. After the integration, this gives

$$\phi_e(t) = \frac{1}{\rho_0} \int_0^t dt \frac{\sqrt{v_\perp^2 - a^2 n_0^2 v_{\perp 0}^2 \sin^2(n_0 \omega_0 t)}}{1 + a \cos(n_0 \omega_0 t)}, \tag{2}$$

where $v_{\perp 0} = \omega_0 \rho_0$. Note that $v_{\perp 0}$ is the velocity of the motion along the circle with radius ρ_0 having the same period T. In particular, we can take $v_\perp = v_{\perp 0}$. In this case, we should additionally require that $a n_0 < 1$ and, in the limit $a \to 0$, the motion under consideration reduces to a uniform circular motion with the period T. For the components of the velocity one finds

$$v_\rho = -n_0 a v_{\perp 0} \sin(n_0 \omega_0 t), \quad v_\phi = \sqrt{v_\perp^2 - v_\rho^2}. \tag{3}$$

The transversal oscillations can be generated by using magnetic structures similar to those employed in planar undulators (see, for example, [20]).

In what follows, we will denote by θ the angle between the wave vector of the radiated photon and the cylinder axis. The frequency of the radiation, $|\omega_n|$, on a given harmonic $n = 1, 2, \ldots$, is given by the expression

$$\omega_n = \frac{n \omega_0}{1 - \beta_{1\parallel} \cos \theta} \tag{4}$$

with the notation $\beta_{1\parallel} = v_\parallel \sqrt{\varepsilon_1}/c$. For the general case of the charge trajectory $(\rho_e(t), \phi_e(t), v_\parallel t)$, at large distances from the cylinder, the angular density of the radiation intensity on the

harmonic n, averaged over the period T, is presented as [19]

$$\frac{dI_n}{d\Omega} = \frac{q^2}{4\pi^3 c} \frac{n^2\omega_0^2\sqrt{\varepsilon_1}}{\left|1 - \beta_{1\|}\cos\theta\right|^3} \sum_{m=-\infty}^{+\infty} \left[\left|\sum_{p=\pm 1} pD_{m,n}^{(p)}\right|^2 + \left|\sum_{p=\pm 1} D_{m,n}^{(p)}\right|^2 \cos^2\theta \right], \qquad (5)$$

where $d\Omega = \sin\theta d\theta d\varphi$ is the solid angle element,

$$
\begin{aligned}
D_{m,n}^{(p)} &= \frac{\pi}{2ic}\left[J_{m+p,m}(n,\lambda_1) - H_{m+p,m}(n,\lambda_1)\frac{W(J_{m+p}, J_{m+p})}{W(J_{m+p}, H_{m+p})} \right] \\
&+ \frac{i\pi\lambda_1}{2ck_z}\left[J_{m,m,z}(n,\lambda_1) - H_{m,m,z}(n,\lambda_1)\frac{W(J_m, J_m)}{W(J_m, H_m)} \right] \\
&+ \frac{pJ_m(\lambda_0\rho_c)J_{m+p}(\lambda_0\rho_c)}{c\rho_c\alpha_m W(J_{m+p}, H_{m+p})}\left[k_z\frac{H_{m,m,z}(n,\lambda_1)}{W(J_m, H_m)} + \frac{\lambda_0}{2}\sum_{l=\pm 1}\frac{H_{m+l,m}(n,\lambda_1)}{W(J_{m+l}, H_{m+l})} \right], \quad (6)
\end{aligned}
$$

and

$$\lambda_0 = \frac{\omega_n}{c}\sqrt{\varepsilon_0 - \varepsilon_1\cos^2\theta}, \;\; \lambda_1 = \frac{\omega_n}{c}\sqrt{\varepsilon_1}\sin\theta, \;\; k_z = \frac{\omega_n}{c}\sqrt{\varepsilon_1}\cos\theta. \qquad (7)$$

