Impact of turbulent phase noise on frequency transfer with asymmetric two-way ground-satellite coherent optical links

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Abstract: Bidirectional ground-satellite laser links suffer from turbulence-induced scintillation and phase distortion. We study how turbulence impacts on coherent detection capacity and on the associated phase noise that restricts clock transfer precision. We evaluate the capacity to obtain a two-way cancellation of atmospheric effects despite the asymmetry between up and down link that limits the link reciprocity. For ground-satellite links, the asymmetry is induced by point-ahead angle and possibly the use, for the ground terminal, of different transceiver diameters, in reception and emission. The quantitative analysis is obtained thanks to refined end-to-end simulations under realistic turbulence and wind conditions as well as satellite cinematic. These temporally resolved simulations allow characterizing the coherent detection in terms of time series of heterodyne efficiency for different system parameters. We show that Tip/Tilt correction on ground is mandatory at reception for the down link and as a pre-compensation of the up link. Good correlation between up and down phase noise is obtained even with asymmetric apertures of the ground transceiver and in spite of pointing ahead angle. The reduction to less than 1 rad^2 of the two-way differential phase noise is very promising for clock comparisons.

1. Introduction

Optical bi-directional links between ground stations and spacecraft in Earth or solar system orbits are of interest for a number of applications ranging from telecommunications to navigation, geodesy, time/frequency metrology and fundamental physics [1-4]. Such links promise high data rates and low phase noise in long distance clock comparison or ranging measurements. In particular, low noise, long distance clock comparisons in Earth orbit and in the solar system are of strong interest for experimental gravitation, relativistic geodesy, and time/frequency metrology [2-5]. The dominating limiting effect for such optical links is likely to be atmospheric turbulence, and in this work we study its impact on two-way clock comparisons by numerical simulations.

Modern long distance clock comparisons take advantage of two-way compensation schemes that allow mitigation of any source of phase noise that is reciprocal on the two channels, as demonstrated in optical fiber links [6, 7] and free space optical links over short horizontal distances [8–10]. Simply put, the phase difference between the two distant clocks is obtained from the half difference of the up and down link measurements, leading to a complete cancellation of any phase noise that is identical on the two links. In our case, it is then essential to characterize the reciprocity of phase fluctuations between the up and down link, and to quantify any residual phase noise resulting from non-reciprocity. As discussed in section 2, one can show that the principle of reciprocity applies to propagation through turbulence [11–13]. Implications in terms of reciprocity of the optical link have been extensively studied [9,14–21]. In practice, there are several reasons that limit the link reciprocity in ground-satellite laser links:

- Large satellite velocities imply a point ahead angle for the ground station, which means that the up and down links will cross different turbulent volumes.
- Technical considerations (available laser power on ground and on board, pointing requirements, stray light,...) may favor different aperture sizes for reception and emission on ground.
- Finite light velocity implies that the turbulent layers are crossed at different times by the up and down signal, implying a change in the turbulent layers due to wind.

Of these asymmetries the first two are largely dominant, since typical propagation times across the turbulent atmosphere are of order 100 μs making the third effect very small. Hence for our study, we aim at providing quantitative estimates of the residual phase noise after two-way compensation due to pointing ahead and transceiver asymmetry. Refined end-to-end simulations in the temporal mode - where the phase screens are moved deterministically following wind profiles and satellite cinematic - are in our opinion the only way to study realistic scenarios, for which the layered turbulent structure (profiles of C_n^2 and L_0) plays a central role.

2. Principle of reciprocity for modelling of two-way ground-satellite optical links

Before discussing the link reciprocity, let us come back to the general principle of reciprocity in propagation through turbulence, a well-known physical property [11–13]. In short, wave propagation being a linear process, there is a unique functional that allows to describe by an integral equation the forward propagation of the complex field from a plane Π_1 (terminal T1) to a plane Π_2 (terminal T2), and the backward propagation from Π_2 to Π_1:

