

# Gaseous bubble oscillations in anisotropic non-Newtonian fluids under influence of high-frequency acoustic field

**R N Golykh**

Chair of Measurement and Automation Methods and Tools, Biysk Technological Institute of FSBEI "Altai State Technical University named after I.I. Polzunov", Biysk city, Russia

E-mail: romang190@gmail.com

**Abstract.** Progress of technology and medicine dictates the ever-increasing requirements (heat resistance, corrosion resistance, strength properties, impregnating ability, etc.) for non-Newtonian fluids and materials produced on their basis (epoxy resin, coating materials, liquid crystals, etc.). Materials with improved properties obtaining is possible by modification of their physicochemical structure. One of the most promising approaches to the restructuring of non-Newtonian fluids is cavitation generated by high-frequency acoustic vibrations. The efficiency of cavitation in non-Newtonian fluid is determined by dynamics of gaseous bubble. Today, bubble dynamics in isotropic non-Newtonian fluids, in which cavitation bubble shape remains spherical, is most full investigated, because the problem reduces to ordinary differential equation for spherical bubble radius. However, gaseous bubble in anisotropic fluids which are most wide kind of non-Newtonian fluids (due to orientation of macromolecules) deviates from spherical shape due to viscosity dependence on shear rate direction. Therefore, the paper presents the mathematical model of gaseous bubble dynamics in anisotropic non-Newtonian fluids. The model is based on general equations for anisotropic non-Newtonian fluid flow. The equations are solved by asymptotic decomposition of fluid flow parameters. It allowed evaluating bubble size and shape evolution depending on rheological properties of liquid and acoustic field characteristics.

## 1. Introduction

In recent decades, non-Newtonian fluids are more and more widely used in various areas of human activity. Examples are epoxy resins which are the basis for the preparation of polymeric composite materials. It is well known that medical implants based on such materials are rarely rejected by the body (in comparison with metal-based implants) and tissue irritation on the contact surface with such implants is minimized. Composite materials in aircraft structures make it possible to combine ease of the aircraft with its resistance to extreme mechanical loads as opposed to metal. Along with polymers, other examples of non-Newtonian fluids used in modern technology are coating materials, liquid crystals, etc. Today, progress of technology and medicine dictates the ever-increasing requirements (heat resistance, corrosion resistance, strength properties, impregnating ability, etc.) for non-Newtonian fluids and materials produced on their basis. Obviously, materials with improved properties are possible to obtain by modification of its physicochemical structure.

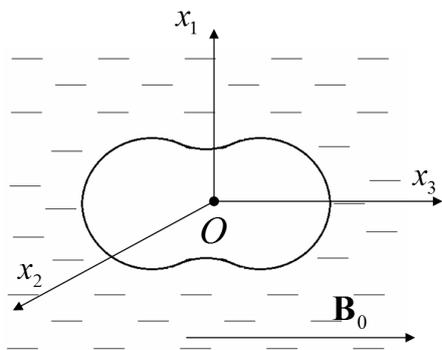


A number of researchers found that one of the most promising approaches to the restructuring non-Newtonian fluids is cavitation generated by high-frequency acoustic vibrations [1]. It allows to concentrate the low density energy (primary influence) in high density energy in the form of shock waves with the amplitude of pressure up to 100 MPa at bubble collapse and increasing the local temperature to 5000 K in cavitation nuclei. For example, this leads to the dispersion of the filler particles mixed in fluid into smaller ones [2], to reduce the viscosity of the liquid base due to breakdown of molecular bonds, etc. [3] Furthermore, associated effects that are stationary swirls acoustic flows and Eckart flows provide additional medium mixing that ensures uniform treatment.

The degree of appearance of each of the presented physical effects is primarily determined by the dynamics of the cavitation bubble or by the change of its size and shape under an external field influence (in particular, under high-frequency acoustic or ultrasonic field). Thus, the task of investigating the dynamics of a gas bubble in a non-Newtonian fluid is important.

## 2. Mathematical problem statement

Due to difficulties of experimental observation of bubble dynamics in non-Newtonian fluids caused by high speed of process, the most appropriate study method is mathematical modeling. The main mathematical difficulties of investigation of bubble dynamics in non-Newtonian fluid are caused by viscosity depending on absolute value and direction of shear rate. Today for the particular case of isotropic non-Newtonian fluids that have viscosity depending only on shear rate Euclidean norm and not depending on shear rate direction, some authors [4, 5, 6, 7] obtained ordinary differential equations for the spherical bubble radius as a function of time. However, for an anisotropic non-Newtonian fluid, whose viscosity depends not only on the module, but also on the shear direction, the problem cannot be reduced to the same ordinary differential equation, because gaseous bubble in anisotropic fluids deviates from spherical shape (Figure 1).



