Journal of Physics: Conference Series **730** (2016) 012014 doi:10.1088/1742-6596/730/1/012014

Cluster physics and the importance of forbiddenness

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Abstract. We review the concept of forbiddenness as introduced by Smirnov and Yu M. Tchuvil'sky [1], which states that most of the properties of nuclear reactions and cluster structure is based on structural effects (Pauli exclusion principle). This becomes also clear when one tries to obtain the ground state of the united nucleus by two heavy clusters, which is not possible when all excitations are put into the relative motion. Smirnov et al. introduced the concept of *forbiddenness*, which is defined as *the minimal number of relative oscillation quanta which have to be shifted to internal excitations of the clusters, such that the ground state can be reached.* We show that Smirnov et al. committed and error in the derivation, leading us to reevaluate the derivation of the forbiddenness. We deduce the correct expression for the forbiddenness, leading to a simple interpretation and implementation in a cluster model and some applications are presented.

1. Introduction

The cluster model in nuclear physics, as introduced by Wildermuth and Kanellopoulos [2, 3], relates the shell model of the united nucleus, with its $SU(3)$ structure for light nuclei, to the shell model of two or more clusters. For heavy clusters, the pseudo-$SU(3)$ model can be applied [4, 5], which takes into account effectively the spin-orbit interaction, allowing still a $SU(3)$ picture. In order to satisfy minimally the *Pauli-exclusion principle*, for example in a two cluster system a *minimal number of oscillation quanta* has to exists in the relative motion. However, still antisymmetrization between the two clusters has to be applied, leading to quite involved calculations. This was resolved in [6, 7], where the $SU(3)$ irreducible representations (irreps) of the clusters were coupled with the irrep of the relative motion and the result finally compared to the complete shell model content. The overlap of the list of the $SU(3)$ coupling with the shell model content gives the microscopic model space of the *Semimicroscopic Alegbraic Cluster Model* (SACM). The SACM was applied with great success to many nuclei. We only mention the study of decay preferences in clusterization of ternary systems [8], hyperdeformed systems [9], energetic considerations in clusterization [8, 10], studies of shape isomers in ^{56}Ni [11] and radiation capture in [12]. In [13] nuclei with the structure core+α where considered, of great importance in understanding fusion processes in heavy stars. These are only the most recent activities but there are many more in the past within the SACM.

Within the SACM, in order to satisfy the Pauli-exclusion principle, all missing oscillation quanta were put into the relative motion. This procedure works as long as the lighter cluster is not "too heavy". Smirnov et al. [1] demonstrated that when the light cluster is too large (starting from A greater than 12), the ground state of the united nucleus cannot be reached, except if one allows internal excitations in the clusters, i.e., part of the missing oscillation quanta are transferred to the internal excitation of the clusters. *The minimal number of quanta needed to achieve an overlap is called the forbiddenness*, n_c. They also illustrated that fusion and fission properties are governed by the forbiddenness. For example, if in a fusion process for a given system of two colliding nuclei the forbiddenness is large (the ground state can not be reached), the complete fusion is more inhibited than when the forbiddenness is lower.

Thus, in order to describe the ground state of heavy nuclei within the SACM, the forbiddenness has to be determined. Unfortunately, it turned out that the derivation of the forbiddenness in [1] is flawed and we were obliged to obtain it from first principles. This flaw forced others to define an alternative forbiddenness [15], which at least allowed to judge approximately which clusterization is more allowed compared to others, see Ref. [8].

In this contribution be will shortly explain the deduction of the forbiddenness, published elsewhere [14]. In section 2 we will discuss the derivation, showing where the considerations in [1] went wrong. As a result we will obtain a very simple rule on how to obtain the value of the forbiddenness, which also allows to do explicit calculations of heavy cluster systems in future. At the end of the same section two applications are presented and in section 3 conclusions are drawn.

2. Determination of the forbiddenness

In [1] a simple example was considered to illustrate the need of the concept of forbiddenness: ^{40}Ca+^{40}Ca $\rightarrow$ ^{80}Zr. The $U(3)$ irreps of these nuclei are $[20,20,20]$ for ^{40}Ca and $[60,60,60]$ for ^{80}Zr, i.e., all $SU(3)$ ireps are $(0,0)$. The number of oscillation quanta in ^{40}Ca is 60, thus the sum of the two clusters is 120. The number of oscillation quanta in ^{80}Zr is 180. The Wildermuth condition states that in this case additional 60 quanta have to be added in order to satisfy the Pauli exclusion principle. If all these quanta are put into the relative motion, as done in the SACM, the $SU(3)$-irrep of the relative motion is $(60,0)$, which has to be coupled with the cluster irrep $(0,0)$, leading to $(\lambda,\mu)=(60,0)$, i.e., *not to the ground state of* ^{80}*Zr* !