$$\mathcal{E}_{2S} = \int h(\mathbf{r}_1, \mathbf{r}_2)\mathcal{E}_{1E}(\mathbf{r}_1)d\mathbf{r}_1 \tag{1}$$
$$\mathcal{E}_{1S} = \int h(\mathbf{r}_1, \mathbf{r}_2)\mathcal{E}_{2E}(\mathbf{r}_2)d\mathbf{r}_2 \tag{2}$$

where, for the forward propagation, $\mathcal{E}_{1E}$ is the emitted complex field in plane Π_1 and $\mathcal{E}_{2S}$ the received complex field in plane Π_2, $\mathcal{E}_{2E}$ and $\mathcal{E}_{1S}$ being the emitted and received fields for the backward propagation. $h(\mathbf{r}_1, \mathbf{r}_2)$ is the functional that characterizes the propagation medium, here the propagation through a given turbulence state at a given time. The implicit assumption is that turbulence does not evolve during the time of propagation through the turbulent volume.

In heterodyne detection one is interested in a complex quantity $\psi(t)$ called "complex coupling": the overlap integral on the pupil aperture P of the received signal electromagnetic field ($\mathcal{E}_S$) and the local oscillator field ($\mathcal{E}_L$) that reads $\psi(t) = \int_P \mathcal{E}_L^\star \mathcal{E}_S d\mathbf{r}$. Using (1) and (2) those couplings can be written as functions of the fields emitted at the opposite terminals

$$\psi_1 = \int \mathcal{E}_{1L}^\star(\mathbf{r}_1)\left(\int \mathcal{E}_{2E}(\mathbf{r}_2)h(\mathbf{r}_1, \mathbf{r}_2)d\mathbf{r}_2\right) d\mathbf{r}_1 \tag{3}$$
$$\psi_2 = \int \mathcal{E}_{1E}(\mathbf{r}_1)\left(\int \mathcal{E}_{2L}^\star(\mathbf{r}_2)\, h(\mathbf{r}_1, \mathbf{r}_2)d\mathbf{r}_2\right) d\mathbf{r}_1 \tag{4}$$

where integral orders have been exchanged to obtain a reciprocal expression of ψ_2, expressed in the plane Π_1. The complex couplings at both sides of the optical link can therefore be expressed as overlap integrals in the T1 aperture plane. They both involve complex fields resulting of the propagation from T2 to T1. These propagated fields, the terms between parenthesis, correspond on one hand to the natural propagation of $\mathcal{E}_{2E}$, the field emitted by T2, and on the other hand to a fictive back propagation through turbulence of $\mathcal{E}_{2L}^\star$, the T2 local oscillator complex conjugate.

It is interesting to note that in the particular case where $\mathcal{E}_{1E} = \mathcal{E}_{1L}^\star$ and $\mathcal{E}_{2E} = \mathcal{E}_{2L}^\star$, symmetry on each terminal between the emission and the local oscillator, then ψ_1 is equal to ψ_2. Under these

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conditions the link reciprocity is perfect, meaning that both sides of the link have strictly identical heterodyne efficiency and phase noise.

Again, these developments are simply a transposition to heterodyne detection of the work of J. Shapiro (see [17] and references therein), where we have local oscillator modes in place of fiber modes, and where our expressions deal with complex coupling instead of its square modulus, since we care about phase noise in our application.

Let us now restrict our discussion to the case of ground-satellite links. Such a link has several specificities:

1. very long distance of propagation (thousands to tens of thousands of kilometers), hence clearly corresponding to a far field case,
2. turbulent volume is concentrated in the first few tens of kilometers near ground; note that the time of propagation through turbulence remains very short (around 100 μs), even if the total propagation time to the satellite is large,
3. high satellite velocity leads to a point ahead angle between the downward and upward beams.

