

# Relativistic geodesy

**J Flury**

Institut für Erdmessung / Institute of Geodesy, Leibniz Universität Hannover, Schneiderberg  
50, 30167 Hannover, Germany

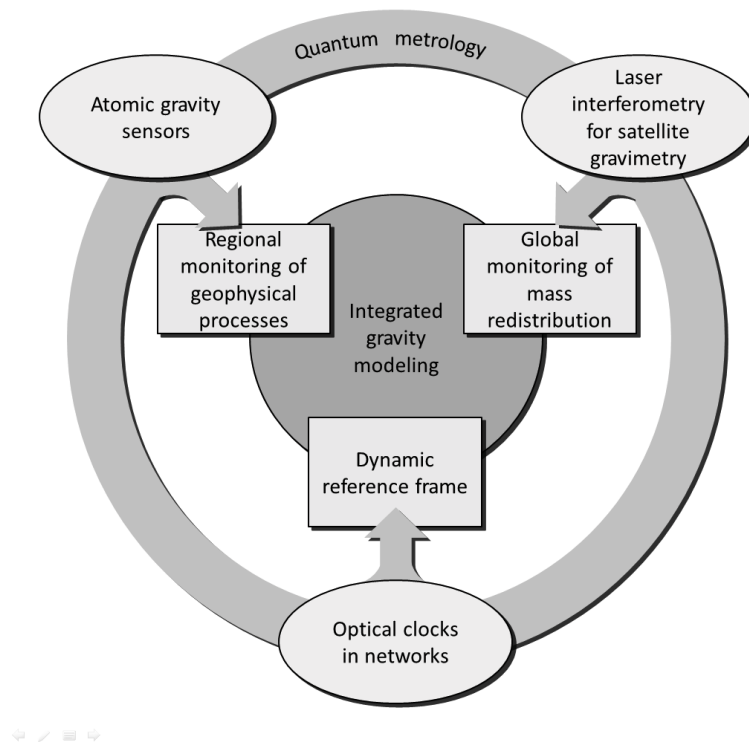
E-mail: [flury@ife.uni-hannover.de](mailto:flury@ife.uni-hannover.de)

**Abstract.** Quantum metrology enables new applications in geodesy, including relativistic geodesy. The recent progress in optical atomic clocks and in long-distance frequency transfer by optical fiber together pave the way for using measurements of the gravitational frequency redshift for geodesy. The remote comparison of frequencies generated by calibrated clocks will allow for a purely relativistic determination of differences in gravitational potential and height between stations on Earth surface (chronometric leveling). The long-term perspective is to tie potential and height differences to atomic standards in order to overcome the weaknesses and inhomogeneity of height systems determined by classical spirit leveling. Complementarily, gravity measurements with atom interferometric setups, and satellite gravimetry with space borne laser interferometers allow for new sensitivities in the measurement of the Earth's gravity field.

The goal of classical geodesy is the measurement and representation of the geometry of the Earth and of its gravity field. Already 100 years ago the theory of general relativity has shown that geometry and gravity cannot be treated separately, and in recent years relativistic effects have become more and more relevant in geodesy. Relativistic geodesy is an emerging field, and we do not want to attempt a formal definition here. For the time being we consider relativistic geodesy rather generically as the representation and use of the principles of general relativity in geodetic theory and in geodetic applications. This includes the description of curved space-time by four-dimensional coordinate systems, together with the description of trajectories, energy and momentum of test bodies such as satellites, and the description of radiation on geodetic scales and for geodetic purposes. Relativistic geodesy addresses the relativistic description of time and frequencies, and thus deals with the measurement and modeling of the relativistic gravitational frequency redshift in the Earth's gravitational field. In particular, it includes chronometric leveling, i.e., the determination of potential differences based on frequency redshift measurements that are obtained from remote comparisons of frequencies generated by ultraprecise atomic clocks. The theory on relativistic geodesy has been studied since several decades [1, 2]. In recent years, optical frequency standards have rapidly improved by orders of magnitude, and breakthroughs have been achieved in long-distance optical frequency transfer. Therefore, chronometric leveling is expected to become very relevant for geodetic applications and geodetic reference frames. It opens the perspective for tying gravitational potential and heights on Earth to an atomic reference, similar to the definition of time through atomic standards. Potsdam as the venue of the 8th FSM conference in 2015 is a very interesting fit for this topic as Helmert House and Einstein Tower both located in close vicinity on Telegraphenberg may be considered as historical symbols for classical and relativistic geodesy.

