Common characteristics of synchrotron radiation and light leaking from a bent optical fiber

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Abstract. Light leaking from a bent optical fiber shares many properties with synchrotron radiation : in ray optics, both lights are emitted tangentially to a *light cylinder* ; in wave optics, the emission mechanism involves a *tunnel effect*. The angular distributions of these two radiations are studied in parallel and found to be similar. The same is done for the impact parameter distributions. The latter show interference fringes of the Airy function type. The far field escaped from the fiber is calculated with the *Volume Current Method*. An optical system observing the impact parameter profile is proposed.

1. Introduction

Synchrotron radiation (SR) is the best known type of radiation emitted by a relativistic electron. Its spectrum and angular distribution is well calculated in classical electrodynamics for most practical purpose. However, its distribution in *impact parameter* space [1, 2, 3, 4], which is relevant in beam profile measurement, is not so well known. Furthermore, the most consistent classical equation of motion including radiation reaction, the so-called Abraham-Lorentz-Dirac (ALD) equation is plagued with unphysical *run-away* solutions. Notwithstanding this difficulty, a link has been established between the impact parameter of an emitted photon, a "side-slip" of the electron and the Schott term of the ALD equation [5, 6].

An heuristic explanation of SR is the following: the whole Coulomb field of the electron cannot follow the curved motion of the latter. Its part outside the "light cylinder" (a concept used in pulsar physics) of radius[1] R/v would go faster than light. It detaches and becomes free radiation. Light escaping a bent optical fiber (LEBF) can be explained is the same way (see figure 1), replacing the Coulomb field by the evanescent wave surrounding the fiber and v by the phase velocity $v_{\rm ph}$ of the internal wave.

In this paper we pursue this analogy quantitatively. We assume that light circulates in the fiber in a definite mode $\mathcal{M}$ (see Ref. [7]) and has a definite frequency ω_0. In Section 2 we compare the far-field angular distributions of both radiations. In Section 3 we analyze the fields in the impact parameter b parallel to the curvature plane and propose an optical system for observing the b-distribution.

[1] R is the orbit radius of the electron or the bending radius of the fiber. In our units the speed of light is 1.

Angular momentum considerations. In SR, the photon angular momentum J_Y about the orbit axis (see figure 2) can be expressed in two ways [4]

$$J_Y = \hbar\omega \, R/v_e \,, \quad (1)$$
$$J_Y = R_{\rm phot} \, \hbar\omega \, \cos\psi \,, \quad (2)$$

$R_{\rm phot}$ is the distance between the light ray (in ray optics) and the orbit axis and ψ is the angle between the ray and the orbit plane. For a photon escaping a fiber bent about the Y-axis we have the same formulae, but replacing v_e by the phase velocity $v_{\rm ph}$. From (1-2) we obtain

$$R_{\rm phot} = R/(v\cos\psi) \quad (3)$$

with $v = v_e$ or $v_{\rm ph}$. We see that the light ray does not come from the electron trajectory (SR case) or from the fiber (LEBF case), but is tangent to a cylinder of radius $R_{\rm phot} > R$. The bundle of rays form a *caustic.* The *classical impact parameter* of the photon is

$$b_{\rm cl} \equiv R_{\rm phot} - R \simeq (\gamma^{-2} + \psi^2)\, R/2 \quad (4)$$

where ψ is the angle between the ray and the curvature plane and $\gamma = (1-v)^{-1/2}$ with $v = v_e$ or $v_{\rm ph}$. We have assumed $\psi \ll 1$ and $\gamma \gg 1$, so that $b_{\rm cl} \ll R$. The finite impact parameter explains the frequency cutoff of SR or the critical bending radius of LEBF. $b_{\rm cl}$ must be less than the transverse size of the electron Coulomb field (for SR) or of the evanescent wave (for LEBF). These are both $\sim \gamma/\omega$. We obtain therefore formally the same condition for a significant SR or LEBF yield :

$$\omega\, R \lesssim \gamma^3 \,. \quad (5)$$

This condition can also be obtained by considering the emission of the photon as a *tunneling* through the centrifugal barrier [8].