Items 1 and 2 imply that the turbulence coherence area in the satellite plane is much larger than the T2 (satellite) pupil size. Therefore, in the previous expressions, on can neglect the dependence in $\mathbf{r}_2$ of $h(\mathbf{r}_1, \mathbf{r}_2)$, and approximate it by $h(\mathbf{r}_1, 0)$ [23]. The complex couplings then read:

$$\psi_1 = \int \mathcal{E}_{1L}^{\star}(\mathbf{r}_1) h(\mathbf{r}_1, 0) d\mathbf{r}_1 \left(\int \mathcal{E}_{2E}(\mathbf{r}_2) d\mathbf{r}_2\right) \quad (5)$$

$$\psi_2 = \int \mathcal{E}_{1E}(\mathbf{r}_1) h(\mathbf{r}_1, 0) d\mathbf{r}_1 \left(\int \mathcal{E}_{2L}^{\star}(\mathbf{r}_2)\, d\mathbf{r}_2\right) \quad (6)$$

In both expressions the terms between parenthesis are constant factors independent of turbulence. This implies that the analysis of turbulent effects does not depend on T2 modes. Looking at (2), one can see that $h(\mathbf{r}_1, 0)$ corresponds to the downward propagation of a plane wave through turbulence. The fact that one can model the downlink with a downward plane wave propagation, neglecting the Gaussian nature of the beam coming from the satellite, is actually a usual approximation in ground-space optical links, both for incoherent and coherent detection. The reciprocal expression of ψ_2 shows that, based on the same approximation, the uplink coupling can also be calculated from this downward plane wave. Actually, this is true only when neglecting the point ahead effect mentioned in item 3. Accounting for point ahead is however easy and only implies that $h(\mathbf{r}_1, 0)$ is not exactly the same term in Eq. (5) and (6): it corresponds to the result of the propagation, through the same turbulent volume, of two downward plane waves with two propagation directions separated by the point ahead angle. Say on-axis for downlink in Eq. (5) and off-axis at point ahead for uplink in Eq. (6). There is therefore no need to simulate the upward propagation of a Gaussian beam.

3. Model and methods

PILOT is our numerical tool based on Monte-Carlo split-step propagation techniques: simulation of optical propagation through discrete turbulent random phase screens and Fresnel propagation. This code is routinely used for endoatmospheric applications [23], astronomy [24], and ground-space telecommunications modelling [25].

Figure 1 displays a sketch describing the optical propagation through turbulence for ground-space optical links. The turbulence volume can be described by discrete layers (perpendicular to the line of sight) introducing a phase perturbation on the optical beam. In a given layer, the turbulent phase statistics are well known (Von Karman spectrum), with turbulence strength characterized locally by the so-called ”refractive index structure parameter” denoted C_n^2 and the turbulence outer-scale L_0. The integrated strength along the line of sight of these wavefront distortions is quantified by the Fried parameter r_0, always expressed in plane wave in this article. Even if turbulence induces locally phase only perturbations, diffraction effects during the beam propagation progressively transform phase effects in amplitude perturbations (so-called scintillation). The scintillation pattern produced by a layer at distance L is granular with a characteristic size $\sqrt{\lambda L}$, where λ is the wavelength. Phase and scintillation effects of course limit the performance of optical time/frequency comparisons.

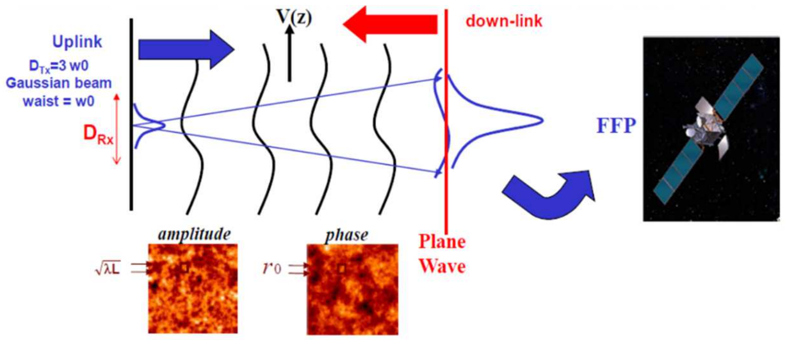

Figure 1: Turbulence effects on ground-space optical propagation: up link (Gaussian beam) and down link (plane wave). Photograph of the satellite from http://commons.wikimedia.org/wiki/File:UFO_satellite_2.jpg. Color online.