**Figure 1.** Schematic diagram of deformed bubble in anisotropic fluid.

Therefore, to solve this problem, a consideration of the general equations of non-Newtonian anisotropic medium flow (1–3) was made based on rheological model presented by V.S. Volkov [8]. These equations together with the speed and pressure of the fluid include the unit vector orientation  $\mathbf{B}$  of the macromolecules, which actually determines the viscosity dependence on shear rate direction. Due to small bubble sizes compared with acoustic wave length, fluid can be assumed incompressible.

$$\operatorname{div} \mathbf{v} = 0; \tag{1}$$

$$\rho \left( \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}, \nabla) \mathbf{v} \right) = -\nabla p + \operatorname{div} \mathbf{T}(\mathbf{D}, \mathbf{B}, t). \tag{2}$$

$$\frac{dB_i}{dt} = \lambda \left( \sum_{s=1}^3 D_{is} B_s - \sum_{m, n=1}^3 D_{mn} B_i B_m B_n \right); \tag{3}$$

where  $\mathbf{v}$  is the velocity of fluid, m/s;  $p$  is the pressure in fluid;  $\mathbf{T}$  is the tensor of viscosity stresses in fluid generally depending on deformation rate tensor  $\mathbf{D}$  ( $s^{-1}$ ), vector of macromolecules orientation  $\mathbf{B}$  (non-dimensional) and time  $t$  (s),  $\lambda$  is the non-dimensional “tumbling” parameter [8].

It is assumed, that at the initial time the vector of macromolecules orientation is uniform in area of fluid and equals to  $\mathbf{B}_0$  (see Figure 1), which is collinear to  $x_3$  axis and has unit norm  $|\mathbf{B}_0| = 1$ . It is easy to check, that unit norm of  $\mathbf{B}$  always remains

$$\begin{aligned} \frac{d|\mathbf{B}|^2}{dt} &= 2 \left( \mathbf{B}, \frac{d\mathbf{B}}{dt} \right) = \lambda \left( \sum_{i, s=1}^3 D_{is} B_s B_i - \sum_{i=1}^3 B_i^2 \sum_{m, n=1}^3 D_{mn} B_m B_n \right) = \\ &= \lambda \left( \sum_{i, s=1}^3 D_{is} B_s B_i - \sum_{m, n=1}^3 D_{mn} B_m B_n \right) = 0; \end{aligned} \quad (4)$$

These equations are supported by boundary conditions on bubble wall

$$\mathbf{v} = \frac{d\mathbf{r}}{dt}; \quad (5)$$

$$\mathbf{T}\mathbf{n} = (p - p_B + 2\sigma K) \mathbf{n}; \quad (6)$$

and infinity

$$\mathbf{v} = 0; \quad (7)$$

$$p = p_{st} + p_A \sin(\omega t); \quad (8)$$

where  $\mathbf{r}$  is the radius vector of bubble wall point, m;  $\mathbf{n}$  is the normal vector to bubble wall;  $K$  is the curvature of bubble wall,  $m^{-1}$ ;  $\sigma$  is the surface tension of liquid, N/m;  $p_{st}$  is the static pressure, Pa;  $p_A$  is the amplitude of acoustic pressure, Pa;  $p_B$  is the gaseous pressure in bubble interior, Pa;  $\omega$  is the angular frequency of acoustic oscillations,  $s^{-1}$ .

Tensor of viscosity stresses in anisotropic non-Newtonian fluid is defined as follows:

$$\sum_{k=1}^3 \sum_{l=1}^3 \tau_{ijkl} \frac{dT_{kl}}{dt} + T_{ij} = \sum_{k=1}^3 \sum_{l=1}^3 \eta_{ijkl} (\mathbf{D}) \left( \frac{\partial v_k}{\partial x_l} + \frac{\partial v_l}{\partial x_k} \right) \quad (9)$$

The 4th range tensors of viscosity  $\eta_{ijkl}$  and relaxation time  $\tau_{ijkl}$  included in equation (9) satisfy the conditions of symmetry:

$$\eta_{ijkl} = \eta_{jikl} = \eta_{ijlk} = \eta_{klij}; \quad \tau_{ijkl} = \tau_{jikl} = \tau_{ijlk} = \tau_{klij}. \quad (10)$$

Depending on subclass of anisotropic non-Newtonian fluid, the viscosity and relaxation time tensors yet satisfy some conditions:

- viscoelastic anisotropic fluids ( $\eta_{ijkl}(\mathbf{D}) = const$ );
- nonlinear viscous anisotropic fluids ( $\tau_{ijkl} = 0$ ).

