On the direct detection of gravitational waves, and some of the problems of improving laser interferometers

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Abstract. In this paper we describe an observational method for determining black holes masses. The study shows that the knowledge of the recorded low-frequency gravitational waves and the period from the beginning of registration till the moment of black holes collapse is sufficient and even preferable in determining the possible values of collapsing black holes masses. The reason for this is that the proportion of the period in the measured interval containing relativistic corrections (i.e. those ones in which the black hole speed is comparable to the speed of light), is smaller if the observed and measured time interval is longer. The values of black holes masses and the measured time interval, obtained as a result of the first observations, according to this model are in a very good agreement.We examine the problem of mirror heating in Fabry-Perot cavity of Michelson interferometer, by incident laser radiation, and we conclude that the problem of heat removal can be solved by a different approach, without use of multilayer reflective openings. As an alternative approach to the creation of highly reflective structures, we suggest using a spatially extended structure with a sinusoidal variation of the refractive index. We consider some of the possible technological methods for producing such structures based on heterogeneous media.The article describes the effects of the incident laser radiation exposure on the periodic structure, and it shows that the volume ponderomotive force may lead to a mirror polarization due to the radiation, and consequently, to appearance of an additional mechanical connection of the mirror with the surrounding mirror suspension design. The article examines the impact of the surface ponderomotive forces on the media boundary with different dielectric permeability and it shows that pressure spatial variables arising at the same time lead to deformation of the media layers, and the deformation and pressure values depend on the difference in the dielectric permeability values of the nearest layers. We also state the possible consequences of such spatially variable pressures effects (violation of synchronism conditions (misalignment) of reflective coatings, and others. It is also noted that our work, in which a laser interferometric technique for the detection of gravitational waves was first proposed, was already known to Joseph Weber in 1963.

The February issue of Physical Review Letters published an article which reported on the direct observation of gravitational waves from the confluence of two black holes [1]. This event is given the abbreviated name GW150914, which stands for: Gravitational Wave 2015, the 9th month, the 14 day, thus "pointing" on the principle of formation of such events in the future. The observation was carried out by means of the two unique independent interferometers LIGO: one in Hanford (Washington, USA), the other in Levingstoune (Louisiana, USA), the distance between them being 3002 km, so the time delay of the signal was about 7 ms, which corresponds to the speed of gravitational wave propagation at the speed of light. The peak amplitude of the gravitational wave (dimensionless value) was about 10^{-21}, which corresponds to a shift of the 4 kilometer arms of the interferometer. We proposed a fundamental scheme of the laser interferometer for the direct registration of gravitational waves as early as 1962 [2] and Rein

Weiss supplemented it with his proposal to set Fabry-Perot cavity in each of the arms of the interferometer, which would extend the arms of the interferometer, increase the phase progression and thereby would improve the sensitivity [3]. The signal obtained with the interferometers, which is given in [1], is shown in Fig.1 (above). The figure shows that the signal is a "chirp" whose frequency ranges from 35Hz to 250Hz.

Taking into consideration a temporary time change in frequency of the signal and making a comparison with the existing database of various scenarios of gravitational waves emission, we made a conclusion that this signal corresponds to the confluence of two black holes, hence, we could identify the masses of the black holes. The expression for the dependence of the observed signal frequency on time, obtained in Newtonian approximation of the general theory of relativity [4] (see also [5]), was the main expression used to determine the mass of the collapsing black holes:

$$\left(\frac{(m_1 m_2)^3}{m_1+m_2}\right)^{\frac{1}{5}} = \frac{c^3}{G}\left(\frac{5}{96}\pi^{-\frac{8}{3}} f^{-\frac{11}{3}} \frac{df}{dt}\right)^{\frac{3}{5}}, \tag{1}$$

where m_1, m_2 – are the masses of confluent black holes, G – is the gravitational constant, c – is the speed of light, $f(t)$ – is the rotation frequency of the masses of black holes around the common center of inertia. Solving the equation (1) with respect to frequency, we obtain

$$\frac{96\pi^{8/3}}{5}\frac{G\,M}{c^3}(t_2 - t_1) = \frac{3}{8}\left(f^{-8/3}(t_1) - f^{-8/3}(t_2)\right), \tag{2}$$

where t_2, t_1 – are the points of time at which the frequency value $f(t_2)$, $f(t_1)$ is measured, respectively $M \equiv \left(\frac{(m_1 m_2)^3}{m_1+m_2}\right)^{\frac{1}{5}}$. Using the obtained relation (2) in [1], the most probable values of the masses of black holes were found according to the obtained frequency values $f(t_2)$, $f(t_1)$. They appeared to be equivalent to $36^{+5}_{-4}M_\odot$ and $29^{+4}_{-4}M_\odot$,where $M_\odot$ is the mass of the sun.