For down links, considering the beam divergence of the laser coming from the satellite and the small size of the ground receiver aperture, one can assume plane wave propagation. The up link channel is slightly more complex since it requires accounting for Gaussian beam propagation through the turbulent atmosphere. In this case, turbulence still induces phase and scintillation perturbations, as well as beam wander effects. However, we make use of the reciprocity principle to compute the up link performance from plane wave downward propagation coupled to a Gaussian reception mode. We thus compute two downward plane wave propagations along two axis separated by the point ahead angle.

We run the wave-optic propagation code PILOT at the working wavelength of λ = 1064 nm. We use a vertical C_n^2 profile covering the [0,20 km] altitudes modeled by a Hufnagel-Valley profile above the boundary layer (at about 1.5 km) linked to the Monin-Obukov similitude laws below. We use 10 phase screens distributed non-uniformly. Five of them are put in the first km, inside the boundary layer, to sample the significant turbulence. The other five are placed at 4, 6, 10, 14 and 18 km. We consider an oblique line of sight with 20° elevation. We study a moderate turbulence condition with r_0 = 13 cm (in plane wave and along this line of sight), corresponding to observation from an astronomical site at high altitude, e.g. the Calern observatory used for lunar and satellite laser ranging (1300 m altitude). We set an outer scale L_0 = 8 m which is a realistic value [26] and we found furthermore a good convergence of the 2 first moments of the heterodyne efficiency m after having varied L_0 on a [1-25] m range. Influence of turbulent parameters r_0 and L_0 is beyond the scope of this article and will be studied in a forthcoming publication [27].

Using our propagation code, we get time series of the electromagnetic fields for the signal. The associated statistical quantities $|\mathcal{E}_S(\mathbf{r},t)|$ (modulus) and $\arg[\mathcal{E}_S(\mathbf{r},t)]$ (phase) are used to derive link metrics such as the phase noise of the signal at the intermediate frequency, and heterodyne efficiency. They can be deduced from respectively the argument and the squared modulus of the complex quantity $\psi(t) = \int_P \mathcal{E}_L^\star \mathcal{E}_S d\mathbf{r}$ the overlap integral on the pupil aperture P of the electromagnetic field propagated and the local oscillator. So, we can derive temporal series of phase noise and heterodyne efficiency through Eq. (7) and Eq. (8), and last but not least, with and without Tip/Tilt correction.

$$\varphi(t) = \arg[\psi(t)] \tag{7}$$

$$m(t) = \frac{|\psi(t)|^2}{|\psi_0(t)|^2} \tag{8}$$

where $\psi_0(t)$ is the overlap integral in the absence of turbulence.

For the down link, the overlap integral computes the coupling between the on-axis downward plane wave and the ground local oscillator mode. While for the up link, and by application of the reciprocity principle (see section 2. above), we actually compute an overlap integral between the ground emission mode and a downward plane wave. Taking an on-axis plane wave corresponds to neglecting the point ahead effect, otherwise one has to use an off-axis downward plane wave at the point ahead angle. We verified consistency of this reciprocal up link modelling by comparing the results with those obtained with a real up link propagation. From these considerations it is obvious that, in the absence of point ahead, up and down link detection is identical if the ground emitter mode is equal to the ground local oscillator mode.

In practice to avoid phase jumps of 2π we will proceed by successive increments:

$$\varphi(t+dt) - \varphi(t) = \arg[\psi(t+dt)\psi^*(t)]. \tag{9}$$

The cumulative sum on all instants of the phase increment in Eq. (9) provides the phase noise temporal trajectory. Simulations can include full Tip-Tilt correction at the ground receiver aperture for the down link. The same Tip-Tilt is used as a pre-compensation of the up link at the ground emitter.

4. Performance evaluation for time/frequency transfer

We consider a satellite in highly elliptic Earth orbit (inspired by the STE-QUEST proposal [4]), at perigee (z = 3200 km) since this corresponds to the worst case in terms of pointing ahead angle (*paa* = 63 μrad), and to the largest velocity of the satellite (v = 9 km/s). To evaluate the effect of the pointing ahead, we compare in the next subsection an on-axis two-way link (with point ahead angle artificially set to zero) to a two-way link (with pointing ahead angle *paa* = 63 μrad). We model a temporal set of 8000 turbulent electromagnetic fields using the propagation code PILOT. The duration of the series is 8 s with 1 ms sampling steps.

