

# Production and decay of radion and Higgs in $e^+e^-$ and $\mu^+\mu^-$ colliders

Dang Van Soa<sup>1</sup>, Dao Thi Le Thuy<sup>2</sup>, Bui Thi Ha Giang<sup>2</sup>

<sup>1</sup> Hanoi Metropolitan University, 98 Duong Quang Ham, Cau Giay, Hanoi, Vietnam

<sup>2</sup>Hanoi National University of Education, 136 Xuan Thuy, Cau Giay, Hanoi, Vietnam

E-mail: [giangbth@hnue.edu.vn](mailto:giangbth@hnue.edu.vn)

**Abstract.** We analyse the production and decay of the scalar particles of the Randall – Sundrum model. The mixture of the original Higgs and radion causes the interesting physical results. In this work, the production and decay of radion  $\phi$  and Higgs  $h$  in high energy  $e^+e^-$  and  $\mu^+\mu^-$  collisions through  $\phi, h$  propagator are studied in detail.

## 1. Introduction

In 1999, the Randall – Sundrum (RS) model was conceived to solve the Higgs hierarchy problem [1], which the gravity scale and the weak scale can be naturally generated. The RS setup involves two three – branes bounding a slice of 5D compact anti-de Sitter space taken to be on an  $S^1/Z_2$  orbifold. Gravity is localized UV brane or Planck brane, while the Standard Model (SM) fields are supposed to be localized IR brane or TeV brane [2]. The Golberger – Wise mechanism is presented to stabilize the radius of the extra dimension without reintroducing a fine tuning. Fluctuations about the stabilized RS model include both tensor and scalar modes. The fluctuations of the size of the extra dimension, characterized by the scalar component of the metric otherwise known as the radion [2,3].

The radion may turn out to be the lightest new particle in the RS model. The phenomenological similarity and potential mixing of the radion and Higgs boson warrant detailed to distinguish between the radion and Higgs signals at colliders. In our earlier works [3,4], we have calculated the production cross - section of radion in the high energy collisions. In this paper, we study the production and decay of radion  $\phi$  and Higgs  $h$  in some high energy colliders. This paper is organized as follows. In Section II, we give a review of the RS model. Section III is devoted to the production and decay of radion and Higgs in high energy  $e^+e^-$  and  $\mu^+\mu^-$  collisions. Numerical evaluation is given in Sec.IV. Finally, we summarize results and make conclusions in Sec.V.

## 2. A review of Randall – Sundrum model

The RS model is based on a 5D spacetime with non – factorizable geometry [1]. The single extradimension is compactified on an  $S^1/Z_2$  orbifold of which two fixed points accommodate two three – branes (4D hyper – surfaces): the Planck brane and TeV brane.

The ordinary 4D Poincar invariance is shown to be maintained by the following classical solution



to the Einstein equation:

$$ds^2 = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu - b_0^2 dy^2, \quad \sigma(y) = m_0 b_0 |y|, \quad (1)$$

where  $x^\mu$  ( $\mu = 0, 1, 2, 3$ ) denote the coordinates on the 4D hyper – surfaces of constant  $y$  with metric  $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$ . The  $m_0$  and  $b_0$  are the fundamental mass parameter and compactification radius, respectively.

The 4D effective Lagrangian is then [5, 6, 7]

$$\mathcal{L} = -\frac{\phi_0}{\Lambda_\phi} T_\mu^\mu - \frac{1}{\hat{\Lambda}_W} T^{\mu\nu}(x) \sum_{n=1}^{\infty} h_{\mu\nu}^{(n)}(x), \quad (2)$$

where  $\Lambda_\phi \equiv \sqrt{6} M_P \Omega_0$  is the VEV of the radion field, and  $\hat{\Lambda}_W \equiv \sqrt{2} M_P \Omega_0$  ( $M_P \equiv 1/\sqrt{8\pi G_N}$ ). The  $T^{\mu\nu}$  is the energy – momentum tensor of the TeV brane localized SM fields. The  $T_\mu^\mu$  is the trace of the energy – momentum tensor, which is given at the tree level as [7, 8]

$$T_\mu^\mu = \sum_f m_f \bar{f} f - 2m_W^2 W_\mu^+ W^{-\mu} - m_Z^2 Z_\mu Z^\mu + (2m_{h_0}^2 h_0^2 - \partial_\mu h_0 \partial^\mu h_0) + \dots \quad (3)$$