Finite element solution of these systems of equations consumes many computing time, especially when it comes to finding the evolution of the position of the free boundary. Therefore, the author has proposed a method of solution based on asymptotical decompositions of flow parameters into a series with powers of Reynolds and Eulerian numbers.

The method is described in the next section.

### 3. Method for solution

For solution of anisotropic non-Newtonian fluid flow equations, firstly the viscosity and relaxation time tensors are decomposed into spectrum (11) that defines viscosity depending on macromolecules orientation vector [8]

$$\eta_{ijkl} = \sum_{\alpha=1}^3 \eta_{\alpha}(\mathbf{D}) a_{ijkl}^{(\alpha)}; \quad \tau_{ijkl} = \sum_{\alpha=1}^3 \tau_{\alpha} a_{ijkl}^{(\alpha)} \quad (11)$$

$$a_{ijkl}^{(1)} = \frac{\delta_{ik}\delta_{je} + \delta_{ie}\delta_{jk}}{2} - \frac{\delta_{ij}\delta_{ke}}{3} - a_{ijkl}^{(2)} - a_{ijkl}^{(3)}; \quad a_{ijkl}^{(2)} = \frac{3}{2} \left( N_{ij} - \frac{1}{3}\delta_{ij} \right) \left( N_{kl} - \frac{1}{3}\delta_{kl} \right)$$

$$a_{ijkl}^{(3)} = \frac{1}{2} \left( \delta_{ik}^{\perp} N_{je} + N_{ik} \delta_{je}^{\perp} + \delta_{ie}^{\perp} N_{jk} + N_{ie} \delta_{jk}^{\perp} \right)$$

where  $N_{ij} = B_i B_j$  is the structural tensor, determined by vector of macromolecules orientation  $\mathbf{B}$ ;  $\delta_{ij}^{\perp} = \delta_{ij} - N_{ij}$  is the transverse Kronecker symbol which makes a projection on the direction orthogonal to the unit vector  $\mathbf{B}$ .

In expression for  $a_{ijkl}^{(1)}$ , the term  $\delta_{ij}\delta_{ke}/3$  can be excluded due to fluid incompressibility. The spectral decomposition follows that viscosity of fluid is determined by four parameters (we assume that fluid has uniaxial anisotropy [8])  $\eta_{\perp} = \eta_1 = \eta_2$ ,  $\eta_{\parallel} = \eta_3$ ,  $\tau_{\perp} = \tau_1 = \tau_2$ ,  $\tau_{\parallel} = \tau_3$ .

Using the viscosity tensor representation, we define Reynolds and Eulerian number as

$$\text{Re} = \rho\omega R_0^2 \sqrt{\frac{2}{\eta_{\perp}^2 + \eta_{\parallel}^2}} \quad (12)$$

$$\text{Eu} = \frac{p_A}{(\rho\omega^2 R_0^2)} \quad (13)$$

where  $R_0$  is an equilibrium radius of bubble, m.

According to the offered method for calculation of bubble shape, the parameters of the flow are decomposed into a series of power of Reynolds and Eulerian numbers. Herewith, the tensor of viscous stresses is represented as

$$\sum_{k=1}^3 \sum_{l=1}^3 \tau_{ijkl} \frac{dT_{kl}}{dt} + T_{ij} = \frac{1}{\text{Re}} \sum_{k=1}^3 \sum_{l=1}^3 E_{ijkl}(\mathbf{D}) \left( \frac{\partial v_k}{\partial x_l} + \frac{\partial v_l}{\partial x_k} \right). \quad (14)$$