Electron side-slipping [5, 6]. In the SR case, the electron "side-slips" in the direction of the force exerted by the field. The displacement

$$b_e = b_{\rm cl}\, \hbar\omega/(\gamma m_e) \quad (6)$$

is such that, just after the emission, the centre-of-inertia of the *photon + electron* system prolongates the initial electron trajectory. $b_{\rm cl}$ may be measurable with a thin enough electron beam, but $b_e \sim \lambda\!\!\!^{-}_c \simeq 400$ fm is practically not visible. Nevertheless, the global side-slipping due to many photon emissions in the $\hbar \to 0$ limit is responsible for the *Schott term* in $d^3X/d\tau^3$ of the ALD equation.

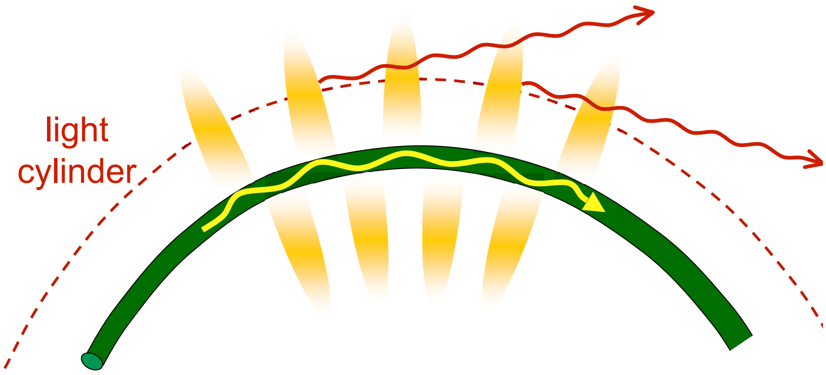

Figure 1. Escape of light from a bent optical fiber. The shaded areas represent the evanescent wave. A similar figure can be drawn for synchrotron radiation.

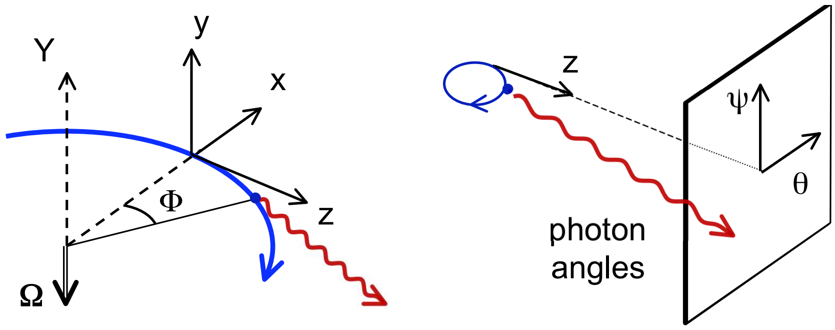

Figure 2. Coordinate system for synchrotron radiation or LEBF. The Y-axis is the bending axis.

2. SR and LEBF amplitudes

Let us treat in parallel SR and LEBF. The electron trajectory or the fiber is bent in the "horizontal" (x, z) plane along the circle of radius R centered at $(-R, 0, 0)$ as shown in figure 2. The angular velocity Ω of the electron or the wave about the Y axis is negative. The right of figure 2 specifies the spherical angular coordinate θ and ψ of the emitted photon momentum, $\mathbf{K} = \omega\,(\cos\psi\sin\theta, \sin\psi, \cos\psi\cos\theta)$. We take as linear polarization vectors $\hat{\mathbf{e}}_\theta(\mathbf{K}) = (\cos\theta, 0, -\sin\theta)$ and $\hat{\mathbf{e}}_\psi(\mathbf{K}) = (-\sin\psi\sin\theta, \cos\psi, -\sin\psi\cos\theta)$.

