

Space-time with a fluctuating metric tensor model

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Abstract. A presented physical time model is based on the assumption that time is a random Poisson process, the intensity of which depends on natural irreversible processes. The introduction of metric tensor space-time fluctuations allowing describing the impact of stochastic gravitational background has been demonstrated. The use of spectral lines broadening measurement for the registration of relic gravitational waves has been suggested.

1. Introduction

A fluctuating physical time model was first proposed in the work [1] to explain the features of ion mobility fluctuations in a small amount of electrolyte under review [2, 3]. Based on the equation binding the entropy production density and the intensity of physical time fluctuations, the work [4] calculates the intensity of fluctuations specified and compares with experimental results.

2. Fluctuating physical time model

Let us consider the fluctuating physical time model, based on the assumption that the physical time under review is a random Poisson process $\tau(t)$ with fluctuations, equal to $\tau_0 = 1/v_\tau$, where v_τ - the intensity of physical time fluctuations.

The one-dimensional characteristic function of a Poisson process under review is given by [5, 6]

$$g_\tau(\mu_\tau; t) = \exp((\exp(i\tau_0\mu_\tau) - 1)v_\tau t), \quad (1)$$

and its n -dimensional characteristic function at $t_n \geq \dots \geq t_2 \geq t_1$ respectively takes the form

$$g_{n\tau}(\mu_{1\tau}, \mu_{2\tau}, \dots, \mu_{n\tau}; t_1, t_2, \dots, t_n) = \exp\left[v_\tau \sum_{j=1}^n t_j \left(\exp\left(i\tau_0 \sum_{k=j}^n \mu_{k\tau}\right) - 1 \right)\right]. \quad (2)$$

The presented characteristic functions (1) and (2) make it possible to register the expectation function $\langle \tau(t) \rangle$ and correlation function $\langle \tau(t_1)\tau(t_2) \rangle$ for arbitrary time points t_1 and t_2 as follows:

$$\langle \tau(t) \rangle = t, \quad (3)$$

$$\langle \tau(t_1)\tau(t_2) \rangle = t_1 t_2 + \tau_0 \min(t_1, t_2). \quad (4)$$

Let us introduce the function

$$\xi_T(t) = \frac{d\tau(t)}{dt}, \quad (5)$$

expectation function $\langle \xi_T(t) \rangle$ and correlation function $\langle \xi_T(t_1)\xi_T(t_2) \rangle$ of which are given by

$$\langle \xi_T(t) \rangle = 1, \quad (6)$$

$$\langle \xi_T(t_1)\xi_T(t_2) \rangle = \tau_0 \delta(t_2 - t_1). \quad (7)$$

The introduction of the function $\xi_T(t)$ allows presenting the differential $d\tau(t)$ in the form of

$$d\tau(t) = \xi_T(t) dt. \quad (8)$$

Please note that the same expression is used in the general relativity theory to establish the relation between proper time τ and time coordinate $x^0 = ct$ [7]

$$d\tau = \frac{1}{c} \sqrt{g_{00}} dx^0, \quad (9)$$

where c - speed of light in vacuum, g_{00} - metric tensor component g_{ik} , $i, k = \overline{0,3}$, describing the space-time time coordinate variation. The comparison of formulas (8) and (9) allows presenting the value g_{00} as follows:

$$g_{00} = \xi_T^2, \quad (10)$$

which indicates a fundamental possibility of making a more general description of a space-time with a fluctuating metric.

3. Space-time with a fluctuating metric tensor model

The statement of the problem of physical processes description within a space-time model with a randomly fluctuating metric tensor is associated not only with the fluctuating physical time model, but also with the possibility of potential fluctuating space curvature due to the composition of a large number of gravitational waves [8]. At the same time, considering gravitational waves generating by different, uncorrelated against each other astrophysical objects such as neutron stars, black holes etc., as well as occurring as a result of gravitational collapse and supernova outburst, the model of randomly fluctuating metric tensor for space-time description is quite reasonable. Moreover, the model proposed may be used to describe the stochastic background caused, in particular, by a relict gravitational radiation [9, 10].

