The models of cosmological inflation in the context of kinetic approximation

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Abstract. In this work the building of models of cosmological inflation with approximate linear dependence of the scalar field kinetic energy on the state parameter is considered. The key parameters of cosmological perturbations are also calculated.

The kinetic approximation

The theory of the cosmological inflation successfully describes the condition of the accelerated expansion of the Universe evolution at early stages [1]. Also inflationary cosmology explains an origin of primary perturbations and predicts their spectrum.

According to the theory of inflation, the source of the primary perturbations is the quantum fluctuations of a scalar field [2]. These fluctuations had essential amplitudes in the scales of Planck length and they came nearer to scales of galaxies with almost same amplitudes during inflation. Thus, inflation connects the large-scale structure of the Universe with microscopic scales.

The resultant spectrum of the perturbations practically doesn't depend on private scenarios of inflation and has a universal form. It leads to unambiguous predictions for the CMB spectrum [3]

The theory of cosmological inflation successfully describes a condition of the accelerated expansion of the Universe at early stages of evolution [1, 2]. Also inflationary cosmology explains an origin of primary not uniformity and predicts their range. The mechanism of the initial inflationary scenario and the repeated accelerated expansion can be described in the assumption of existence of the scalar field or inflaton, defining stage of inflation and the field of quintessence which is a source of the observed accelerated expansion of the Universe.

According to the theory of inflation, the source of the primary perturbations is the quantum fluctuations a of a scalar field. These fluctuations had essential amplitudes in the scales of Planck length and they came nearer to scales of galaxies with almost same amplitudes during inflation. Thus, inflation connects the large-scale structure of the Universe with microscopic scales. The resultant spectrum of the perturbations practically doesn't depend on private scenarios of inflation and has a universal form. It leads to unambiguous predictions for the CMB spectrum.

Cosmological acceleration specifies that now in the Universe evenly distributed slowly changing space liquid with negative pressure, called by dark energy, dominates.

For the specification of various types of cosmic liquid the phenomenological ratio between a pressure p and an energy density ε, $p = w\varepsilon$ is usually used. Modern experiments [4] testify that the Universe is spatially flat and now the parameter of a condition of dark energy is $w = -1 \pm 0.1$.

Standard way of obtaining time-dependent the parameter of a state is inclusion of scalar fields in cosmological model. Under sufficiently general assumptions, within four-dimensional quintessential model with one scalar field $-1 < w < -1/3$ and phantom models of $w < -1$.

Models of inflation are set by a type of effective potential $V=V(\phi)$. In this case, potential determines the behavior of a scalar field ϕ which comes down to $V(\phi)$ minimum. The end of inflation leads to violation of conditions of slow rolling, the field oscillates about a minimum and process of reheating begins. This process includes at once some various stages, such as the decay of inflation condensate, the birth of particles of standard model and their thermalization.

The equations of scalar field dynamics in flat Friedmann-Robertson-Walker Universe in units $8\pi G=c=1$ are [3]

$$3H^2=\frac{1}{2}\dot{\phi}^2+V(\phi), \tag{1}$$

$$\ddot{\phi}+3H\dot{\phi}=-\frac{dV(\phi)}{d\phi}, \tag{2}$$

$$\dot{H}=-\frac{1}{2}\dot{\phi}^2 . \tag{3}$$

From three equations of the scalar field dynamics are independent two only.

The building of consistent model of cosmological inflation requires performance of the following conditions: the existence of the stage of accelerated expansion which means $-1<w<-1/3$, the stage of the accelerated expansion comes to the end with a reheating with the subsequent birth of photons, that is transition to a stage of radiation domination to which corresponds $w=1/3$, the correspondence of received cosmological parameters to the observations.

The usual method for studying of inflationary dynamics is slow roll approximation, which means simplification of the equations of the scalar field dynamics by neglecting of kinetic energy [3].

Thus, in the case of the slow roll approximation $\frac{1}{2}\dot{\phi}^2\approx 0$, that limits a form of potential and provides the existence of an inflationary stage.

The energy density and the pressure for a scalar field

$$\varepsilon=\frac{1}{2}\dot{\phi}^2+V=X+V, \tag{4}$$

$$p=\frac{1}{2}\dot{\phi}^2-V=X-V, \tag{5}$$

where X is kinetic energy of the scalar field ϕ.

The state parameter for the slow roll approximation

$$w=\frac{\frac{1}{2}\dot{\phi}-V}{\frac{1}{2}\dot{\phi}+V}\approx -1 \tag{6}$$

and Hubble parameter $H\approx const$.

It is also possible to receive the exact solutions of the equations (1) – (3) by means of various methods for arbitrary potentials [5-17]. In case of exact solutions, for which restrictions for a form of potential are absent [9], the proof of existence of a stage of the accelerated expansion and an exit from inflation, proceeding from a type of a scale factor or the state parameter is required.

In this work the possibility of solutions of the scalar field dynamical equations without the restrictions for a form of potential and kinetic energy with exit from the inflationary stage is considered.