The gravity – scalar mixing arises at the TeV – brane by [9]

$$S_\xi = -\xi \int d^4x \sqrt{-g_{\text{vis}}} R(g_{\text{vis}}) \hat{H}^+ \hat{H}, \quad (4)$$

where  $R(g_{\text{vis}})$  is the Ricci scalar for the induced metric on the visible brane or TeV brane,  $g_{\text{vis}}^{\mu\nu} = \Omega_b^2(x) (\eta^{\mu\nu} + \varepsilon h^{\mu\nu})$ .  $\hat{H}$  is the Higgs field before re-scaling, i.e.  $H_0 = \Omega_0 \hat{H}$ . The parameter  $\xi$  denotes the size of the mixing term. With  $\xi \neq 0$ , there neither a pure Higgs boson nor pure radion mass eigenstate. This  $\xi$  term mixes the  $h_0$  and  $\phi_0$  fields into the mass eigenstates  $h$  and  $\phi$  as given by [6, 9]:

$$\begin{pmatrix} h_0 \\ \phi_0 \end{pmatrix} = \begin{pmatrix} 1 & -6\xi\gamma/Z \\ 0 & 1/Z \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} h \\ \phi \end{pmatrix} = \begin{pmatrix} d & c \\ b & a \end{pmatrix} \begin{pmatrix} h \\ \phi \end{pmatrix}, \quad (5)$$

where

$$\gamma \equiv v_0/\Lambda_\phi, Z^2 \equiv 1 - 6\xi\gamma^2(1 + 6\xi) = \beta - 36\xi^2\gamma^2, \beta = 1 - 6\xi\gamma^2, \quad (6)$$

$$a = \frac{\cos\theta}{Z}, b = -\frac{\sin\theta}{Z}, c = \sin\theta - \frac{6\xi\gamma}{Z} \cos\theta, d = \cos\theta + \frac{6\xi\gamma}{Z} \sin\theta. \quad (7)$$

The mixing angle  $\theta$  is defined by

$$\tan 2\theta = 12\gamma\xi Z \frac{m_{h_0}^2}{m_{h_0}^2(Z^2 - 36\xi^2\gamma^2) - m_{\phi_0}^2}. \quad (8)$$

The new fields  $h$  and  $\phi$  are mass eigenstates with masses

$$m_{h,\phi}^2 = \frac{1}{2Z^2} \left[ m_{\phi_0}^2 + \beta m_{h_0}^2 \pm \sqrt{(m_{\phi_0}^2 + \beta m_{h_0}^2)^2 - 4Z^2 m_{\phi_0}^2 m_{h_0}^2} \right]. \quad (9)$$

The mixing between the states enable decays of the heavier eigenstate into the lighter eigenstates if kinematically allowed. Overall, the production cross – sections, widths and relative branching fractions can all be affected significantly by the value of the mixing parameter  $\xi$  [8, 9, 10]. We

now discuss the previous estimations on some model parameters. All phenomenological signatures of the RS model including the radion – Higgs mixing are specified by five parameters:

$$\Lambda_\phi, \frac{m_0}{M_P}, m_h, m_\phi, \xi. \quad (10)$$

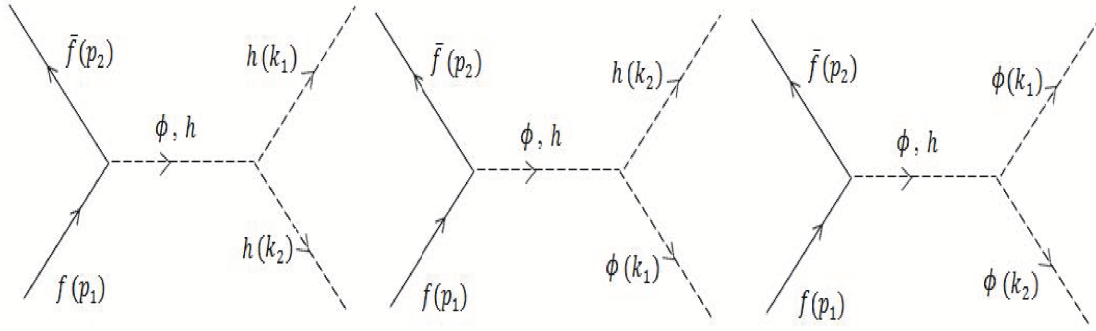
For the reliability of the RS solution, the ratio  $\frac{m_0}{M_P}$  is usually taken around  $0.01 \leq \frac{m_0}{M_P} \leq 0.1$  to avoid too large bulk curvature. Therefore, we consider the case of  $\Lambda_\phi = 5\text{TeV}$  and  $\frac{m_0}{M_P} = 0.1$ , where the effect of radion on the oblique parameters is small [11]. In the following, let us choose  $\xi = 0, \pm 1/6$ , in agreement with those in Ref.[9] with  $\xi\gamma \ll 1, Z^2 \approx 1$ .

### 3. The properties of radion $\phi$ , Higgs $h$

#### 3.1. The creation of radion $\phi$ , Higgs $h$

In this section, we consider the production of radion and Higgs in the high energy  $e^+e^-$  and  $\mu^+\mu^-$  collisions. The Feynman diagram of the process collision is shown in Fig.1.

We have amplitude squared of this collisions in the following cases:



**Figure 1.** Feynman diagrams for  $f\bar{f}$  collision through  $\phi, h$  propagator, where  $f$  is  $e^-, \mu^-$  respectively ( $p_i, k_i$  stand for the momentum)

for  $f\bar{f} \rightarrow \phi h$  through  $\phi, h$  propagator

$$M^2(f\bar{f} \rightarrow \phi h) = 4 \left[ \bar{g}_{f\bar{f}h}^2 \bar{g}_{\phi h h}^2 \frac{1}{(q^2 - m_h^2)^2} + \bar{g}_{f\bar{f}h} \bar{g}_{f\bar{f}\phi} \bar{g}_{\phi\phi h} \bar{g}_{\phi h h} \frac{1}{(q^2 - m_h^2)(q^2 - m_\phi^2)} + \bar{g}_{f\bar{f}\phi}^2 \bar{g}_{\phi\phi h}^2 \frac{1}{(q^2 - m_\phi^2)^2} \right] [(p_1 p_2) - m_f^2], \quad (11)$$

for  $f\bar{f} \rightarrow \phi\phi$  through  $\phi, h$  propagator

$$M^2(f\bar{f} \rightarrow \phi\phi) = 4 \left[ \bar{g}_{f\bar{f}h}^2 \bar{g}_{\phi\phi h}^2 \frac{1}{(q^2 - m_h^2)^2} + \bar{g}_{f\bar{f}h} \bar{g}_{f\bar{f}\phi} \bar{g}_{\phi\phi h} \bar{g}_{\phi\phi\phi} \frac{1}{(q^2 - m_h^2)(q^2 - m_\phi^2)} + \bar{g}_{f\bar{f}\phi}^2 \bar{g}_{\phi\phi\phi}^2 \frac{1}{(q^2 - m_\phi^2)^2} \right] [(p_1 p_2) - m_f^2], \quad (12)$$