The current sources. SR is emitted by the current density

$$\mathbf{j}(t, \mathbf{X}) = -e\,\mathbf{v}_{\rm e}(t)\,\delta^3[\mathbf{X} - \mathbf{X}_{\rm e}(t)]\,, \tag{7}$$

where $\mathbf{X}_{\rm e}(t) = (R\cos\phi - R, 0, R\sin\phi)$, $\phi = vt/R$ and $\mathbf{v}_{\rm e} = (-\sin\phi, 0, \cos\phi)$.

LEBF can be considered as emitted by the *polarization current* $\mathbf{j}_{\rm pol}$ induced by the channeled wave in the fiber medium. For the mode $\{\mathcal{M}, \omega\}$ defined in [7] the current density is

$$\mathbf{j}_{\rm pol}(\mathbf{X}, t) = 2\,{\rm Re}\{\mathbf{j}_{\{\mathcal{M},\omega\}}(\mathbf{r})\,\exp(ipz - i\omega t + i\eta)\}\,, \tag{8}$$

η being a "classical" phase. *Macroscopic* Maxwell equations in medium give $\mathbf{j}_{\rm pol} = (\varepsilon - 1)\,\partial_t\mathbf{E}_{\rm real}$, whence

$$\mathbf{j}_{\{\mathcal{M},\omega\}}(\mathbf{r}) = -i\omega\,[\varepsilon(\mathbf{r}) - 1]\,\mathbf{E}_{\{\mathcal{M},\omega\}}(\mathbf{r})\,. \tag{9}$$

$\mathbf{j}_{\{\mathcal{M},\omega\}}$ is complex, like $\mathbf{E}_{\{\mathcal{M},\omega\}}$ and $\vec{\mathcal{E}}_{\{\mathcal{M},\omega\}}$ of Ref. [7]. In a straight fiber the current density $\mathbf{j}_{\rm pol}$ is in uniform translation motion at speed $v_{\rm ph}$, therefore does not radiate. In a bent fiber it emits synchrotron-like radiation, the retarded field obeying the *microscopic* Maxwell equations

$$\begin{aligned} \nabla\times\mathbf{B} - \partial_t\mathbf{E} &= \mathbf{j}_{\rm pol}\,, \qquad & \nabla\cdot\mathbf{E} &= \rho_{\rm pol}\,, \\ \nabla\times\mathbf{E} + \partial_t\mathbf{B} &= 0\,, \qquad & \nabla\cdot\mathbf{B} &= 0\,. \end{aligned} \tag{10}$$

The *Volume Current Method* [9, 10] consists in taking the $\mathbf{j}_{\rm pol}$ of the *straight tangent fiber* in (10), replacing the variable z in (8) by the curvilinear abscissa along the fiber.

2.1. Angular distributions

The radiation field emitted by the current density $\mathbf{j}$ is expanded in free plane waves:

$$\mathbf{E}_{\rm rad}(t, \mathbf{r}) = \int \frac{d^3\mathbf{K}}{(2\pi)^3}\,f(\omega)\;{\rm Re}\left\{\tilde{\mathbf{E}}(\mathbf{K})\;e^{i\mathbf{K}\cdot\mathbf{X} - i\omega t}\right\}\,, \tag{11}$$

$$f(\omega)\,\tilde{\mathbf{E}}(\mathbf{K}) = -\int dt\, d^3\mathbf{X}\ \exp(i\omega t - i\mathbf{K}\cdot\mathbf{X})\ \mathbf{j}_\perp(t,\mathbf{X})\,, \tag{12}$$

where $\omega = |\mathbf{K}|$, $f(\omega) = 1$ for a polychromatic source (SR case), $f(\omega) = 2\pi\delta(\omega-\omega_0)$ for a monochromatic source (LEBF case); $\mathbf{j}_\perp$ is the component of $\mathbf{j}$ orthogonal to $\mathbf{K}$. In SR one measures the **spectral**-angular distribution,

$$\frac{dW}{d\omega\, d\Omega} = \frac{\omega^2}{16\pi^3}\sum\nolimits_\mu |\hat{\mathbf{e}}^*_\mu(\mathbf{K})\cdot\tilde{\mathbf{E}}(\mathbf{K})|^2\,, \tag{13}$$

and in LEBF, the angular **power** distribution,

$$\frac{dW}{dt\, d\Omega} = \frac{\omega_0^2}{8\pi^2}\sum\nolimits_\mu |\hat{\mathbf{e}}^*_\mu(\mathbf{K})\cdot\tilde{\mathbf{E}}(\mathbf{K})|^2\,; \tag{14}$$

$\mu = \theta$ or ψ denotes the linear polarization axis.