We will consider kinetic energy X of the scalar field as linear function of the state parameter so that at $w=-1$the kinetic energy $X=0$, which corresponds to the accelerated expansion caused by the cosmological constant

$$X=\beta\left(w+1\right). \tag{7}$$

The state parameter

$$w=\frac{X-V}{X+V}\ . \tag{8}$$

Substituting (8) in the equation (7), taking into account the equation (1), we will receive $3H^2=X+V=2\beta$, $H=const$.

For the phantom fields $\beta<0$ and $w=\left(X+V\right)/\left(X-V\right)$what again leads to result $H=const$.

Therefore, determination of kinetic energy of the scalar field in the form of $X\approx\beta\left(w+1\right)$, which we will call *kinetic approximation*, provides the accelerated expansion $H\approx const$ and gives the method of solution of the scalar fields dynamical equations by the choice of the state parameter w.

One can rewrite the equations (1) and (3) in the other form

$$V=3H^2-X\ , \tag{9}$$

$$\dot{H}=V-3H^2, \tag{10}$$

$$\dot{\phi}=\pm\sqrt{2}\sqrt{3H^2-V}\ . \tag{11}$$

Substituting the expression for kinetic energy, we get

$$V=3H^2-\beta\left(w+1\right), \tag{12}$$

$$\dot{H}=-\beta\left(w+1\right), \tag{13}$$

$$\dot{\phi}=\pm\sqrt{2\beta\left(w+1\right)}\,. \tag{14}$$

Now we will consider various cases of the state parameter as the function of Hubble parameter and time.

The state parameter as the function of Hubble parameter

For the state parameter $w\left(H\right)=-1+\dfrac{AH^n}{\beta}$ we will receive the following solutions of the equations (12) – (14)

$$V\left(\phi\right)=\frac{A^{3/2}}{\sqrt{8}}\left(2-n\right)\left(\phi+B\right)^{2/(2-n)}\left(\frac{3}{8}\left(2-n\right)^2\left(\phi+B\right)^2-\left(\frac{A}{8}\right)^{n/2}\left(2-n\right)^n\left(\phi+B\right)^n\right), \tag{15}$$

$$\phi(t)=\sqrt{\frac{8}{A}}\,\frac{\left[A(n-1)(t+C)\right]^{(2-n)/2(1-n)}}{\left(2-n\right)}-B\,, \tag{16}$$

$$H(t)=\left[A\left(n-1\right)\left(t+C\right)\right]^{1/(1-n)}, \tag{17}$$

$$a(t) = a_0 \exp\left\{ \frac{(1-n)A\left[(n-1)\right]^{1/(1-n)} (t+C)^{(2-n)/(1-n)}}{2-n} \right\}, \tag{18}$$

$$w(t) = -1 + A\left[A(n-1)(t+C)\right]^{n/(1-n)}, \tag{19}$$

where A, B, β and C are arbitrary constants, the constant n accepts the integer values ($n \neq 1, 2$).

The solutions (15) – (18) correspond to the solutions received in the paper [8] with another state parameter.

The stage of inflation is provided by approximate linear dependence of kinetic energy on the state parameter and an exit from inflation is provided by means of the special choice of parameters A, B, C and n.

The state parameter as the function of time

Now we consider solutions for the state parameter as the function of time

$$w(t) = -1 + \frac{A_1}{\beta\left(A_1 t + B_1\right)^2}, \tag{20}$$

$$V(\phi) = (3 - A_1) B_1^2 \exp\left(\pm\sqrt{2A_1}\phi\right), \tag{21}$$

$$\phi(t) = \pm\sqrt{\frac{2}{A_1}} \ln\left(\frac{1}{C_1\left(A_1 t + B_1\right)}\right), \tag{22}$$

$$H(t) = \frac{1}{\left(A_1 t + B_1\right)}, \tag{23}$$

$$a(t) = a_0\left[\left(A_1 t + B_1\right)\right]^{1/A}. \tag{24}$$

This is the model of power-law inflation which is considered in [6,14]. It is necessary the special choice of parameters for exit on the radiation domination stage, also.

The model with graceful exit

We will consider, in the context of kinetic approximation, model of cosmological inflation with graceful exit which we will set, determining the state parameter as function of time

$$w(t) = -1 - \frac{2\alpha^6\lambda^2}{\beta}\coth^2\left(\frac{2\lambda t}{\sqrt{\beta}}\right) + \frac{2\alpha^2\lambda^2}{\beta}. \tag{25}$$

The solutions of equations (12) – (14) are

$$V(\phi) = \alpha^2\lambda^2\left[\left(3\alpha^2 - 2\right)\cosh^2\left(\frac{\phi}{\alpha}\right) + 2\right], \tag{26}$$

$$\phi(t) = \alpha \ln\left[\tanh\left(\frac{\lambda t}{\sqrt{\beta}}\right)\right], \tag{27}$$

$$H(t) = \alpha^2\lambda\coth\left(\frac{2\lambda t}{\sqrt{\beta}}\right), \tag{28}$$

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$$a(t) = a_0 \sinh\left(\frac{2\lambda t}{\sqrt{\beta}}\right)^{\alpha^2/2}. \tag{29}$$

Graceful exit from inflation, reheating and further correct transition to the stage of radiation domination $(w = 1/3)$ occurs at the choice of parameter $\lambda = \pm\frac{1}{3\alpha}\sqrt{\frac{6\beta}{1-\alpha^4}}$, where $\alpha^4 < 1$.