Here and below, $J_m(x)$ and $H_m(x)$ are the Bessel and Hankel (of the first kind) functions, and

$$W(J_{m+p}, F_{m+p}) = J_m(\lambda_0\rho_1)\frac{\partial F_m(\lambda_1\rho_1)}{\partial\rho_1} - F_m(\lambda_1\rho_1)\frac{\partial J_m(\lambda_0\rho_1)}{\partial\rho_1}, \qquad (8)$$

where $F = J$ and $F = H$. Other notations are defined by the relations

$$
\begin{aligned}
F_{m',m,l}(n,\lambda_1) &= \frac{1}{T}\int_0^T dt\, v_l(t)F_{m'}(\lambda_1\rho_e(t))e^{-im\phi_e(t)+in\omega_0 t}, \\
F_{m+p,m}(n,\lambda_1) &= F_{m+p,m,\phi}(n,\lambda_1) - ipF_{m+p,m,\rho}(n,\lambda_1), \qquad\qquad (9)
\end{aligned}
$$

where $l = \rho, \phi, z$ and

$$\alpha_m = \frac{\varepsilon_0}{\varepsilon_1 - \varepsilon_0} - \frac{\lambda_0}{2}J_m(\lambda_0\rho_1)\sum_{l=\pm 1}\frac{lH_{m+l}(\lambda_1\rho_1)}{W(J_{m+l}, H_{m+l})}. \qquad (10)$$

The equation $\alpha_m = 0$ determines the eigenmodes of the dielectric cylinder for the electromagnetic field. The information on the charge motion is encoded in the functions $F_{m',m,l}(n,\lambda_1)$ with $m' = m, m \pm 1$.

3. Features of the radiation
First let us consider the special case of the charge motion along a circular helix. In this case one has $v_l(t) = \text{const}$, $\rho_e(t) = \rho_0$, $\phi_e(t) = \omega_0 t$ and, hence, $F_{m',m,l}(n,\lambda_1) = v_l F_{m'}(\lambda_1\rho_0)\delta_{mn}$. The only nonzero contribution to the series over m in (5) comes from the term with $m = n$ and we obtain the result of [17] (for the radiation emitted by a charged particle moving along a circular helical orbit inside a dielectric cylinder see [16]). By using Debye's asymptotic expansions for the Bessel and Neumann functions with large values of the order, in [17] it has been shown that, under the condition $|\lambda_1|\rho_c < n$, at points where the real part of the function α_n, given by formula (10), is equal to zero, the contribution of the imaginary part of this function into the coefficients $D_{n,n}^{(p)}$ in the expression of the radiation intensity can be exponentially large for large values n. This gives rise to strong narrow peaks in the angular distribution for the radiation intensity at a given harmonic n. The condition for the real part of the function α_n to be zero is obtained

from the equation determining the eigenmodes for the dielectric cylinder by the replacement $H_n \to Y_n$, where $Y_n(x)$ is the Neumann function. This equation has no solutions for $\lambda_0^2 < 0$, which is possible only for $\varepsilon_0 < \varepsilon_1$. Hence, the peaks do not appear for the case $\lambda_0^2 < 0$ which corresponds to the angular region $\cos^2\theta > \varepsilon_0/\varepsilon_1$. As necessary conditions for the presence of the strong narrow peaks in the angular distribution for the radiation intensity one has $\varepsilon_0 > \varepsilon_1$, $\widetilde{v}\sqrt{\varepsilon_0}/c > 1$, were $\widetilde{v} = \sqrt{v_\parallel^2 + \omega_0^2\rho_c^2}$ is the velocity of the charge image on the cylinder surface. The second condition is the Cherenkov condition for the velocity of the charge image on the cylinder surface and dielectric permittivity of the cylinder.

In figure 1, by the full curves we have plotted the angular density of the number of the radiated quanta per period T,

$$\frac{dN_n}{d\Omega} = \frac{T}{\hbar|\omega_n|}\frac{dI_n}{d\Omega}, \tag{11}$$