4.1. Heterodyne efficiency vs system parameters

We study here the heterodyne efficiency for various system parameters under moderate turbulence condition. We consider two values of the ground telescope diameter $D_{\text{g-rec}}$ = [0.15, 0.4] m (receiver). The down link is coupled to a Gaussian LO of waist w_0, using the usual truncation rule of Gaussian beams $w_0 = D/3$. For the up link, the two Gaussian beams emitted from ground have waists respectively w_0 = 0.05 m and w_0 = 0.133 m, corresponding to two values of the emitter diameter $D_{\text{g-emit}}$ = 0.15 m and $D_{\text{g-emit}}$ = 0.4 m. As recalled in Sect. 2, due to reciprocity the up and down link have by construction identical performance, in particular they have the same heterodyne efficiency, if the ground emitter mode is equal to the ground local oscillator mode (diameter and waist) and in the absence of point ahead.

Table 1: Synthesis of the mean heterodyne efficiency $\langle m \rangle$ and its fluctuation $[\sigma_m^2/\langle m \rangle^2]$ for the up and down link, the two Gaussian modes, with and without Tip/Tilt correction. Performance of the up link is given with/without pointing ahead.

Gaussian modes **LO for down link or waist of uplink**	**Down link & Up link** **no pointing ahead**		**Up link** **pointing ahead**
$\boldsymbol{w_0}$ **= 0.050 m**	Turbulent	0.60 [0.14]	0.60 [0.14]
	Tip/Tilt control	0.79 [0.03]	0.78 [0.03]
$\boldsymbol{w_0}$ **= 0.133 m**	Turbulent	0.18 [0.84]	0.18 [0.84]
	Tip/Tilt control	0.53 [0.07]	0.51 [0.07]

The performance of the heterodyne efficiency in various cases is summarized in Table 1. Full turbulence performance presents lower mean value and larger variance than in the case with Tip/Tilt control (in this work we assume ideal tip/tilt control, the effect of imperfections being the subject of future work). Without Tip/Tilt control again, we observe relatively large variances of the heterodyne efficiency for up and down links using 0.4 m aperture (w_0 = 0.133 m). For such aperture, Tip/Tilt correction is mandatory to maintain reasonable signal to noise ratio (S/N), even at astronomical sites with moderate turbulence. Applying down link Tip/Tilt to up link is efficient, and essential for reciprocity. Note that the efficiency of Tip/Tilt pre-compensation is slightly minored at the percent level in presence of pointing ahead, but remains nonetheless very efficient. The slight reduction of performance is caused by tip-tilt anisoplanatism that induces a difference between on-axis and off-axis tip-tilt at point ahead. The fact that this effect is minor may seem surprising since point-ahead angle is larger than the isoplantic angle in our case. One has however to remember that the isoplanatic angle is not a relevant parameter to describe the modal normalized angular correlations of Zernike modes. Tip-tilt is known for instance to have a large angular correlation (see for instance Fig. 3 in [24]): in our case the normalized correlation is better than 95% at point ahead angle. *Figure 2* shows, as an example, the heterodyne efficiencies for the 40 cm diameter receiver downlink, with and without tip/tilt correction.