for  $f\bar{f} \rightarrow hh$  through  $\phi, h$  propagator

$$M^2(f\bar{f} \rightarrow hh) = 4 \left[ \bar{g}_{f\bar{f}h}^2 \bar{g}_{h h h}^2 \frac{1}{(q^2 - m_h^2)^2} + \bar{g}_{f\bar{f}h} \bar{g}_{f\bar{f}\phi} \bar{g}_{\phi h h} \bar{g}_{h h h} \frac{1}{(q^2 - m_h^2)(q^2 - m_\phi^2)} + \bar{g}_{f\bar{f}\phi}^2 \bar{g}_{\phi h h}^2 \frac{1}{(q^2 - m_\phi^2)^2} \right] [(p_1 p_2) - m_f^2], \quad (13)$$

where

$$\begin{aligned} \bar{g}_{\phi hh} = & \frac{1}{\Lambda_\phi} [\{6b\xi(\gamma(ad+bc)+cd)+ad^2\}(k_1^2+k_2^2) \\ & + d\{12ab\gamma\xi+2bc+ad(6\xi-1)\}k_3^2 - 4d(ad+2bc)m_{h_0}^2 - 3\gamma^{-1}cd^2m_{h_0}^2], \end{aligned} \quad (14)$$

$$\bar{g}_{hhhh} = \frac{1}{\Lambda_\phi} [bd\{[12b\gamma\xi+d(6\xi+1)](k_1^2+k_2^2+k_3^2)-12dm_{h_0}^2\} - 3\gamma^{-1}d^3m_{h_0}^2], \quad (15)$$

$$\bar{g}_{\phi\phi\phi} = \frac{1}{\Lambda_\phi} [ac\{[12a\gamma\xi+c(6\xi+1)](k_1^2+k_2^2+k_3^2)-12cm_{h_0}^2\} - 3\gamma^{-1}c^3m_{h_0}^2], \quad (16)$$

$$\begin{aligned} \bar{g}_{\phi\phi h} = & \frac{1}{\Lambda_\phi} [\{6a\xi(\gamma(ad+bc)+cd)+bc^2\}(k_1^2+k_2^2) \\ & + c\{12ab\gamma\xi+2ad+bc(6\xi-1)\}k_3^2 - 4c(2ad+bc)m_{h_0}^2 - 3\gamma^{-1}c^2dm_{h_0}^2], \end{aligned} \quad (17)$$

$$\bar{g}_{f\bar{f}\phi} = -\frac{g}{2} \frac{m_f}{m_W} (c + \gamma a), \quad (18)$$

$$\bar{g}_{f\bar{f}h} = -\frac{g}{2} \frac{m_f}{m_W} (d + \gamma b). \quad (19)$$

Here,  $q = p_1 + p_2 = k_1 + k_2$  and  $s = (p_1 + p_2)^2$  is the square of the collision energy. We work in the center - of - mass frame:  $p_1 = (E_1, \vec{p})$ ,  $p_2 = (E_2, -\vec{p})$ ,  $k_1 = (E_3, \vec{k})$ ,  $k_2 = (E_4, -\vec{k})$  and denote the scattering angle by  $\theta = (\vec{p}, \vec{k})$ .

From the expressions of the differential cross - section:

$$\frac{d\sigma}{d(\cos\theta)} = \frac{1}{32\pi s} \frac{|\vec{k}|}{|\vec{p}|} |\bar{M}|^2, \quad (20)$$

where  $M$  is the scattering amplitude, we will assess the number and make the identification, evaluation of the results obtained from the dependence of the differential cross - section (DCS) by  $\cos\theta$ , the total cross - section fully follows  $\sqrt{s}$  in Sec.IV.