And the decompositions into the series are

$$\mathbf{v} = \sum_{n=0}^{\infty} \frac{\mathbf{v}_n}{\text{Re}^n}; \quad \mathbf{v}_n = \sum_{m=1}^{\infty} \text{Eu}^m \mathbf{v}_{nm}; \quad \text{div } \mathbf{v}_{nm} = 0; \quad (15)$$

$$p = \sum_{n=0}^{\infty} \frac{p_n}{\text{Re}^n}; \quad p_n = \sum_{m=1}^{\infty} \text{Eu}^m p_{nm}; \quad (16)$$

$$\mathbf{B} = \mathbf{B}_0 + \sum_{n=0}^{\infty} \frac{\mathbf{B}^{(n)}}{\text{Re}^n}; \quad \mathbf{B}^{(n)} = \sum_{m=1}^{\infty} \text{Eu}^m (\lambda_{nm} \mathbf{n}_{\parallel} + \mu_{nm} \mathbf{n}_{\perp}); \quad (17)$$

$$\mathbf{T} = \sum_{n=0}^{\infty} \frac{\mathbf{T}^{(n)}}{\text{Re}^n}; \quad \mathbf{T}^{(n)} = \sum_{m=1}^{\infty} \text{Eu}^m \mathbf{T}^{(nm)}; \quad (18)$$

where  $\mathbf{n}_{\parallel} = \mathbf{B}_0$ ,  $\mathbf{n}_{\perp} = (x_1\mathbf{e}_1 + x_2\mathbf{e}_2) / (x_1^2 + x_2^2)$ ,  $\lambda_{nm}$  and  $\mu_{nm}$  are the coefficients satisfying norm conditions

$$\forall N \in \mathbb{N} \cup \{0\}, \forall M \in \mathbb{N} \quad \left( \sum_{n=0}^N \sum_{m=1}^M \lambda_{nm} \right)^2 + \left( \sum_{n=0}^N \sum_{m=1}^M \mu_{nm} \right)^2 = 1 \quad (19)$$

The convergence of the series is guaranteed at  $\text{Re} \geq 3$  and  $\text{Eu} \geq 1$ , that is at high frequency acoustic field.

At  $\text{Re} \rightarrow \infty$  the equations (1, 2) reduce to the equations of flow of ideal isotropic fluid

$$\text{div } \mathbf{v}_0 = 0; \quad (20)$$

$$\rho \left( \frac{\partial \mathbf{v}_0}{\partial t} + (\mathbf{v}_0, \nabla) \mathbf{v}_0 \right) = -\nabla p_0 \quad (21)$$

with spherical symmetric solution

$$\mathbf{v}_0 = R_0^2 \frac{\partial R_0}{\partial t} \frac{\mathbf{r}}{r^3}; \quad (22)$$

$$p_0 = p_{st} + p_A \sin(\omega t) + R_0 \frac{p_B - 2\sigma K - p_{st} - p_A \sin(\omega t)}{|\mathbf{r}|}; \quad (23)$$

Using the power series, the fluid flow around the bubble is calculated by following algorithm.

- (i) Choose  $N$  and  $M$  which are maximum of  $\text{Re}$  and  $\text{Eu}$  powers in series respectively.
- (ii) To calculate the  $p_0$ ,  $\mathbf{v}_0$  in spherical symmetry representation (22, 23) and  $\mathbf{B}^{(0)}$  through  $\mathbf{v}_0$  by equation (3).

For  $(n, m)$

- (iii) To solve Poisson equation

$$\Delta p_{nm} = \text{div div } \mathbf{T}^{(nm)} - \rho \text{div} \left( \sum_{k=0}^n \sum_{l=1}^{m-1} (\mathbf{v}_{kl}, \nabla) \mathbf{v}_{(n-k)(m-l)} \right); \quad (24)$$

- (iv) To calculate viscosity tensor

$$\mathbf{T}^{(nm)} = \mathbf{T}^{(nm)}[\mathbf{v}_0, \mathbf{v}_1, \dots, \mathbf{v}_{n-1}, \mathbf{v}_{n0}, \mathbf{v}_{n1}, \dots, \mathbf{v}_{n(m-1)}] \quad (25)$$

- (v) To calculate velocity  $\mathbf{v}_{nm}$  through  $\mathbf{v}_0, \mathbf{v}_1, \dots, \mathbf{v}_{n-1}, \mathbf{v}_{n0}, \mathbf{v}_{n1}, \dots, \mathbf{v}_{n(m-1)}$  by equation (2) and the bubble wall point radius vector  $\mathbf{r}$  by boundary condition (5);
- (vi) Go to step iii for  $(n + 1, m)$ , when  $n < N$ , for  $(0, m + 1)$ , when  $n = N$  and  $m < M$ , and stop the algorithm when  $n = N$  and  $m = M$ .