Synchrotron case. We consider the radiation from one single pass of orbital motion. From (7) and (12),

$$\tilde{\mathbf{E}}(\mathbf{K}) = eR\int_{-\pi}^{\pi} d\phi\ \exp\{iKR(\phi/v_\mathrm{e} - \cos\psi\,\sin\phi)\}\,[\sin\phi\,\hat{\mathbf{e}}_\theta + \sin\psi\,\cos\phi\,\hat{\mathbf{e}}_\psi]\,. \tag{15}$$

Using the ultra-relativistic approximations $\gamma \gg 1,\ \ \psi \ll 1$ we obtain

$$\tilde{\mathbf{E}}(\mathbf{K}) = e\,\theta_0 R\left[-i\theta_0\,\mathcal{A}'(\xi)\,\hat{\mathbf{e}}_\theta + \psi\,\mathcal{A}(\xi)\,\hat{\mathbf{e}}_\psi\right]\, e^{i\zeta}\,, \tag{16}$$

where

$$\theta_0 = (\omega R)^{-1/3}\,,\quad \xi = (\gamma^{-2}+\psi^2)\,\theta_0^{-2}/2\,, \tag{17}$$

$$\zeta = \omega R\,(\cos\psi\sin\theta - \theta/v) \tag{18}$$

and $\mathcal{A}(\xi)$ is a re-scaled Airy function

$$\mathcal{A}(\xi) = 2^{4/3}\pi\ \mathrm{Ai}\left(2^{1/3}\xi\right) = \int_{-\infty}^{+\infty} d\tau\ \exp(i\xi\tau + i\tau^3/6)\,. \tag{19}$$

Optical fiber case. We will assume that the fiber is sufficiently thin so that $v \equiv v_\mathrm{ph}$ is close to 1. Then we can use the same "ultrarelativistic" approximations as in SR and obtain

$$\begin{aligned}\tilde{\mathbf{E}}(\mathbf{K}) \;&=\; -\theta_0 R\Big\{\ \Big[\tilde{\mathbf{j}}_x\,\mathcal{A}(\xi) + i\,\tilde{\mathbf{j}}_z\,\theta_0\,\mathcal{A}'(\xi)\Big]\ \hat{\mathbf{e}}_\theta \\ &\quad +\ \Big[\tilde{\mathbf{j}}_y - \psi\,\tilde{\mathbf{j}}_z\Big]\,\mathcal{A}(\xi)\,\hat{\mathbf{e}}_\psi\ \Big\}\ e^{i\zeta}\end{aligned} \tag{20}$$

where

$$\tilde{\mathbf{j}} = \int dx\, dy\ \exp(-iK_y\, y)\,\mathbf{j}_{\{\mathcal{M},\omega\}}(x,y) \tag{21}$$

is calculated for a straight fiber and we use again the definitions (17-18).