The solution is to allow excitations in the cluster system: Let $[n_r,0,0]$ be the irrep of the relative motion (its $SU(3)$-irrep is $(n_r,0)$), in order to couple to the ground state of the Zr-nucleus, we need an $SU(3)$-irrep $(0,n_r)$, which is obtained by the cluster $U(3)$-irrep $[40+n_r,40+n_r,40]$. Thus the number of excitation quanta in the cluster system is $n_c=2n_r$. When n_0 denoted the minimal number of oscillation quanta to add according to Wildermuth ($n_0=n_c+n_r$), then $n_0=3n_r=60$, or $n_c=2n_r=40$. This number was not obtained in [1].

We thus needed to reanalyze the calculations in order to obtain a new answer.

The coupling of the cluster system, with the $SU(3)$-irrep (λ_c,μ_c), with the relative motion $(n_r,0)$ to a total irrep (λ,μ) can be cast into the following expression of $U(3)$-irreps:

$$\begin{aligned} & [\lambda_c+\mu_c+a,\mu_c+b,c]\otimes[n_r,0,0] \\ \rightarrow & \ [\lambda_c+\mu_c+a+k_1,\mu_c+b+k_2,c+k_3] \quad , \end{aligned} \tag{1}$$

where we skipped all possible columns with three boxes in the Young diagram. For example, in the case of ^{40}Ca the $U(3)$ Young diagram is given by $[20,20,20]$, which is equivalent to $[0,0,0]$ and both give the $SU(3)$ irrep (0,0). The cluster irrep is given by $[\lambda_c+\mu_c,\mu_c,0]$ to which we

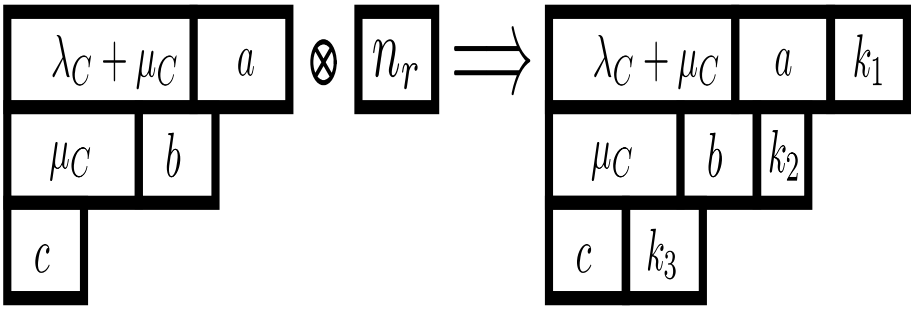

Figure 1. Graphical representation of the direct product of Eq. (1). The $a+b+c$ boxes added to the cluster irrep are distributed within the three rows. The number of quanta from the relative motion $n_c = k_1 + k_2 + k_3$ are also distributed after having applied the direct product. The arrow indicates the coupling to the Young diagrams with the structure shown. In general a sum of many irreps appear.

add a certain number of boxes, the sum given by the excitation quanta $n_c = a + b + c$. a boxes are added in the first row, b in the second and c in the third one. Finally, the relative motion is added such that k_i boxes are added to the i'th row, with $n_r = k_1 + k_2 + k_3$. In all steps, the number of boxes in the first row of a Young diagram has to be larger or equal than those in the second row and these larger or equal than those in the third row. An illustration of the coupling procedure is given in Figure 1. This leads to the following relations:

$$
\begin{aligned}
0 &\leq k_2 \leq \lambda_c + \lambda_0 \ , \ 0 \leq k_3 \leq \mu_c + \mu_0 \\
\lambda &= \lambda_c + \lambda_0 + k_1 - k_2 \ , \ \mu = \mu_c + \mu_0 + k_2 - k_3 \\
\lambda_0 &= a - b \ , \ \mu_0 = b - c \quad ,
\end{aligned}
\tag{2}
$$

where λ is given by the difference in the number of boxes of the first row to the second one and μ by the difference in the number of boxes of the second row to the third one.