The method allows us obtaining the bubble shapes at different acoustic field characteristics and fluid rheological properties. Obtained results for two subclasses of fluids (nonlinear viscous anisotropic and viscoelastic anisotropic) are presented in next section.

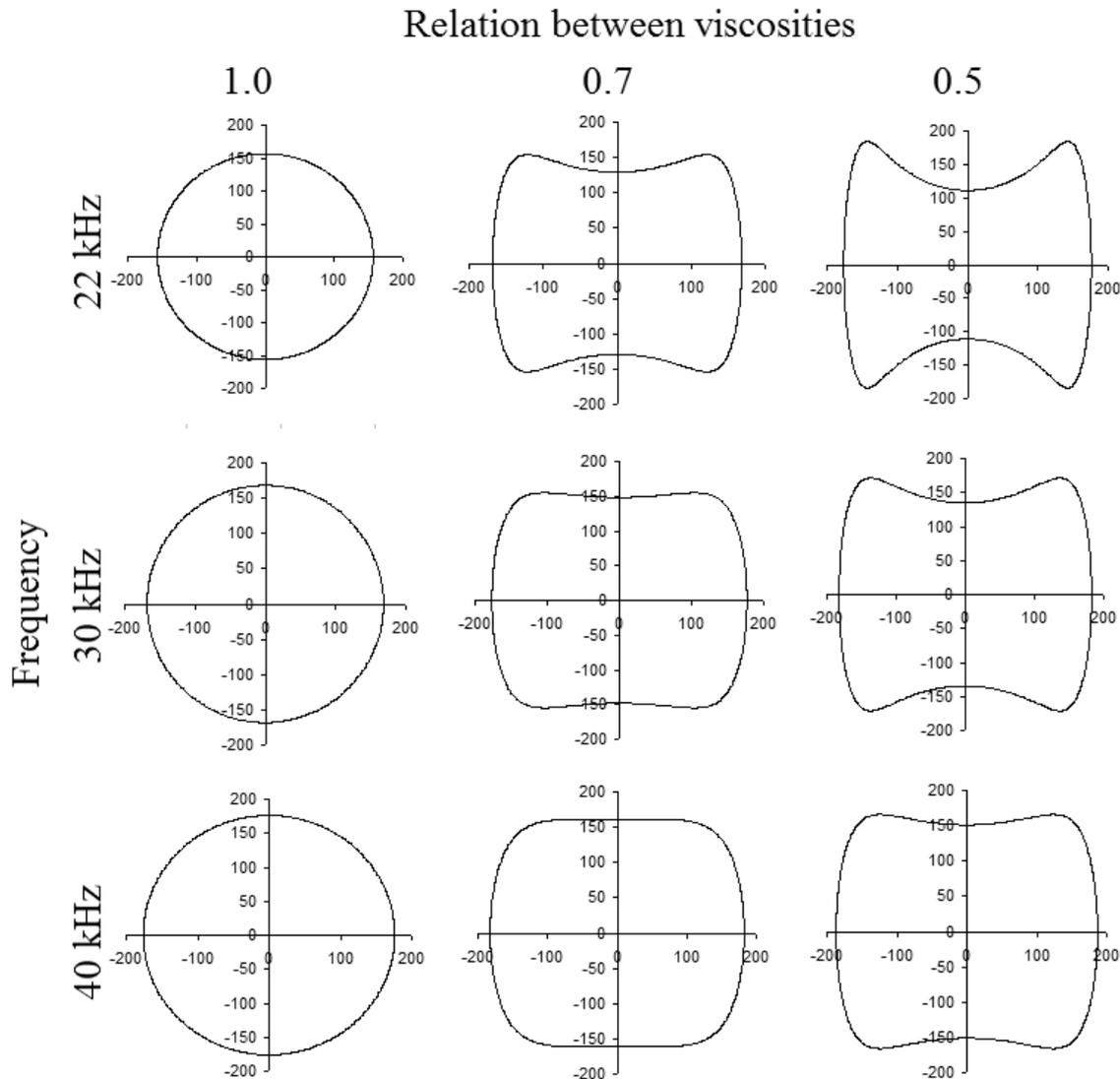
#### 4. Obtained results

##### 4.1. Nonlinear viscous anisotropic fluids case

As previously mentioned, the viscosity tensor in nonlinear viscous anisotropic fluids is presented as