Quantum sensors such as optical atomic clocks are highly relevant for relativistic geodesy





**Figure 1.** Optical atomic clocks in networks for relativistic geodesy are complementary to other new techniques from the field of quantum metrology and quantum sensors for the measurement and modeling of the Earth's gravity field. The complementarity of techniques is in the focus of the Hannover Collaborative Research Center “Relativistic Geodesy and Gravimetry with Quantum Sensors (geo-Q)” (colour online)

because they can achieve extreme sensitivity levels. We consider relativistic geodesy and gravimetry with quantum sensors in a broader context as key elements in the development of new gravitational measurement techniques (Fig. 1). Optical atomic clocks are closely interlinked with atom interferometers with respect to quantum metrology. Atomic gravity sensors based on atom interferometry are being developed for regional monitoring of mass changes with a dedicated sampling of gravity change in space and time. Relativistic geodesy is also strongly linked to the progress in satellite gravimetry where laser interferometry is evolving as extremely precise tool for tracking testmasses in space, in order to determine the large-scale mass variations due to climate, to the Earth's water cycle and to processes in the Earth's interior.

Relativistic geodesy on Earth is characterized by the properties of the Earth's gravity field. On Earth surface, the gravity anomalies - the spatial variations of gravity acceleration - typically are in the order of  $10^{-3} \text{ m/s}^2$ , reflecting the internal density structure of the Earth. The geoid - an equipotential surface of the gravity potential that approximates the sea level - deviates from a reference ellipsoid by up to 100 m. Gravity anomalies and geoid undulations are related, the gravity acceleration vector being the gradient of the gravity potential field. Heights on Earth are defined through the gravity potential field, e.g., as the length of the curved plumbline between the Earth surface and the geoid. The determination of heights therefore requires the knowledge of the gravity field. Increasingly, also the temporal change of the gravity field is becoming relevant for geodesy and Earth sciences. Changes in the mass distribution lead to seasonal geoid variations in the order of mm, with long term (secular) changes in the order of mm/y. Temporal

change of gravity acceleration is typically in the  $10^{-8}$  m/s<sup>2</sup> range.

According to General Relativity, a clock at lower altitude - and therefore in a stronger gravity potential - is slowed down with respect to a clock at higher altitude by the relative relativistic frequency shift  $\Delta f/f$  that is related to the difference in gravity potential  $\Delta W$  through

$$\Delta W = \frac{\Delta f}{f} c^2 \quad (1)$$

where  $c$  is the velocity of light. The measurement of  $\Delta f/f$  using optical atomic clocks connected by optical frequency transfer, enables chronometric leveling (Fig. 2a). A frequency inaccuracy of  $10^{-18}$  corresponds to an error in  $\Delta W$  of  $0.1$  m<sup>2</sup>/s<sup>2</sup> or a height error of 1 cm. The potential difference  $\Delta W$  is a quantity that we also obtain from classical geodetic spirit leveling (Fig. 2b). For precision leveling, e.g., in the national first order networks,  $\Delta W$  between points  $P$  along the path of leveling lines is obtained by integrating the product of geometric leveling increments  $dn$  measured with leveling rods and gravity  $g$  measured with gravity meters:

$$\Delta W_{P_1 P_2} = - \int_{P_1}^{P_2} g \, dn \quad (2)$$

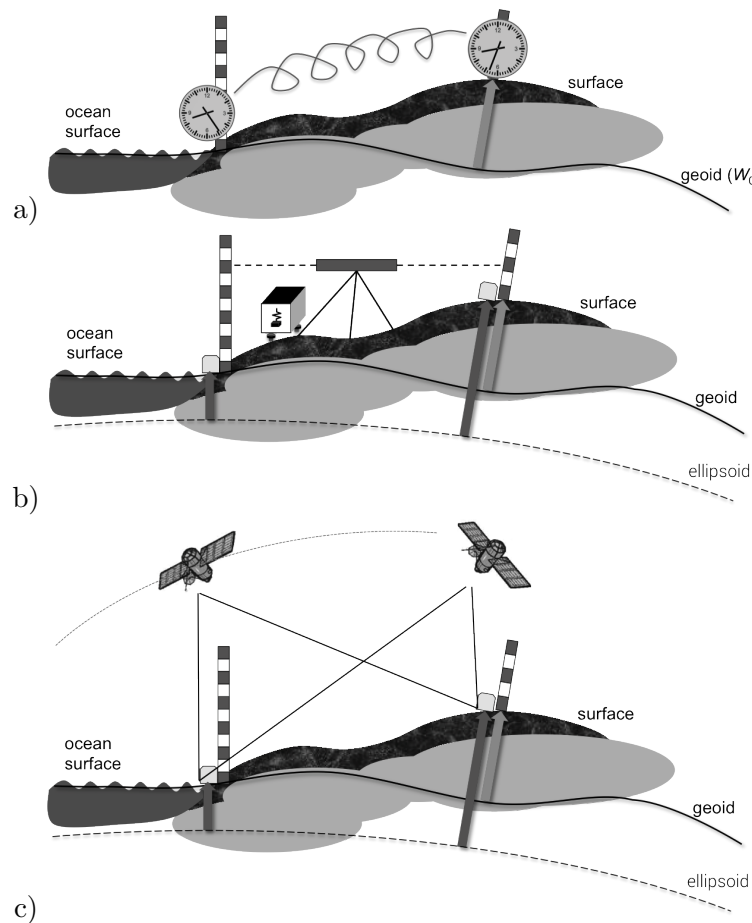
Classical precision spirit leveling and chronometric leveling both aim at the observation of potential differences in gravity potential networks. For height networks, a conversion to metric quantities is applied. Orthometric heights  $H$  at a point  $P$  are derived by dividing the potential difference  $\Delta W_{P_0 P}$  to a datum point  $P_0$  - such as a tide gauge - by the mean gravity  $\bar{g}$  along the plumbline in  $P$

$$H_P = - \frac{\Delta W_{P_0 P}}{\bar{g}} \quad (3)$$

Alternatively, normal heights are obtained by dividing the potential difference by the mean normal gravity  $\bar{\gamma}$  that is determined from a normal gravity field model

$$H_P^N = - \frac{\Delta W_{P_0 P}}{\bar{\gamma}} \quad (4)$$

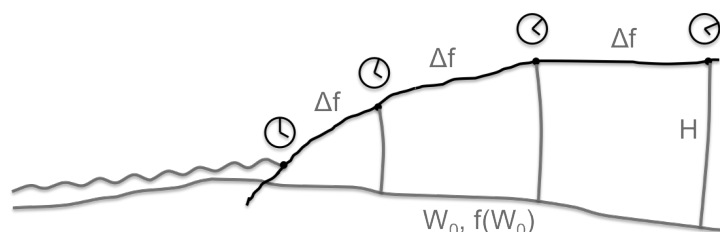
Classical spirit leveling involves a large effort, and national networks are often checked and renewed in intervals of decades only. Over large distances, error propagation in leveling networks is unfavorable. In addition, the choice of tide gauges as datum points is rather arbitrary. Tide gauges do not refer to the geoid but to sea level, which deviates from the geoid by up to 2 meters due to the dynamic sea surface topography related to ocean circulation. Even between neighboring tide gauges, sea level differences may be in the decimeter range. Vertical land movement in some regions further adds to the complexity of providing accurate and stable height networks. As a result, today still a multitude of national and regional height systems exists, which are difficult to combine and which are inhomogeneous in terms of their standards. Homogenization of height systems using space geodesy such as GPS is possible but needs geoid models to derive heights from geometric positions (Fig. 2c). This approach is called gravity field approach to height determination, or GPS-leveling. The determination of the involved geoid model depends on the combination of many different data types such as satellite gravimetry, terrestrial gravimetry and topographic mass distribution. The result is an inherently complex height reference, with a quality that is today typically in the range of few centimeters to few decimeters, although this is expected to further improve. A famous example for height inconsistencies are the height differences between the tide gauges along the Atlantic coast of the US and Canada. Here, inconsistencies between the classical leveling and oceanographic sea level models of up to 60 cm are found. Inconsistencies between the gravity field approach