In general, the statement of the problem of physical processes description in the space-time with a fluctuating metric tensor model can be formulated as follows. Let us assume that any physical process, for example, electromagnetic wave propagation, proceeds in a space-time model. Let us consider that the metric tensor $g_{ik}(x^0, x^1, x^2, x^3)$ constitutes a random coordinate function $x^l = \{x^0, x^1, x^2, x^3\}$. It is required to determine the history of the physical process under review in case of metric tensor $g_{ik}(x^l)$ fluctuating randomly with time and space shift.

To create equations describing the physical processes proceeding in the space-time with a fluctuating metric tensor model, the method, described in the work [6], can be applied. Electromagnetic field equations in a curved space-time allow analyzing, in particular, the character of electromagnetic waves propagation in the space-time with a fluctuating metric tensor model [11].

4. Light propagation in the space-time with a fluctuating metric tensor model

Let us further consider the case of light propagation description with geometrical optics approximation. This case may occur if short light wavelength λ_0 condition is met, in comparison to the characteristic correlation length δL of the metric tensor fluctuations: $\lambda_0 \ll \delta L$. The equation is describing the light propagation in a curved space-time with geometrical optics approximation is given by [7]:

$$\frac{dk^i}{d\chi} + \Gamma_{kl}^i k^k k^l = 0, \quad (11)$$

where: k^i - four-dimensional wave vector, χ - parameter, fluctuating along a light ray, Γ_{kl}^i - Christoffel symbols, expressed through the metric tensor as follows:

$$\Gamma_{kl}^i = \frac{1}{2} g^{im} \left(\frac{\partial g_{mk}}{\partial x^l} + \frac{\partial g_{ml}}{\partial x^k} - \frac{\partial g_{kl}}{\partial x^m} \right). \quad (12)$$

Formulas (11) and (12) and further assume the summation over repeated indices, and the metric tensor g_{ik} becomes symmetric: $g_{ik} = g_{ki}$.

Let us further consider the case when the metric tensor g_{ik} is considered little different from the Galilean metric $g_{ik}^{(0)}$:

$$g_{ik} = g_{ik}^{(0)} + h_{ik}, \quad (13)$$

where: $|h_{ik}(x^l)| \ll 1$, $g_{00}^{(0)} = 1$, $g_{11}^{(0)} = g_{22}^{(0)} = g_{33}^{(0)} = -1$; $g_{ik}^{(0)} = 0$ at $i \neq k$. Then, first approximation Christoffel symbols can be presented as follows:

$$\Gamma_{kl}^i = \frac{1}{2} g_{(0)}^{im} \left(\frac{\partial h_{mk}}{\partial x^l} + \frac{\partial h_{ml}}{\partial x^k} - \frac{\partial h_{kl}}{\partial x^m} \right), \quad (14)$$

where $g_{(0)}^{im} = g_{im}^{(0)}$.

Following the work [12], the solution of the equation (11) will be presented in expanded form:

$$k^i = k_{(0)}^i + k_{(1)}^i + \dots \quad (15)$$

Then, zero-order approximation equation (11) takes the form:

$$\frac{dk_{(0)}^i}{d\chi} = 0. \quad (16)$$

Therefore, $k_{(0)}^i = \text{const}$.

First approximation equation (11) integration results in

$$k^i(\chi) = k_{(0)}^i + k_{(1)}^i(0) - k_{(0)}^k k_{(0)}^l \int_0^\chi \Gamma_{kl}^i d\chi. \quad (17)$$

The formula for the four-dimensional vector $k_{(1)}^i(0)$ can be derived from the condition [7]:

$$g_{ik} k^i k^k = 0 \quad (18)$$

and in the first approximation is given by:

$$k_{(1)}^i = -\frac{1}{2} g_{(0)}^{im} h_{ml}(0) k_{(0)}^l. \quad (19)$$

Then, the solution (17) takes the final form:

$$k^i(\chi) = k_{(0)}^i - \frac{1}{2} g_{(0)}^{im} h_{ml}(0) k_{(0)}^l - k_{(0)}^k k_{(0)}^l \int_0^\chi \Gamma_{kl}^i d\chi. \quad (20)$$