### 3.2. Decay of radion and Higgs

We calculate the decay widths of the radion to the SM particles as follows:

$$\Gamma(\phi \rightarrow gg) = \frac{m_\phi^3 \alpha_s^2}{8\pi(4\pi\Lambda_\phi)^2} (b_3 a)^2, \quad (21)$$

$$\Gamma(\phi \rightarrow \gamma\gamma) = \frac{m_\phi^3 \alpha_s^2}{32\pi(2\pi\Lambda_\phi)^2} [(b_2 + b_Y) a]^2, \quad (22)$$

$$\Gamma(\phi \rightarrow f\bar{f}) = \frac{1}{32\pi m_\phi} g^2 \frac{m_f^2}{m_W^2} (c + \gamma a)^2 (m_\phi^2 - 4m_f^2). \quad (23)$$

Similarly, we calculate the decay widths of the Higgs to the SM particles as follows:

$$\Gamma(h \rightarrow gg) = \frac{1}{32\pi} \frac{(-\alpha_s)^2}{(4\pi v_0)^2} m_h^3 (-2b_3 \gamma b)^2, \quad (24)$$

$$\Gamma(h \rightarrow \gamma\gamma) = \frac{1}{32\pi} \frac{\alpha^2}{(2\pi v_0)^2} m_h^3 (b_2 + b_Y)^2 (\gamma b)^2, \quad (25)$$

$$\Gamma(h \rightarrow f\bar{f}) = \frac{m_f^2}{32\pi m_W^2} \frac{g^2}{m_h} (d + \gamma b)^2 (m_h^2 - 4m_f^2). \quad (26)$$

#### 4. Numerical evaluation and discussion

We choose  $\sqrt{s} = 3\text{TeV}$  (CLIC),  $m_e = 0.00051\text{ GeV}$ ,  $m_\mu = 0.1057\text{ GeV}$ ,  $m_\phi = 10\text{ GeV}$ ,  $m_h = 125\text{ GeV}$  (CMS),  $\Lambda_\phi = 5\text{ TeV}$ ,  $b_2 = 19/6$ ,  $b_Y = -41/6$ ,  $b_3 = 7$ ,  $g = \frac{2m_W}{v_0}$ ,  $\xi = 1/6$ ,  $v_0 = 246\text{ GeV}$  [7, 10], to estimate for the cross - section as follows:

$$DCS(e^+e^- \xrightarrow{\phi, h} \phi\phi) = 8.63543 \times 10^{-18}(\text{pbar}), \quad (27)$$

$$DCS(e^+e^- \xrightarrow{\phi, h} \phi h) = 9.00065 \times 10^{-11}(\text{pbar}), \quad (28)$$

$$DCS(e^+e^- \xrightarrow{\phi, h} hh) = 2.16804 \times 10^{-13}(\text{pbar}), \quad (29)$$

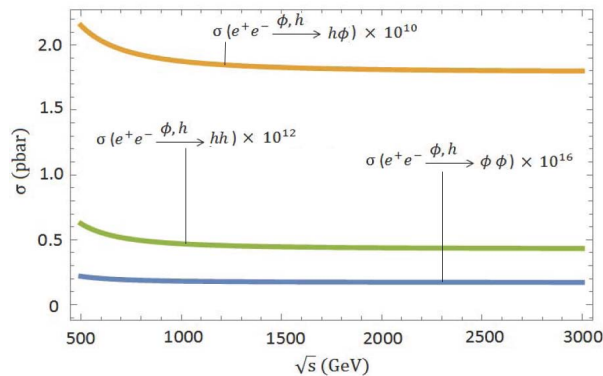
$$DCS(\mu^+\mu^- \xrightarrow{\phi, h} \phi\phi) = 3.77275 \times 10^{-13}(\text{pbar}), \quad (30)$$

$$DCS(\mu^+\mu^- \xrightarrow{\phi, h} \phi h) = 3.93231 \times 10^{-6}(\text{pbar}), \quad (31)$$

$$DCS(\mu^+\mu^- \xrightarrow{\phi, h} hh) = 9.47199 \times 10^{-9}(\text{pbar}). \quad (32)$$

Therefore, the differential cross - section is independent on the  $\cos\theta$ . DCS is minimum value in  $e^+e^- \rightarrow \phi\phi$  collision and maximum value in  $\mu^+\mu^- \rightarrow \phi, h$  collision. The total cross - section is shown as the function of the collision energy  $\sqrt{s}$  in Fig.2 and Fig.3. We show that the total cross - section is approximately independent on  $\sqrt{s}$  when  $\sqrt{s} > 1\text{ TeV}$ . Therefore, it is difficult to collect radion at very high energies.