In this work we propose a slightly different method for determining the most probable blackhole mass values, and we need to know only one, the lowest, frequency of the observed gravitational waves and the time lag between the point of measuring the lowest frequency and the point of the double black hole collapse. In this case, we do not need to know the highest frequency of gravitational radiation. The fact is that the maximum frequency value occurs immediately before the collapse when the velocity values of the confluent black holes are comparable with the speed of light (for example, in the case of events recorded in [1], this value reaches 0.4-0.5 the speed of light) and as it is shown in [5], relativistic corrections become significant. Knowing the lower frequency corresponds exactly to Newtonian approximation used in deriving the ratio (2). Certainly, the time interval in a strong gravitational field of a black hole is known [4] to be changing, however, time is the integral value and the longer the time lag in which the observations are carried out is, the smaller is the proportion of time changes. In contrast, the oscillation frequency in a gravitational wave is an instantaneous parameter and its value in the strong gravitational field depends strongly on the gravitational field value [4,5].

We now obtain a slightly different expression for determining the masses of collapsing black holes. We consider the motions in Newtonian approximation, when we meet the conditions of black hole slowness compared to the speed of light, i.e. the condition $GM/r \ll c^2$ is fulfilled. Then, using the relationship between the rotation frequency $\omega(t)$ of the two masses around the common center of inertia

$$\omega^2(t)\, r^3(t) = G(m_1 + m_2)\,, \tag{3}$$

where $r(t)-$ is the radius of the orbit, as well as the expression for the binding energy of the two masses $E(t) = -G\dfrac{m_1\, m_2}{2r(t)}$ and the expression for the energy loss due to the emission of gravitational waves

$$\frac{dE(t)}{dt} = -\frac{32\,G}{5c^5}\left(\frac{m_1\, m_2}{m_1+m_2}\right)^2 r^4(t)\,\omega^6(t) \tag{4}$$

we obtain an equation for the time change in the rotation radius [3]

$$\frac{dr(t)}{dt} = -\frac{64\,G^3\, m_1\, m_2(m_1 + m_2)}{5c^5} r^{-3}(t)\,. \tag{5}$$

Equation (5) allows us to find the time lag from the start of the measurement t_1 until the moment when the collapse occurred t_2 :

$$\Delta t = \frac{5c^5}{256\, G^{5/3} m_1\, m_2\, (m_1 + m_2)}\left(r^4(t_1) - r^4(t_2)\right), \qquad \Delta t \equiv t_2 - t_1. \tag{6}$$

Equation (6) and formula (2) are very similar, if in (2) we drop member $f(t_2)$, and in (6) we drop member $r^4(t_2)$, they coincide. The latter is the direct consequence of Newtonian approximation, when approaching the collapse $r \to 0, \quad f \to \infty,$ but near the collapse the speeds of black holes are strongly relativistic and Newtonian approximation is no longer valid. However, in the time lag (6), the proportion of non-Newtonian relativistic time lag becomes smaller, as the time lag of the observation is longer. For point masses the collapse means that the distance between the rotating mass tends to zero and, consequently, the frequency f tends to infinity (in non-relativistic approximation, see formula (3)). After the two masses collapse and the value of the quadrupole moment of the system decreases significantly, as it is clear from the observations [1], the intensity of gravitational waves emission falls sharply, which is quite clear from the general physical considerations. This circumstance makes it possible to more accurately determine the point of time when the confluence of two black holes occurred.

The given graph of gravitational waves frequency and its changes depending on the time clearly shows that this point of time corresponds to the change of the derivative sign $\dfrac{df}{dt}$ or, more precisely, to the point of time t_2 at which the second derivative is equal to zero. Thus, the observations give an opportunity to more accurately determine the time when there was collapse and thus more accurately determine the interval $\Delta t = t_2 - t_1$. According to the observations presented in [1], we conclude that the interval is approximately $\Delta t \simeq 0{,}1754\,\text{sec}$, and considering that the rotation frequency of the two black holes around the total moment of inertia is equal to one half of the frequency of gravitational wave emission, i.e. 17.5 Hz, for the mass values $36^{+5}_{-4}\ M_{\odot}$ and $29^{+4}_{-4}\ M_{\odot}$ given in [1], we obtain the value for the interval $\Delta t \simeq 0{,}1754 сек$ from the formula (6). Thus, there is a good agreement between observations and estimates for the masses of collapsing black holes given in [1].