as a function of the angle θ, in the case of a circular motion along the orbit with $\rho_c/\rho_0 = 0.7$ around a dielectric cylinder with permittivity $\varepsilon_0/\varepsilon_1 = 3.75$. The numbers near the curves correspond to the values of the harmonic n ($n = 6, 8$ for figure 1(a) and $n = 10, 12$ for figure 1(b)). The dashed curves correspond to the radiation in a homogeneous medium with permittivity ε_1. For the other parameters we have taken $\beta_{1\parallel} = 0.4$, $\beta_{1\perp} = v_\perp\sqrt{\varepsilon_1}/c = 0.9$, $v_{\perp 0} = v_\perp$. For the peak at $\theta \approx 0.69$ one has $dN_n/d\Omega \approx 2.88q^2\sqrt{\varepsilon_1}/\hbar c$. Note that, for given values of $\beta_{1\parallel}$ and $\beta_{1\perp}$, the quantity $\varepsilon_1^{-1/2}dN_n/d\Omega$ depends on the permittivities ε_0 and ε_1 in the form of the ratio $\varepsilon_0/\varepsilon_1$. When the charge moves in the vacuum one has $\varepsilon_1 = 1$ and for the graphs presented in figure 1 $\varepsilon_0 = 3.75$. The latter corresponds to a cylinder made of quartz. In figure 2 we have presented the selected peaks from figure 1b for $n = 10$ and $n = 12$.
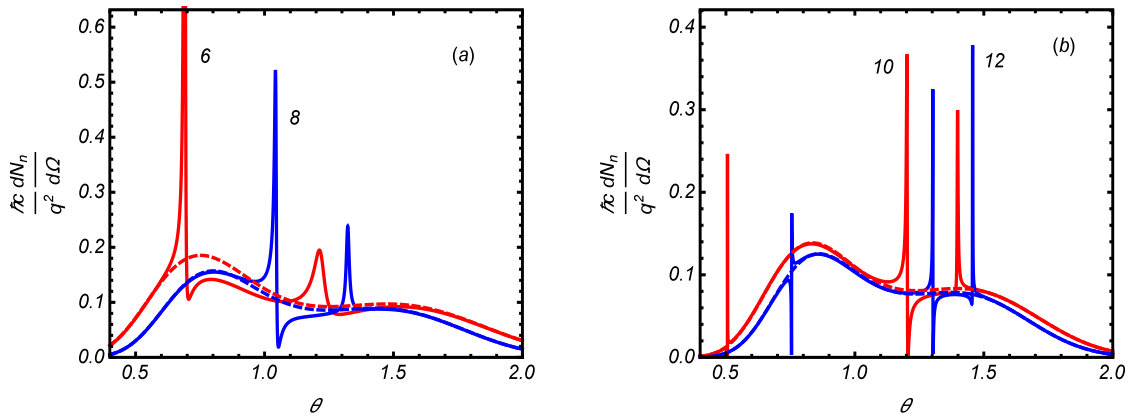

Figure 1. The angular density of the number of the quanta, radiated per period T for the motion along a circular helix on the harmonics $n = 6, 8, 10, 12$ (numbers near the curves), as a function of the angle θ. The dashed curves correspond to the radiation in a homogeneous medium with dielectric permittivity ε_0 immersed in the same medium. The other values of the parameters are $\beta_{1\parallel} = 0.4$, $\beta_{1\perp} = 0.9$, $\rho_c/\rho_0 = 0.7$.

In the case of the motion described by (1) and (2), the transversal oscillations of the charge serve as an additional tool to control the height and width of the peaks in the angular distribution of the radiation intensity. In addition, the oscillations may induce new angular peaks. Let us denote by $dN_{n,m}/d\Omega$ the contribution of the term with a given m in (5) to the angular density of the number of the radiated quanta. In table 1 we present this contribution at the angle $\theta \approx 0.69$ in the case $m = n = 6$, corresponding to the peak in the left panel of figure 1, for $n_0 = 3$ and for
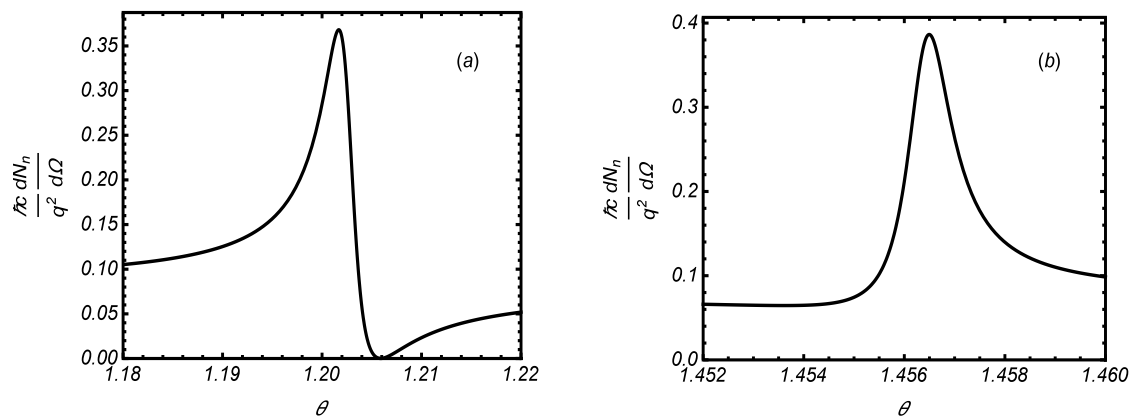
Figure 2. The angular density of the number of the quanta near the selected peaks for $n = 10$ (a) and $n = 12$ (b). The values of the parameters are the same as those for figure 1(b).