$$T_{ij} = \sum_{k=1}^3 \sum_{l=1}^3 \eta_{ijkl}(\mathbf{D}) \left( \frac{\partial v_k}{\partial x_l} + \frac{\partial v_l}{\partial x_k} \right) \quad (26)$$

and fluid viscosity ( $\eta_{ijkl}$ ) is determined by two parameters  $\eta_{\parallel}$  and  $\eta_{\perp}$ . Obtained shapes of gaseous bubble at different frequencies of acoustic field and relations between viscosities ( $\eta_{\parallel}/\eta_{\perp}$ ) in different directions is presented in following Figure 2.



**Figure 2.** Bubble shapes in nonlinear viscous anisotropic fluids (unit division in axes is 1  $\mu\text{m}$ ; horizontal axis is  $x_3$ ; vertical axis is  $x_1$ ).

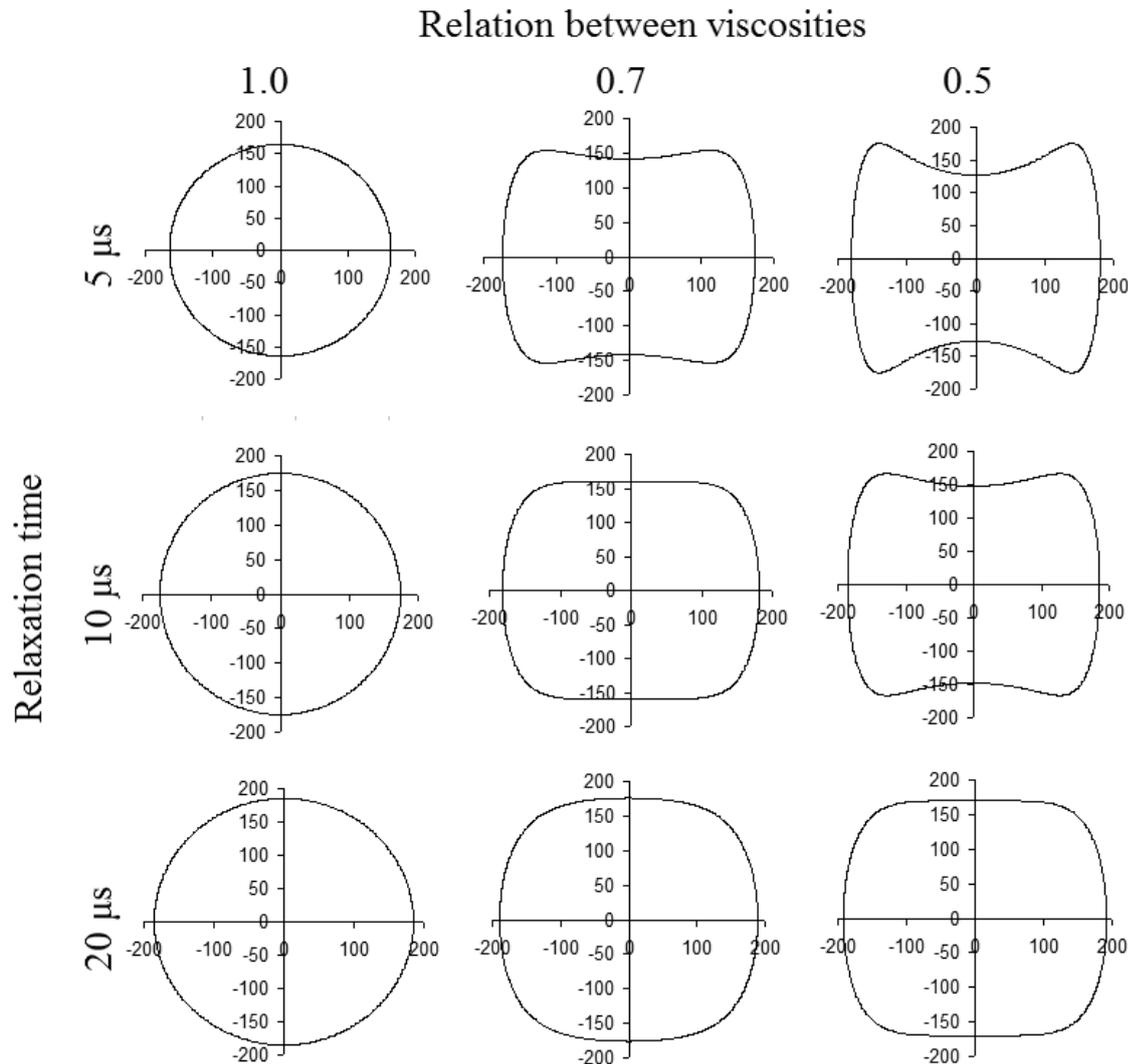
As follows from the figures, decreasing of the ratio between viscosities in the longitudinal and transverse directions leads to increase of deformation of the cavitation bubble. Thus, gaseous bubble has the largest dimensions in the longitudinal direction, since the fluid at shear in a given direction has the smallest viscosity. However, increasing of the acoustic field frequency causes the cavitation bubble shape is close to spherical. This happens due to the Reynolds number increasing and the viscosity influence weakening.