Given that the value of the rotation radius at the time of the collapse approximates to the value of Schwarzschild radius for the total mass of the black holes, taking into account the weight loss in the emission of gravitational waves, we can assume that $r\ (t_1) \gg r\ (t_2)$, and then the formula (6) is even simpler:

$$\Delta t \simeq \frac{5c^5(m_1+m_2)^{1/3}}{256\, G^{5/3} m_1\, m_2\, \omega_{\min}{}^{8/3}(t_1)}, \tag{7}$$

where $\omega_{\min}\ (t_1)$ is the minimum value of the rotation frequency of black holes, which is equal to the half of the frequency of the observed gravitational waves. This simple formula is more convenient for selecting the values of the masses of the collapsing black holes.

As we continue to improve the sensitivity of LIGO type gravitational antennas, we need to express our view concerning the use of mirrors in Fabry-Perot cavity. We aim at improving the reflectance coefficient of laser light mirrors in Fabry-Perot cavity and we result in a significant increase in circulating laser power, which now reaches the values of 100 kW, and in the future is likely to increase even more - up to 830 kW. It is clear that even with a small absorption in the reflective multilayer structures, there is not only the problem of heat dissipation, but also the problem of misalignment of the cavity itself due to nonuniform mirror heating, and therefore it is necessary to introduce the additional heat in order to prevent misalignment of the system.

Taking this into account it is necessary to consider other approaches to creating highly reflective mirrors that can withstand high power laser radiation. In this regard, we should pay attention and explore the possibility of obtaining highly reflective extended structures by means of creating the refractive index in the medium (i. e. correct periodic) of sinusoidal change structure. The amplitude of the change in refractive index is small, there are no sharp jumps, and high reflectance is achieved through the use of an extended periodic structure (sinusoidal Bragg mirror).

Such structures were studied in detail previously (see [6-8]), when in the mediumthere is a propagating sound wave and due to the effect of photoelasticity, there occurs periodic variation of the refractive index, on which the incident laser light diffracts "back". Certainly, the creation of such a periodic structure by means of a sound wave is unlikely to be suitable for LIGO type interferometers. However, the use of heterogeneous media, or one-dimensional photonic crystals, in which the use of a particular technology provides the necessary periodic structure of changes in the dielectric permeability of the medium, allows us [9] to solve many problems in the mirror interferometers such as LIGO. Let us briefly consider the "back" diffraction phenomenon of the of the incident laser light on the mediaum, when the dielectric permiability of the medium $\varepsilon(x)$ varies according to the law:

$$\varepsilon(x) = \varepsilon_0 + \Delta\varepsilon(x), \qquad \Delta\varepsilon(x) = \Delta\varepsilon\, \mathrm{Cos}(qx), \quad \Delta\varepsilon(x) \ll \varepsilon_0. \tag{8}$$

Here $\Delta\varepsilon$ is the "amplitude" of changes in the dielectric permeability of the medium, $q-$ is the wave vector of the periodic structure of the medium. Maxwell's equations for the incident and reflected waves propagating in the opposite direction, in the approximation of slowly varying amplitudes

$$E^{+}(x,t)=\frac{1}{2}E_{0}^{+}(x)Exp(i\,\omega t-i\,k\,x)+k.c.\;\; E^{-}(x,t)=\frac{1}{2}E_{0}^{-}(x)Exp(i\,\omega t+i\,k\,x)+k.c.$$

are written in the form:

$$\frac{dE^{+}(x)}{dx}=i\,k\,\Delta\varepsilon\,E^{-}(x)Exp(i\,\Delta k\,x),\qquad \frac{dE^{-}(x)}{dx}=-i\,k\,\Delta\varepsilon\,E^{+}(x)Exp(-i\,\Delta k\,x),\qquad (9)$$

ω, k – are frequency and wave vector of the laser radiation propagating within Fabry-Perot cavity, $\Delta k=2k-q$ – is the departure from synchronism conditions. Solving the equation (9) with the boundary conditions: $E^{+}(0)=E_{0}$, $E^{-}(L)=0$ where L is the size of the structure, for the amplitude of the reflected wave propagating in the opposite direction with respect to the direction of the incident wave, we obtain the following expression:

$$E^{-}(\Delta k)=\frac{2i\,k_{0}\Delta\varepsilon E_{0}}{\Delta k-i\sqrt{-\Delta k^{2}+4k_{0}^{2}\Delta\varepsilon^{2}}\,\mathrm{Coth}\left(\frac{1}{2}L\sqrt{-\Delta k^{2}+4k_{0}^{2}\Delta\varepsilon^{2}}\right)}\qquad (10)$$