various values of the relative amplitude of the oscillations. The values of the other parameters are the same as those for figure 1. For small values of a the contribution of the term with $m = n$ dominates. For example, in the case $a = 0.1$ for the contribution $(\hbar c/q^2\sqrt{\varepsilon_1})dN_{6,m}/d\Omega$ at the peak $\theta \approx 0.69$ one has 3.6×10^{-4}, 1.1×10^{-3}, 9.4×10^{-4}, 3.4×10^{-5} for $m = 4, 5, 7, 8$, respectively.

Table 1.

a	0	0.05	0.1	0.15	0.2	0.25	0.28
$(\hbar c/q^2\sqrt{\varepsilon_1})dN_{6,6}/d\Omega$	2.88	2.90	2.93	2.78	1.97	0.42	0.016

As is seen from the table, the dependence on the amplitude of the oscillations is not monotonic. After an initial increase, it decreases with increasing a.

4. Conclusion

We have investigated the radiation intensity from a transversal oscillator rotating around a dielectric cylinder. The radial oscillations are described by (1). We have assumed that the energy of the charge is fixed. From this condition the time dependence of the azimuthal coordinate of the charge is obtained. The latter is given in the integral form (2). For general functions $\rho_e(t)$ and $\phi_e(t)$, the angular density of the radiation intensity on a given harmonic is given by the expression (5) with the coefficients defined by (6). In the case of a circular helix coaxial with the cylinder axis, in the summation over m in (5) the term $m = n$ survives only and we recover the results previously discussed in [17]. For this case, under certain conditions for the velocity of the charge and for the dielectric permittivity of the cylinder, strong narrow peaks may appear in the angular distribution of the radiation intensity. We have shown that these peaks appear also in the presence of additional transversal oscillations of the charge if the oscillations amplitude is relatively small.

Refernces

[1] Ter-Mikaelian M L 1972 *High Energy Electromagnetic Processes in Condensed Media* (New York: Wiley Interscience)

[2] Baier V N, Katkov V M and Strakhovenko V M 1998 *Electromagnetic Processes at High Energies in Oriented Single Crystals* (Singapore: World Scientific)
[3] Rullhusen R, Artru X and Dhez P 1998 *Novel Radiation Sources Using Relativistic Electrons* (Singapore: World Scientific)
[4] Potylitsyn A P 2011 *Electromagnetic Radiation of Electrons in Periodic Structures* (Berlin: Springer)
[5] Tsytovich V N 1951 *Westnik MGU* **11** 27 (in Russian)
[6] Kitao K 1960 *Prog. Theor. Phys.* **23** 759
[7] Erber T, White D and Latal H G 1976 *Acta Phys. Austriaca* **45** 29
[8] Schwinger J, Tsai W-Y and Erber T 1976 *Ann. Phys.* **96** 303
[9] Erber T, White D, Tsai W-Y and Latal H G 1976 *Ann. Phys.* **102** 405
[10] Rynne T M, Baumgartner G B and Erber T 1978 *J. Appl. Phys.* **49** 2233
[11] Villaroel D and Fuenzalida V 1987 *J. Phys. A: Math. Gen.* **20** 1387
[12] Villaroel D 1987 *Phys. Rev. A* **36** 2980
[13] Villaroel D and Milan C 1987 *Phys. Rev. D* **38** 383
[14] Grigoryan L Sh, Kotanjyan A S and Saharian A A 1995 *Sov. J. Contemp. Phys.* **30** 239
[15] Kotanjyan A S, Khachatryan H F, Petrosyan A V and Saharian A A 2000 *Sov. J. Contemp. Phys.* **35** 1
[16] Saharian A A and Kotanjyan A S 2005 *J. Phys. A: Math. Gen.* **38** 4275
[17] Saharian A A and Kotanjyan A A 2009 *J. Phys. A: Math. Theor.* **42** 135402
[18] Kotanjyan A A and Saharian A A 2013 *NIMB* **309** 177
[19] Kotanjyan A S and Saharian A A 2014 *J. Phys. Conf. Ser.* **517** 012025
[20] Hofmann A 2004 *The Physics of Sinchrotron Radiation* (Cambridge: Cambridge University Press)