The relations in (2) have to be resolved with the constraint that n_c is *minimal.* In [14] the explicit derivation is given and here we restrict to the main result:

$$
\begin{aligned}
n_c &= \max\left[0, \frac{1}{3}\left\{n_0 - (\lambda - \mu) - (2\lambda_c + \mu_c)\right\}\right] \\
&\quad + \max\left[0, \frac{1}{3}\left\{n_0 - (\lambda + 2\mu) + (\lambda_c - \mu_c)\right\}\right] \quad .
\end{aligned}
\tag{3}
$$

The n_0 is given by the Wildermuth condition and (λ, μ) is determined by the ground state or any other excited state of the united nucleus, thus these numbers are fixed. The only numbers which still can be changed are λ_c and μ_c. In order to obtain a minimal value for n_c the $(2\lambda_c + \mu_c)$ has to be *maximized* and $(\lambda_c - \mu_c)$ has to be *minimized.* The first relation implies a large deformed system, while the last one requires that this system is as oblate as possible ($\mu_c > \lambda_c$), which can be interpreted as a large most compact, oblate system.

To determine the forbiddenness is now straight forward. For example, for the case, given above, we have $n_0 = 60$ and $(\lambda, \mu) = (0, 0)$ and $(\lambda_c, \mu_c) = (0, 0)$. Thus, $n_c = \max\left[0, \frac{60}{3}\right] + \max\left[0, \frac{60}{3}\right] = \frac{120}{3} = 40$.

Table 1. A series of 2-cluster systems, all belonging to the nucleus ${}^{236}_{92}U_{144}$ are enumerated (first column). In the last column the *forbiddenness* n_c for these systems is evaluated.

No.	Two cluster system	United cluster	n_c
1	${}^{4}_{2}He_2 + {}^{232}_{90}Th_{142}$	${}^{236}_{92}U_{144}$	0
2	${}^{20}_{10}Ne_{10} + {}^{216}_{82}Pb_{134}$	${}^{236}_{92}U_{144}$	4
3	${}^{24}_{10}Ne_{10} + {}^{212}_{82}Pb_{130}$	${}^{236}_{92}U_{144}$	4
4	${}^{26}_{10}Ne_{16} + {}^{210}_{82}Pb_{128}$	${}^{236}_{92}U_{144}$	2
5	${}^{28}_{12}Mg_{16} + {}^{208}_{80}Hg_{128}$	${}^{236}_{92}U_{144}$	4
6	${}^{30}_{12}Mg_{18} + {}^{206}_{80}Hg_{126}$	${}^{236}_{92}U_{144}$	4
7	${}^{32}_{14}Si_{18} + {}^{204}_{78}Pt_{126}$	${}^{236}_{92}U_{144}$	0
8	${}^{34}_{14}Si_{20} + {}^{202}_{78}Pt_{124}$	${}^{236}_{92}U_{144}$	6
9	${}^{40}_{22}Ti_{18} + {}^{296}_{70}Yb_{126}$	${}^{236}_{92}U_{144}$	16
10	${}^{66}_{36}Kr_{30} + {}^{170}_{56}Ba_{114}$	${}^{236}_{92}U_{144}$	20
11	${}^{66}_{22}Ti_{44} + {}^{170}_{70}Yb_{100}$	${}^{236}_{92}U_{144}$	32
12	${}^{128}_{50}Sn_{78} + {}^{108}_{42}Mo_{66}$	${}^{236}_{92}U_{144}$	28
13	${}^{132}_{50}Sn_{82} + {}^{104}_{42}Mo_{62}$	${}^{236}_{92}U_{144}$	36

The forbiddenness will play an important role in extending the SACM, allowing that the light clusters have a large mass and the treatment of heavy nuclei. In the Hamiltonian of the model a dependence of n_c has to be included and also for the calculation of the spectroscopic factor. In [16] an algebraic expression for the spectroscopic factor was given, which is able to reproduce for cluster+α systems the within the $SU(3)$ model theoretically determined spectroscopic factors of light nuclei [16] by 1 to 3 percent. The formula depends on (λ_i, μ_i) (i=1,2), the $SU(3)$ irreps of the clusters in their ground state, (λ_c, μ_c), the cluster irrep, (λ, μ), the final $SU(3)$ irrep and $n_\pi = n_0 + \Delta n_\pi$, the number of quanta in the relative motion. In the new expression, (λ_c, μ_c) is changed to $(\lambda_c + \lambda_0, \mu_c + \mu_0)$ and n_π to $n_0 - n_c + \Delta n_\pi$, where Δn_π is the number of relative excitation quanta with respect to $0\hbar\omega$.