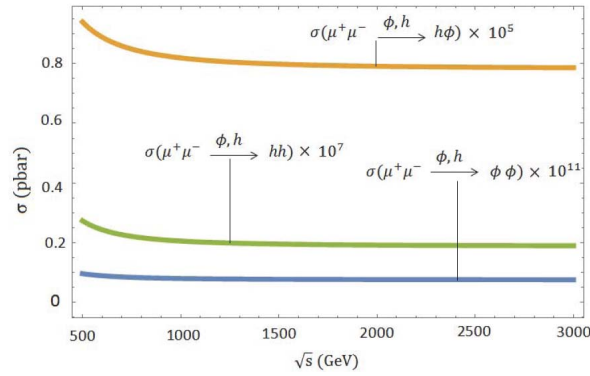
The widths of the radion and Higgs decay channels dependence on the radion mass are shown in Fig.4 and Fig.5, respectively. The mass range is chosen as  $10\text{ GeV} \leq m_\phi \leq 100\text{ GeV}$ . The dominant decay modes are  $\phi \rightarrow gg$  and  $h \rightarrow gg$ . The widths of the radion and Higgs decay channels increase or the liveable time of radion and Higgs decrease when the mass radion increases.



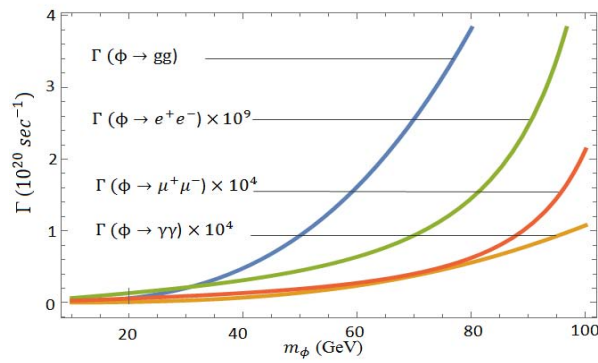
**Figure 2.** The total cross - section as a function of the collision energy  $\sqrt{s}$  in  $e^+e^-$  collision

#### 5. Conclusion

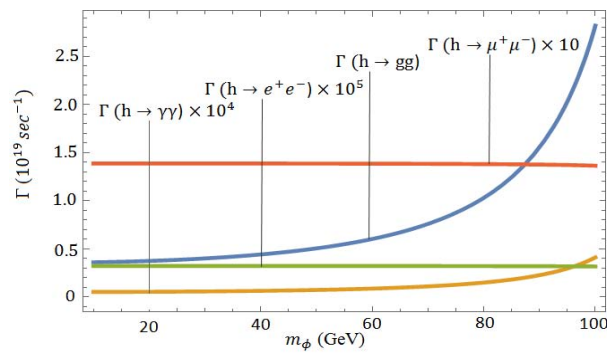
In this work, the properties of radion and Higgs in their production and decay are evaluated in detail. At high energy values, the total scattering cross section does not depend on the collision energy  $\sqrt{s}$ . However, the total scattering cross section is very small. We also have studied the decay of radion and Higgs into the SM particles. The result shows that, the  $h \rightarrow gg$  and  $\phi \rightarrow gg$  modes dominate over the other decay channels.



**Figure 3.** The total cross - section as a function of the collision energy  $\sqrt{s}$  in  $\mu^+\mu^-$  collision



**Figure 4.** The widths of the radion decay channels as the function of the radion mass  $m_\phi$



**Figure 5.** The widths of the higgs decay channels as the function of the radion mass  $m_\phi$  with  $m_h = 125\text{GeV}$

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