**Figure 2.** a: chronometric leveling, b: classical spirit leveling, c: leveling with the gravity field approach. Source: P.O.Schmidt (colour online)

and oceanographic models are still up to 30 cm [3]. In Europe, the comparison of leveling networks with the gravity field approach show similar inconsistencies [4]. A consistent world height system at the 1 cm level would be needed for many purposes [5]: for sea level studies, as fundamental geodetic height reference, for engineering, and for geophysical and environmental studies involving land uplift and subsidence. Such a world height system in cm accuracy will, however, be difficult to achieve with classical methods.

Before this background, the current progress in frequency measurement and frequency transfer with optical clocks and phase-stabilized optical fiber is considered as major breakthrough for the determination of heights. Long-distance comparison of the Sr lattice clocks at SYRTE, Paris, and PTB, Braunschweig, has demonstrated a  $3 \cdot 10^{-17}$  frequency uncertainty level [6]. The uncertainty of the height difference and the corresponding gravitational redshift is part of this uncertainty. Chronometric leveling with clocks that have been calibrated side-by-side is expected to achieve an accuracy at this level or better, which would correspond to an accuracy of 30 cm or better for long-distance height differences. The next important step will be to use a transportable optical clock for side-by-side calibration with a stationary optical clock and subsequent long-distance frequency comparison through phase-stabilized optical fiber. It is important to note that with calibrated clocks, the quality of chronometric leveling will be determined by the frequency reproducibility instead of the absolute frequency accuracy of the involved clocks. In this respect, the requirements in standard frequency metrology and in relativistic geodesy are



**Figure 3.** Frequency redshift network providing access to the relativistic geoid definition

substantially different. If proof-of-principle experiments of chronometric leveling are successful, this will allow to tie observations of potential and height differences to atomic standards. Clock networks measuring relativistic frequency differences would make the relativistic geoid definition - the surface where clocks run with the same speed - applicable in geodesy and could provide a well defined height reference that is accessible within continents (Fig. 3). Clock networks are not expected to replace the GPS-leveling approach for height determination. They could evolve as the superior technique for the fundamental networks, whereas GPS-leveling would provide for an economic continuous coverage. The far-reaching perspective may be a global atomic clock network as a homogeneous and stable reference for gravity potential and height systems. This would be strongly complementary to the International Terrestrial Reference Frame that provides based on space geodetic measurements a globally homogeneous geometric reference at the 1 cm level or better [7].

Observations of gravity potential differences in clock networks will be complementary to satellite gravimetry (Fig.1). The satellite gravimetry missions GRACE and GOCE have provided quantifications for global and regional mass redistribution such as polar ice mass loss and ocean mass increase, as well as precision geoid models for geodesy and modeling of ocean circulation. The GRACE Follow-On satellite mission to be launched in 2017 will further improve the knowledge of the Earth's gravity field through satellite-to-satellite tracking with a laser ranging interferometer. The advantage of satellite observations is the global and dense coverage of measurements, which is used to determine complete spherical harmonic gravity models. Chronometric leveling in clock networks will provide completely independent and possibly very precise observations of gravity potential differences in specific points. The fact that chronometric leveling results refer to an atomic standard could also be valuable for the quality assessment of continuous, satellite-based models. The combination of point values with continuous gravity models is, however, not straightforward, and it will be an interesting research question whether and how high quality ground reference for the gravity potential from clock networks can stabilize or improve the satellite models.

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