In Table 1 the forbiddenness is listed for different partitions of ^{236}U. When the light cluster has $A < 12$, the forbiddenness is zero and only in rare example it might be zero for larger clusters. In figure 2 the forbiddenness is plotted versus the mass of the light cluster. As can be appreciated, the forbiddenness increases toward larger clusters, which reflects the property that in order the system can fission into the two clusters, both clusters have to be in general in an excited state. The inverse process, i.e. fusion, also is largely inhibited allowing molecular states, but reducing the probability for fusion.

The same data are listed in Table 2 for ^{252}Cf and the forbiddenness is plotted versus the mass of the light cluster in Figure 3.

In both systems, we see the structural effect (the Pauli principle) on the structure of fission and fusion processes, as discussed in [1]. The implication of the extension of the SACM is clear: Not only we have to include in the Hamiltonian a dependence on the forbiddenness but also the algebraic expression of the spectroscopic factor, as proposed in [16].

3. Conclusions

We have derived an expression for the forbiddenness, as defined in [1]. The result is easy to implement within the SACM, for the inclusion of systems where the light cluster has a large mass. Using the pseudo-$SU(3)$ model [4, 5], this concept can also be introduced easily for heavy nuclei, to which we will dedicate our effort in near future.

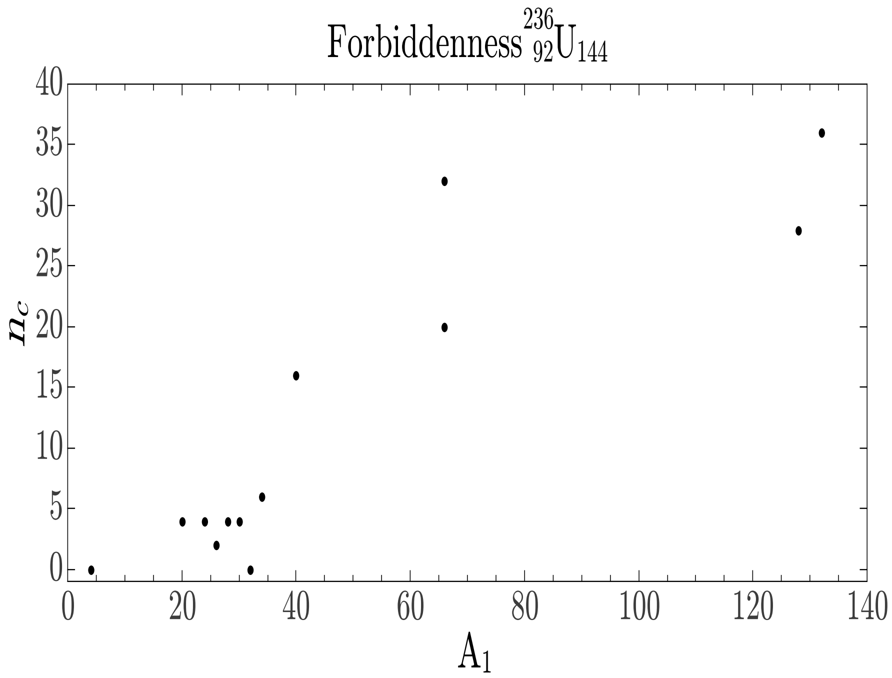

Figure 2. Graphical representation of the results from Table 1. The forbiddenness is depicted versus the mass of the lightest cluster.