Since vector k^i can be presented as [7]

$$k^i = \frac{dx^i}{d\chi}, \quad (21)$$

in zero-order approximation it takes the form:

$$x^i(\chi) = k_{(0)}^i \chi + x^i(0). \quad (22)$$

Let us consider the case of light ray propagation in the direction of the axis x^1 . Then we may assume that the components of vector $k_{(0)}^i$ have the following meanings: $k_{(0)}^0 = k_{(0)}^1 = k_0$, $k_{(0)}^2 = k_{(0)}^3 = 0$, where k_0 - wave number. Considering the distance l , passed by the light, equal to: $l = x^1(\chi) - x^1(0)$, from the formula (22) at $i = 1$, we obtain an expression for the parameter χ :

$$\chi = \frac{l}{k_0}. \quad (23)$$

Based on assumptions, from the formula (22) the following expressions were obtained:

$$\frac{k^\alpha(l)}{k_0} = 1 - \frac{1}{2}(-1)^\alpha (h_{0\alpha}(0) + h_{1\alpha}(0)) - \int_0^l (\Gamma_{00}^\alpha + 2\Gamma_{01}^\alpha + \Gamma_{11}^\alpha) dl, \quad \alpha = 0, 1, \quad (24)$$

$$\frac{k^\beta(l)}{k_0} = \frac{1}{2}(h_{0\beta}(0) + h_{1\beta}(0)) - \int_0^l (\Gamma_{00}^\beta + 2\Gamma_{01}^\beta + \Gamma_{11}^\beta) dl, \quad \beta = 2, 3, \quad (25)$$

where the Christoffel symbols $\Gamma_{kl}^i = \Gamma_{lk}^i$ symmetry is taken into account.

Formulas (24) and (25), after inserting an expression for the Christoffel symbols (14), allow to make an expression for relative frequency $\delta\omega$ and the wave vector δk^α fluctuations at the point of observation:

$$\frac{\delta\omega}{\omega_0} = \frac{\delta k^0}{k_0} = -\frac{1}{2}(h_{00}(0) + h_{01}(0)) + \int_0^l \left(\frac{1}{2} \frac{\partial(h_{11} - h_{00})}{\partial x^0} - \frac{\partial(h_{00} + h_{01})}{\partial x^1} \right) dl, \quad (26)$$

$$\begin{aligned} \frac{\delta k^\alpha}{k_0} &= \frac{1}{2}(h_{0\alpha}(0) + h_{1\alpha}(0)) + \\ &+ \int_0^l \left(\frac{\partial(h_{0\alpha} + h_{1\alpha})}{\partial x^0} + \frac{\partial(h_{0\alpha} + h_{1\alpha})}{\partial x^1} - \frac{1}{2} \frac{\partial(h_{00} + 2h_{01} + h_{11})}{\partial x^\alpha} \right) dl, \quad \alpha = \overline{1, 3}. \end{aligned} \quad (27)$$

Given that according to the formulas (22) and (23), we can state the following: $dx^1 = dl$, let us integrate the addend within the integral of the expression (26). Then we obtain the following:

$$\frac{\delta\omega}{\omega_0} = \frac{1}{2}(h_{00}(0) + h_{01}(0)) - h_{00}(l) - h_{01}(l) + \frac{1}{2} \int_0^l \frac{\partial(h_{11} - h_{00})}{\partial x^0} dl. \quad (28)$$

Let us introduce the tensor h_{ik} in the form of

$$h_{ik} = h\xi_{ik}, \quad i, k = 0, 1, \quad (29)$$

where: h - tensor h_{ik} fluctuations amplitude, $\xi_{ik} = \xi_{ik}(x^0, x^1)$ - random field with a unit amplitude and a correlation length $\delta L < \lambda_0$. In this case, all the addends preceding the integral within expression (28), will have the order h size.