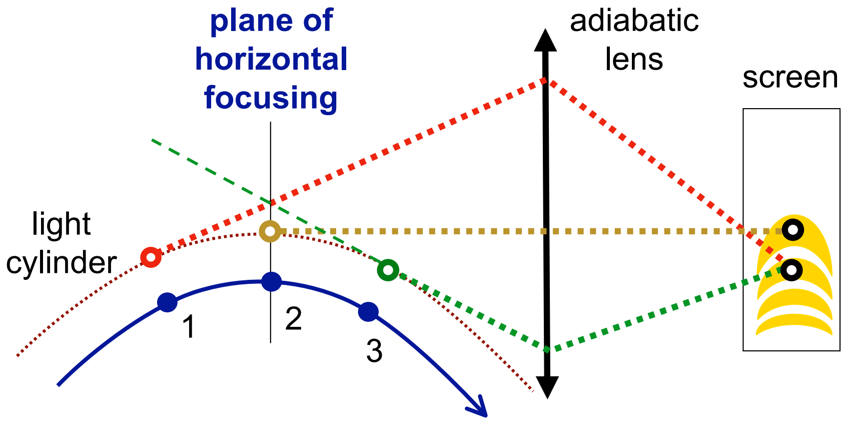

Figure 3. Optical device reproducing the hybrid impact parameter distribution of SR or LEBF on a screen. For the horizontal (resp. vertical) coordinate, the adiabatic lens is focused on the $z = 0$ (resp. $-\infty$) plane (the "vertical" axis $\hat{\mathbf{y}}$ is out of the figure plane). The hollow dots on the light cylinder are the classical emission points when the electron is in positions 1, 2 and 3. Rays emitted in 1 and 3 reach the same point on the screen, therefore interfere.

2.2. Impact parameter representation

We collect the light with an optical device, shown in figure 3, centered on the z axis and of aperture O such that $\theta_0 \ll O \ll 1$. Thus $\hat{\mathbf{e}}_\theta(\mathbf{K}) \simeq \hat{\mathbf{x}}$ and $\hat{\mathbf{e}}_\psi(\mathbf{K}) \simeq \hat{\mathbf{y}}$. We are interested only in the component b of the impact parameter parallel to the curvature plane, so we choose an astigmatic device horizontally focused at $z = 0$ and vertically focused at $z = -\infty$. We thus observe the partial Fourier transform

$$\check{\mathbf{E}}(\omega, x, K_y) = \int \frac{dK_x}{2\pi} \exp(iK_x x)\, \tilde{\mathbf{E}}(\mathbf{K}) \tag{22}$$

$$\simeq [\omega\theta_0/(2\pi)]\, \tilde{\mathbf{E}}(0, K_y, \omega)\, \mathcal{A}(\chi) \;, \tag{23}$$

with $\chi = [b_{\rm cl} - x]/b_0$ and $b_0 = R\,\theta_0^2$. The factor $\mathcal{A}(\chi)$ comes from the Fourier transform of $e^{i\zeta}$. According to the Parseval-Plancherel formula one passes from the K_x-distribution to the x-distribution by the substitution $|\tilde{\mathbf{E}}|^2 \to |\check{\mathbf{E}}|^2$ and $dK_x/(2\pi) \to dx$. We obtain thus the hybrid (x, ψ) distribution

$$\frac{dW}{dx\, d\psi\, (d\omega \text{ or } dt)} = \frac{\omega}{2\pi}\, \theta_0^2\, \mathcal{A}^2(\chi)\, \frac{dW}{d\theta\, d\psi\, (d\omega \text{ or } dt)} \,, \tag{24}$$

which applies both to SR (with $d\omega$) and LEBF (with dt). The simplicity of this result comes from the uniform rotation of the source, which gathers the θ dependence in the phase factor $e^{i\zeta}$.

The Airy fringes. The oscillating Airy function in (24) gives the crescent-shaped fringes on the screen. These can be explained by the interference between waves emitted from points of the light cylinder which are symmetric about the (x, y) plane, like the hollow points above 1 and 3 of figure 3. The same oscillations are present in the full (x, y) impact parameter distribution (see figure A5 of [2]). Note that the fringes are characteristic of the impact parameter representation of a *caustic* wave [11]. It is therefore possible to gather the fringes in *one single peak* in x, with an optical system which transforms the caustic into a true focus, for instance a curved mirror with a cubic deformation. This single peak would be more convenient and give a better resolution for beam profile measurements.