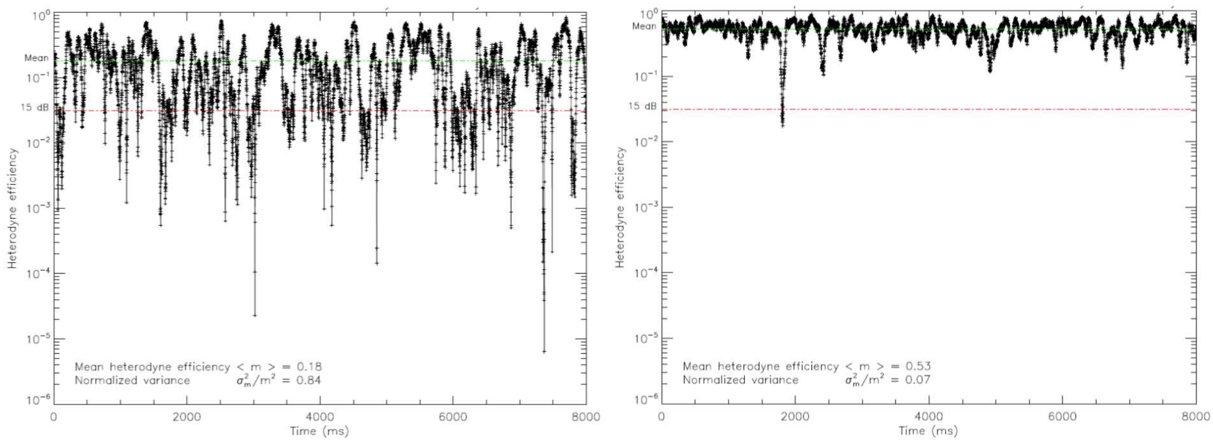

Figure 2: Down link (D_{g-rec} = 0.4 m, w_0 = 0.133 m) time series of heterodyne efficiency, without (left) and with (right) tip/tilt control. Horizontal line in red shows the detection level. Green horizontal line shows the mean heterodyne efficiency $\langle m \rangle$. Due to reciprocity (see sect. 2) uplink performance (with point ahead) is similar with only a slight degradation due to a minor reduction of tip/tilt correction efficiency (see table 1). Color online.

We indicate on *Figure 2* the loss of 15 dB in S/N which we consider as a "conservative" extinction threshold. Obviously extinctions of the signal happen more often without Tip/Tilt control and can be very deep, leading to a S/N loss of up to 10^6. With tip/tilt control the situation is much improved with only very moderate loss of S/N.

4.2. Two-way phase noise compensation

High performance clock comparison techniques use two-way signals to compensate for any phase noise that is reciprocal in the two directions. Consider two clocks A and B whose phase difference is to be determined. A typical two-way link then consists of a signal going from A to B whose phase at emission is measured on clock A and whose phase at reception is measured on clock B using e.g. a heterodyne detection. Provided the phase shift accumulated during propagation $\delta\phi_{A\to B}$ is known the phase difference of the clocks $\Delta\Phi_{AB}$ can be deduced from the measurements. Similarly, the return signal allows deducing the quantity $\Delta\Phi_{BA} = -\Delta\Phi_{AB}$ from a knowledge of $\delta\phi_{B\to A}$. Forming the half difference of the two links gives the clock phase difference affected by the link noise via the quantity $(\delta\phi_{A\to B} - \delta\phi_{B\to A})/2$, i.e. any noise on the links that is reciprocal cancels in the two-way clock comparison. As mentioned earlier, reciprocity of turbulent phase noise is perfect provided that the links are symmetric.

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But this is never the case in practice, because of the necessary point ahead angle which can be much larger than the isoplanatic angle (here ≈25 μrad), and technical considerations that may require different apertures for emission and reception on the ground.

Figure 3 shows individually the effect of the two asymmetries. In the presence of a point ahead angle of 63 μrad, the phase noise of the up and down links as well as their difference is plotted for the case where the apertures are symmetric ($D_{g-emit} = D_{g-rec} = 0.4$ m) with tip/tilt correction on the down link and the same tip/tilt correction applied as pre-compensation on the up link. Also plotted is the case of asymmetric apertures on the ground ($D_{g-emit} = 0.15$ m, $D_{g-rec} = 0.4$ m) and no point ahead angle. The latter is not a realistic situation as point ahead is unavoidable, but it shows the different residual noise behaviour coming from the aperture asymmetry. We see that the up and down links are highly correlated (ie. good reciprocity) in both cases. The residual noise due to the asymmetry has variances of less than 1 rad^2 in both cases, significantly less than on the individual up or down links. The structure of the residual phase noise is different in the two cases, but with similar magnitude.