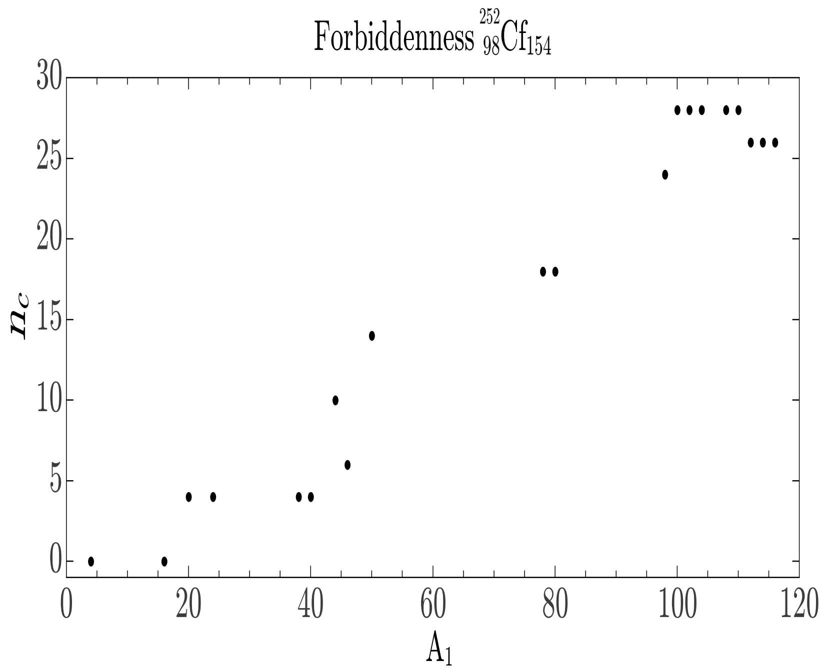

Figure 3. Graphical representation of the results from Table 2. The forbiddenness is depicted versus the mass of the lightest cluster.

Table 2. A series of 2-cluster systems, all belonging to the nucleus $^{252}_{98}\mathrm{Cf}_{154}$, $(\lambda,\mu)=(56,10)$, are enumerated (first column). In the last column the *forbiddenness* n_c for these systems is evaluated.

No.	Two cluster system	United cluster	n_c
1	$^{4}_{2}He_2 + ^{248}_{96}Cm_{152}$	$^{252}_{98}Cf_{154}$	0
2	$^{16}_{8}O_8 + ^{236}_{90}Th_{146}$	$^{252}_{98}Cf_{154}$	0
3	$^{20}_{6}C_{14} + ^{232}_{92}U_{140}$	$^{252}_{98}Cf_{154}$	4
4	$^{24}_{10}Ne_{14} + ^{228}_{88}Ra_{140}$	$^{252}_{98}Cf_{154}$	4
5	$^{38}_{14}Si_{24} + ^{214}_{84}Po_{130}$	$^{252}_{98}Cf_{154}$	4
6	$^{40}_{16}Sn_{24} + ^{212}_{82}Pb_{130}$	$^{252}_{98}Cf_{154}$	4
7	$^{44}_{16}Sn_{28} + ^{208}_{82}Pb_{126}$	$^{252}_{98}Cf_{154}$	10
8	$^{46}_{18}Ar_{28} + ^{206}_{80}Hg_{126}$	$^{252}_{98}Cf_{154}$	6
9	$^{50}_{18}Ar_{32} + ^{206}_{80}Hg_{122}$	$^{252}_{98}Cf_{154}$	14
10	$^{78}_{30}Zn_{48} + ^{174}_{68}Er_{106}$	$^{252}_{98}Cf_{154}$	18
11	$^{80}_{30}Zn_{50} + ^{172}_{68}Er_{104}$	$^{252}_{98}Cf_{154}$	18
12	$^{98}_{38}Sr_{60} + ^{152}_{60}Nd_{92}$	$^{252}_{98}Cf_{154}$	24
13	$^{100}_{38}Sr_{62} + ^{152}_{60}Nd_{92}$	$^{252}_{98}Cf_{154}$	28
14	$^{100}_{40}Zr_{60} + ^{152}_{58}Ce_{94}$	$^{252}_{98}Cf_{154}$	28
15	$^{102}_{40}Zr_{62} + ^{150}_{58}Ce_{92}$	$^{252}_{98}Cf_{154}$	28
16	$^{104}_{40}Zr_{64} + ^{148}_{58}Ce_{90}$	$^{252}_{98}Cf_{154}$	28
17	$^{104}_{42}Mo_{62} + ^{148}_{56}Ba_{92}$	$^{252}_{98}Cf_{154}$	28
18	$^{108}_{42}Mo_{66} + ^{144}_{56}Ba_{88}$	$^{252}_{98}Cf_{154}$	28
19	$^{110}_{44}Ru_{66} + ^{142}_{54}Xe_{88}$	$^{252}_{98}Cf_{154}$	28
20	$^{112}_{44}Ru_{68} + ^{140}_{54}Xe_{86}$	$^{252}_{98}Cf_{154}$	26
21	$^{114}_{44}Ru_{70} + ^{138}_{54}Xe_{84}$	$^{252}_{98}Cf_{154}$	26
22	$^{116}_{46}Pd_{70} + ^{136}_{52}Te_{84}$	$^{252}_{98}Cf_{154}$	26

Acknowledgments

The authors acknowledge financial help from DGAPA-PAPIIT (IN100315).

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