The amplitude and phase dependance of the reflected wave on the extent of the departure from synchronism condition in accordance with the formula (1) is shown in Fig. 2. It also shows numerical values which were used when making the graphs. We see that at relatively low values of refractive index changes at a sufficient length of the structure we have a reflection coefficient which is very close to the unity. Withvalues indicated in Fig. 2, the amplitude of the reflected wave at $\Delta k=0$ will be $E^{-}=0.9999930E_{0}$ (without radiation attenuation in the medium). Formula (10) is easily generalized in the case of an absorbing medium. Thus, if we use periodic structures in such a way, when radiation occupies almost the entire volume of the mirror, the heat is more easily removed. It should be noted that formula (10) was obtained under the condition of a spatially homogeneous distribution of the sinusoidal periodic structure, but there exist more interesting practical approaches when the structure itself is still modulated (see[7]). At a certain modulation type (see [7] as an example), there occur exponentially narrow radiation transmission lines with controlled intensity, which can be very useful for the semireflecting Fabry-Perot mirror, as well as in the problem of the laser mode purification. It is necessary to point out that if Fabry-Perot cavity has such mirrors with a very narrow bandwidth line, it allows us to create new optical schemes for direct detecting of the gravitational waves.

With regard to the technology of periodic structures, we can point out the following possibilities. a) First, we create a periodic structure consisting of alternating layers (sheets) of a transparent insulator such as fused quartz, and in its sheets there were sequentially and alternatively inserted nano-particles of impurities (metals or insulators with the large dielectric permeability value), changing the dielectric permeability of the sheet. Then, we heat the stack of the sheets to the desired temperature and pass this workpiece through a series of cylindrical waves, reducing its cross-sectional dimension to the desired value. Such technology is already used to create a gradient optical fiber. b) There is another approach, based on the fact that the heated heterogeneous medium containing the necessary quantity of nano particles is placed in a microwave resonator of electromagnetic oscillations, in which nano-particles affected bythe electric field of a standing electromagnetic wave, are redistributed in space in accordance with the structure of the field, and then such a workpiece passes through a series of waves changing its cross-sectional dimensions to the desired value. There are other methods of obtaining

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periodic structures associated with exposure of high-power laser or acoustic waves on heterogeneous media.

Concerning the problem of hanging the mirrors the following should be noted. It is known that the sufficiently powerful optical radiation emitted on the free-suspended mirror, polarizes it and the mirror appears to have dipole moment. This polarization occurs when a mirror has a number of external charges [10], which may result from many causes: space radiation affects the mirror itself, as well as the surrounding structures, residual gas molecules hit the surface of the mirror, and other reasons. Occurrence of polarization in an insulator, a semiconductor or even in some metals having oxidized film on the surface has been regularly observed experimentally [11].

The appearance of dipole moment in a mirror should lead to a further link with the surrounding equipment (dipole - dipole interaction), especially with conductive metal structures. This relationship can be very undesirable, particularly in a laser beam modulation. This is a new effect, which unlike the usual Coulomb charge interaction on the mirror, interacts with metal structures, when there is laser radiation.

The effect of appearance of dipole moment in the mirror, or in other words, its polarization, apparently, has not been observed until now. We believe this effect may be of significant importance for the problem of alignment of the optical interferometer scheme and it is not limited to recording the light pressure on the mirror. In this case we are talking about the effect of volume ponderomotive forces from the laser emissionon the mirror itself, and its multilayered periodic structure.