The cosmological parameters

During inflation, quantum fluctuations of the scalar field created perturbations of metric. In a linear order, for scalar and tensor perturbations the basic parameters of cosmological perturbations on crossing of Hubble radius $(k = aH)$ for exact solutions were calculated in works [18-19]

The power spectra of scalar and tensor perturbations

$$\mathcal{P}_S(k) = -\left.\frac{H^4}{8\pi^2 \dot{H}}\right|_{k=aH}, \tag{30}$$

$$\mathcal{P}_T(k) = -\left.\frac{H^4}{2\pi^2}\right|_{k=aH}. \tag{31}$$

The spectral indexes

$$n_S(k) - 1 = \left.\frac{4\dot{H} - \frac{H\ddot{H}}{\dot{H}^2}}{\dot{H} + H^2}\right|_{k=aH}, \tag{32}$$

$$n_T(k) = \left.\frac{2\dot{H}}{\dot{H} + H^2}\right|_{k=aH}. \tag{33}$$

The tensor-to-scalar ratio

$$r = -4\frac{\dot{H}}{H^2}. \tag{34}$$

Values of cosmological parameters are limited to observational data [4]

$$10^9 \mathcal{P}_S = 2.142 \pm 0.049,$$

$$\mathcal{P}_T = r\mathcal{P}_S,$$

$$n_S = 0.9667 \pm 0.0040,$$

$$r < 0.113.$$

One can calculate parameters of cosmological perturbations from the received solutions. Also important parameter is the number of e-folds

$$N = \int_{t_i}^{t_e} H(t)dt \ge 60, \tag{35}$$

Where t_i - the time of the beginning and t_e - the time of the end of the inflationary stage.

For recalculation of the power spectra on a modern stage it is necessary to multiply their values on crossing of Hubble radius on the transfer function square. The transfer function $T(k)$ is found to be well approximated by the form [20]

$$T(k)=\frac{3j_1(k\eta_0)}{k\eta_0}\sqrt{1+1.34\left(\frac{k}{k_{eq}}\right)+2.5\left(\frac{k}{k_{eq}}\right)^2}\,, \qquad (36)$$

where η_0 - the conformal time corresponding to the modern stage, $k_{eq}=0.073\Omega_m h_0^2\ Mpc^{-1}$ is the wave number corresponding to the Hubble radius at the time of the matter-radiation equality.

Thus, the model parameters must be set taking into account the observational data.

Conclusion

In this work the method of the solution of the scalar field dynamical equations on the inflationary stage based on representation of kinetic energy by linear function of the state parameter was considered. Such representation of kinetic energy is entered for providing a stage of the accelerated expansion. Within the offered approach the model of cosmological inflation for the set state parameter with the graceful exit to a stage of radiation domination has been constructed.

Advantage of kinetic approximation approach concerning of slow roll approximation is the accounting of a scalar field kinetic energy and existence of an inflationary stage for arbitrary potentials.

Also solutions of the equations (1) – (3) for arbitrary potentials can be received by methods of exact solutions [5-17]. The existence of the stage of accelerated expansion is provided by means of kinetic approximation.

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References

[1] Starobinsky A A *Phys. Lett.* B **91** p 99
[2] Mukhanov V F, Feldman H A and Brandenberger R H *Phys. Rep.* **215** p 203
[3] Liddle A R and Lyth D H *Phys. Rep.* **231** p 1
[4] *Ade P et al. Planck 2015 results. XIII. Cosmological parameters, Astronomy & Astrophysics manuscript No. planck parameters* (Preprint astro-ph/1502.01589)
[5] Barrow J D *Phys. Lett.* B **187** p 12
[6] Lucchin F and Matarrese S *Phys. Rev.* **32** p 1316
[7] Barrow J D *Phys. Rev.* **49** 3055
[8] Schunck F E and Mielke E W *Phys. Rev.* **50** 4794
[9] Zhuravlev V M, Chervon S V and Shchigolev V K *JETP* **87** p 223
[10] Norbury J W *Phys. Lett.* B **433** p 263
[11] Starkovich S P and Cooperstock F I *Astrophysical Journal* **398** p 1
[12] Lidsey J E *Phys. Lett.* B **273** p 42
[13] Ellis G and Madsen M S *Class. Quantum Grav.* **8** p 667
[14] Salopek D S and Bond J R *Phys. Rev.* **42** 3936
[15] Muslimov A G *Class. Quantum Grav.* **7** p 231
[16] Lobo S N, Mak M K *Int. J. Mod. Phys.* **23** 1450063
[17] Hawkins R M, Lidsey J E *Phys.Rev.* **66** 023523
[18] Chervon S V, Novello M and Triay R *Grav. & Cosm.* **11** no. 4 (44) p 329
[19] Chervon S V, Fomin I V *Grav. & Cosm.* **14** no. 2 p 163
[20] Kosowsky A and Turner M S *Phys. Rev.* **52** 1739