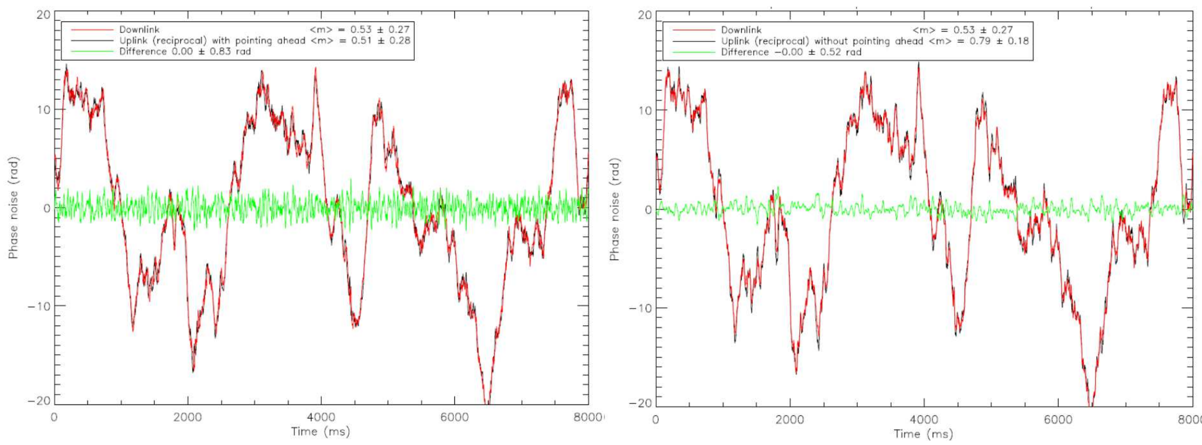

Figure 3: Two-way compensation on phase noise (phase noise/rad vs. time/ms) in the case of symmetric apertures with point ahead angle (left) and asymmetric apertures without point ahead angle (right), with tip/tilt correction for both. Color online.

Finally, *Figure 4* shows the modified Allan deviation of the down link, and the two-way combination, (up – down)/2, for the case of symmetric apertures on the ground (0.4 m) and a point ahead angle of 63 μrad. We see a very steep decrease of the frequency noise with averaging time, reaching $1.4\text{x}10^{-17}$ in fractional frequency after only 1 s averaging time. For asymmetric apertures the long term performance is somewhat degraded ($7\text{x}10^{-17}$ at 1 s averaging) [27], nonetheless these results are very promising for optical free space time/frequency comparisons.

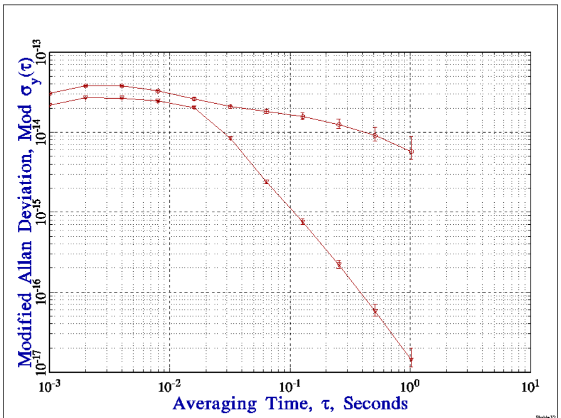

Figure 4: Modified Allan deviation for the down link, (top) and the two way combination (bottom) for symmetric apertures and a point ahead angle of 63 μrad. Color online.

5. Conclusion

In summary, we show that the down link must be tip/tilt corrected to improve the heterodyne efficiency, thus avoiding large fluctuations in signal to noise ratio with frequent extinctions. This is usually the case in all optical links as tip/tilt correction is used for active tracking. We also show that the same tip/tilt correction can be efficiently applied as pre-compensation on the up link. Then good correlation between up and down phase noise is obtained even with asymmetric apertures of the ground transceiver and in spite of the point ahead angle. For a moderate turbulence site, we quantify the reduction of the two-way differential phase noise at less than 1 rad^2. These simulation results are very promising for the feasibility of ground-satellite coherent optical links for time/frequency transfer application with outstanding performance.

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