#### 4.2. Viscoelastic anisotropic fluids case

As previously mentioned, the viscosity tensor in viscoelastic anisotropic fluids is presented as

$$\sum_{k=1}^3 \sum_{l=1}^3 \tau_{ijkl} \frac{dT_{kl}}{dt} + T_{ij} = \sum_{k=1}^3 \sum_{l=1}^3 \eta_{ijkl} \left( \frac{\partial v_k}{\partial x_l} + \frac{\partial v_l}{\partial x_k} \right) \quad (27)$$

and fluid rheological properties are determined by four parameters  $\eta_{\parallel}$ ,  $\eta_{\perp}$ ,  $\tau_{\parallel}$  and  $\tau_{\perp}$ .



**Figure 3.** Bubble shapes in viscoelastic anisotropic fluids (unit division in axes is  $1 \mu\text{m}$ ; horizontal axis is  $x_3$ ; vertical axis is  $x_1$ ).

Obtained shapes of gaseous bubble at different intensities, relaxation times ( $\tau_{\perp}$ ) and relations between viscosities ( $\eta_{\parallel}/\eta_{\perp}$ ) in different directions is presented in following Figure 3.

As follows from the figures, decreasing of the ratio between viscosities in the longitudinal and transverse directions leads to an increase of deformation of the cavitation bubble, that is similar to nonlinear viscous anisotropic fluid case. However, increasing of the relaxation time causes the cavitation bubble shape is close to spherical. This is due to decreasing of viscosity tensor components absolute values, because in this case viscosity stresses is inertial, and its values is  $\sim \frac{\eta}{\omega\tau}$ .

### 5. Conclusion

Thus, the model of gaseous bubble oscillations in anisotropic non-Newtonian fluid was developed. To calculate bubble shape, the method based on asymptotical decomposition into Reynolds and Eulerian numbers power series of flow parameters was developed. This allowed evaluating the bubble size and shape achieved during oscillation and depending on rheological properties of

anisotropic non-Newtonian fluids and acoustic field characteristics. It can be used for further investigations aimed at determination of characteristics of shock wave generated during cavitation bubble collapse. Shock wave characteristics will allow to analyse cavitation influence on molecular structure of fluid and evaluate acoustic influence modes providing required properties of the fluids and materials produced on their basis.

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### References

- [1] Khmelev V N, Golykh R N, Shalunov A V, Bazhin V E and Nesterov V A 2015 *16th Int. Conf. of Young Specialists on Micro/Nanotechnologies and Electron Devices EDM Conf. Proc.* (IEEE Russia) pp 208–12
- [2] Tolochko N K 2008 *Advanced materials* [In Russian] **2** 5–9
- [3] Huang Y D, Liu L, Qiu J H and Shao L 2002 *Composit. Sci. Techn.* **62** 2153–9
- [4] Golykh R N, Shalunova A V, Nesterov V A and Ilchenko E V 2014 *Americal Journal of Engineering Research* **12** 124–33
- [5] Khmelev V N, Golykh R N, Shalunov A V, Khmelev S S and Karzakova K A 2014 *15th Int. Conf. of Young Specialists on Micro/Nanotechnologies and Electron Devices EDM Conf. Proc.* (IEEE Russia) pp 203–7
- [6] Brujan E A and Williams P R 2005 *Rheology reviews* pp 147–72
- [7] Tanasawa I and Yang W-J 1970 *Journal of Applied Physics* **41**(11) 124–33
- [8] Volkov V S 2010 *Polymer Science Series A* **52**(11) 1105–13