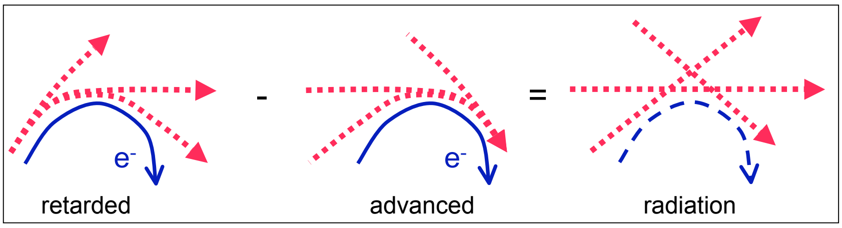

Figure 4. Relation between the fields $\mathbf{E}_{\text{rad}}$, $\mathbf{E}_{\text{ret}}$ and $\mathbf{E}_{\text{adv}}$.

"Far-field" conditions. The x- or (x, y)-distribution of $\mathbf{E}_{\text{rad}}$ cannot be measured with a probe put in the $z = 0$ plane, close to the electron beam or to the fiber : one would record the intensity of the *retarded* field $\mathbf{E}_{\text{ret}}$ (of the electron for SR, of the polarization current for LEBF). Due to the identity $\mathbf{E}_{\text{rad}} = \mathbf{E}_{\text{ret}} - \mathbf{E}_{\text{adv}}$ pictured in figure 4 the error would be equal to the *advanced* field $\mathbf{E}_{\text{adv}}$. If, for the purpose of deflecting the light toward a focusing optics, one puts a mirror close to the electron beam, one adds a parasitic *diffraction radiation* by the mirror edge. We must therefore put any collecting system far enough from the source. $\mathbf{E}_{\text{adv}}$ is still there, but propagating at large angle to the optical axis, as can be guessed from figure 4; therefore it cannot reach the image screen.

3. Conclusion

The strong similarity between synchrotron radiation (SR) and light escaping a bent fiber (LEBF) allows a deeper understanding of both processes. For example the cutoff frequency of SR and the critical curvature of a fiber are both derived from angular momentum considerations and reveal a tunneling mechanism. The Airy-type fringes of the impact parameter distribution, yet observed neither in SR nor in LEBF, could provide a tool for beam position monitoring. The principle of the optical system could be cheaply tested with LEBF.

References

[1] Hofmann A and Méot F 1982, *Nucl. Inst. Meth.* A **203** 483-493
[2] Andersson Å1997, Electron beam profile measurements and emitance manipulation at the MAX-laboratory, *doctoral dissertation*, Lund University
[3] Chubar O, Elleaume P, Snirigev A, 1999, *Nucl. Inst. Meth. in Phys. Research* A **435** 495
[4] Artru X 2006, Impact parameter profile of synchrotron radiation, *Advanced Radiation Sources and Applications*, ed Wiedemann H, NATO Science Series II **199** 387 ; arXiv:hep-ph/0503182
[5] Artru X and Bignon G 2002, A semiclassical approach to the radiation damping force, in *Electron-Photon Interaction in Dense Media*, ed Wiedemann H, *NATO Science series* II **49** pp. 85-90
[6] Artru X, Bignon G and Qasmi T 2001, Side-slipping of a radiating particle, *Problems of Atomic Science and Technology* **6** 98 ; arXiv:physics/0208005
[7] Artru X and Ray C 2015, Coherent light induced in optical fiber by a charged particle, this proceedings.
[8] Artru X 2015, Classical and quantum phenomenology in radiation by relativistic electrons in matter or in external fields, *Nucl. Inst. Meth. in Phys. Research* B **355** 11
[9] White I A 1979, Radiation from bends in optical waveguides: the volume-current method, *IEE Journal on Microwaves, Optics and Acoustics* 186.
[10] Kuznetsov M and Haus H 1983, Radiation loss in dielectric waveguide structures by the volume current method, *Quantum Electronics, IEEE* **10** 1505
[11] Landau L and Lifshitz E 1951, *The Classical Theory of Fields*, Addisson-Wesley