It is well known (see. [4], §15, and [10], §§32-34), that an insulator is acted upon the volume force from the electric field.

$$\mathbf{f}(x)=\frac{1}{8\pi}\nabla\left(<\mathbf{E}^2(x,t)>\rho\frac{\partial\varepsilon}{\partial\rho}\right)-\frac{<\mathbf{E}^2(x,t)>}{8\pi}\nabla\varepsilon+<q\,\mathbf{E}(x,t)>, \quad (11)$$

here $\mathbf{f}(x)$ is the volume force density acting on the medium, $<\mathbf{E}^2(x,t)>$ is the time average of the square of the laser radiation amplitude, $\frac{\partial\varepsilon}{\partial\rho}$ is the density derivative of the dielectric permeability of the medium (the medium is assumed to be isotropic), q is the volume density of external charges induced in the mirror. Expression (11), unlike the equations for ponderomotive forces given in the monographs [3,11], includes the first two time averaged terms, because it is believed that there are no free light particles (electrons), which could follow the wave field oscillations. As for the last term in (11), ifthe mirror has free charges, it is necessary to carry out its averaging considering the fact that the charge density $q=q(x,t)$ under the action of the wave field can be redistributed (see references [12-14]). The force (11) is of the greatest problem for the time modulated laser signal, in this case different resonances are quite possible, and they need to be given special attention.

It is known [10,12], in addition to the volume force, there is also a surface force which can be obtained from the formula (11) by integrating according to the volume of the mirror. Then we can show that there is a surface density of the force acting on the boundary of two media with different dielectric permeability values in each of the layers. This surface force density "puts pressure" on the boundary, and this pressure is determined by the following:

$$f(x)=\frac{1}{2}<E(x,t)^2>\left(\varepsilon_1-\varepsilon_2\right), \quad (12)$$

where $\varepsilon_1, \varepsilon_2$ are the values of dielectric permeability of the adjacent layers of the periodic structure. With the spread of the radiation through the periodic structure, the value of the tangential field component is reduced, and the pressure sign, as it can be seen from the formula (12) varies along the structure, creating different voltages. Such spatial variation of the pressure can lead to a number of effects: changes in the thickness of the layers and, as a consequence, deviation from the synchronism conditions, or the destruction of the periodic structure itself, and if there is temporal modulation of laser emission intensity, it can result in appearance of acoustic vibrations. (Note, that such an obvious mechanism of high-frequency acoustic vibrations excitation in a medium with a periodic change of dielectric permeability by the modulated laser radiation, apparently, has never been examined before. It is clear that this problem has to be studied separately).

Thus, at the boundary between two media with different dielectric permeability there appears a force, leading to the deformation of these layers, and in the presence of the laser intensity modulation, there can be high-frequency acoustic waves. All these effects need to be considered when taking measurements.

4. Concerning the question, whether our work [2] is known to professionals working on creation of resonant antennas for the reception of gravitational waves, and making the first attempts to detect gravitational waves (J.Weber, 1960), it is necessary to note the following.

In 1963, my teacher VL Ginzburg participated in a conference on gravitation in Poland (Warsaw) [15]. There he gave a talk in which he criticized the resonance detectors of gravitational waves relying on our work. It is known that the resonance gravitational wave detectors are most sensitive at the well-defined resonance frequency, and have a very narrow spectral band. The frequency of possible sources of gravitational waves is unknown, moreover, the direction from which gravitational waves can arise or their polarization is unknown as well. It is these features of resonance antennas, in our opinion, that led to the failure of detecting gravitational waves by such a method, and we noted this in our work [2], in which we also proposed to use broadband laser interferometers.

This conference was attended by Joseph Weber and in August 1963 he published a report [16], which contained a response to this "criticism" in our work [2]. J. Weber referred to V.L. Ginzburg'swork [15] and our work [2]. Unfortunately, Joseph Weber' paper [16] is not cited by many researchers, but Felix Pirani' report [17] was apparentlycited for the first time by Joseph Weber in his work [16], and it is sometimes referred to as the work relating to the problem of the direct detection of gravitational waves.

Joseph Weber's pioneer experiment presented the antenna as an aluminum cylinder 2 m long and 1 m in diameter, suspended on steel wires; resonance frequency of the antenna was 1660 Hz, the amplitude sensitivity of piezoelectric detectors was about10^{-14} sm. J. Weber used two detectors working in coincidence, and he reported the detection of a signal, whose source was most likely the center of the Galaxy. However, no independent experiments confirmed Weber's observations. Still it should be noted that J. Weber's pioneering work played an important role in developing and understanding the problem of direct detection of gravitational waves, and attracted much attention from researchers. Among detectors currently operating on this principle we can mention the spherical antenna MiniGRAIL (Leiden University, The Netherlands), the antenna ALLEGRO (Baton Rouge, Louisiana, USA), with which Joseph Weber worked he past, AURIGA (Rome, Italy), the receiver temperature in this antenna is $0{,}1^{o}K$, NAUTILUS (Rome, Italy), the EXPLORER (Switzerland) and Agra (Russia).

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