Let us estimate the characteristic values of the integral within the expression (28). Since the geometrical optics approximation application is limited to the limit value of characteristic dimensions of correlation $\delta L \approx \lambda_0 = 1/k_0$ and characteristic time of correlation $\delta \tau \approx 1/\omega_0$, the frequency fluctuations variance estimate described by the integral in the expression (28), takes the following form:

$$\sigma_\omega^2 \approx h^2 l k_0 \quad (30)$$

or

$$h \approx \frac{\sigma_\omega}{\sqrt{l k_0}}. \quad (31)$$

In case of $l \gg 1/k_0$, the integral contribution for the light frequency fluctuations becomes decisive.

Let us estimate the spectral line broadening when light passes through the space with a fluctuating metric. If it is assumed that the value l is approximately equal to the characteristic size of the space: $l \approx 10^{25} \dots 10^{26}$ m (1...10 billion light years), $k_0 = 10^7$ m⁻¹, and the minimum registered spectral line broadening equals to $\sigma_\omega \approx 10^{-6} \dots 10^{-8}$, the fluctuations amplitude h , calculated from the formula (31), takes the following form: $h \approx 10^{-22} \dots 10^{-24}$.

Thus, when light propagates over a distance comparable to the size of the space, in the case of fluctuating metric with an amplitude $h \approx 10^{-22} \dots 10^{-24}$ and frequency, upper-bounded with a light wave frequency, the spectral lines broadening shall be observed. The registration of an irreversible process of spectral lines broadening during the light propagation can evidence the existence of fluctuations of the space-time metric tensor and, consequently, the time fluctuations.

5. The calculation of the gravitational waves effect on light spectral lines broadening

If the description is carried out in a synchronous reference system [7], as usually done when calculating the plane gravitational waves effect within the linear theory framework [12], the Lorentz gage presents the following:

$$h_{00} = h_{0i} = 0. \quad (32)$$

Then the expression (28) for the light fluctuations relative frequency takes a simple form:

$$\frac{\delta \omega}{\omega_0} = \frac{1}{2} \int_0^l \frac{\partial h_{11}}{\partial x^0} dl. \quad (33)$$

Assuming that the gravitational waves filling the space have the frequency limit ω_g and a corresponding wave number k_g , the variance of the minimum detectable fluctuations of the space-time metric when measuring the light spectral lines broadening can be estimated by the following formula:

$$\sigma_h \approx \sqrt{\frac{c}{c \omega_g}} \sigma_\omega, \quad (34)$$

or based on measurement results averaging over a period of T :

$$\sigma_h \approx \frac{1}{\sqrt{T\omega_g}} \sqrt{\frac{c}{c\omega_g}} \sigma_\omega. \quad (35)$$

If $T = 3 \cdot 10^7$ sec, then

$$\sigma_h \approx 10^{-31} \dots 10^{-32}. \quad (36)$$

To calculate the gravitational waves $\Omega_{GW}(\omega)$ energy density in terms of their amplitude h , the formula given in the work [13] can be used:

$$\Omega_{GW}(\omega) = \frac{1}{6\pi H_0^2} \omega^3 G_h(\omega), \quad (37)$$

where H_0 - Hubble parameter. Then the relic gravitational waves $\omega_g = 10^9 \dots 10^{10} \text{ s}^{-1}$ frequency limit formula is given by:

$$G_h(\omega) \approx 2 \cdot (10^{-61} \dots 10^{-64}) \Omega_{GW}(\omega) \quad (38)$$

or based on estimate of the value $\Omega_{GW}(\omega)$ on indicated frequencies within the quintessential inflation model framework [9]:

$$\sigma_h \approx 10^{-32} \dots 10^{-33}. \quad (39)$$

The resulting value (39) is close to the minimum observed value (36).

6. Conclusion

Thus, the space-time with a fluctuating metric tensor model proposed allows describing potential physical time fluctuations. It was demonstrated that the measurement of the broadening of light spectral lines spreading over the space filled with high-frequency relic gravitational waves allows us to estimate the upper limit of their amplitude and possibly explore their spectral composition.

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