Torsion of circular rods at anisotropic creep

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Abstract. The torsion problems for aluminum alloy rods with circular cross section are solved. The rods are cut out from a transversely-isotropic plate with reduced resistance to creep strain in the direction of 45° to the normal direction of plate. Approximate estimates and finite element analysis show that the warping of the cross-section occurs when the rods are cut out in the longitudinal direction. In this case the value of torsion angle exceeds more than three times the value of torsion angle calculated in solving the isotropic problem. The analytical solution and finite element analysis for the rods cut out in the normal direction of plate show that the warping of the cross-section is missing, but the value of torsion angle s greater by an order of the value of torsion angle calculated in solving of the isotropic problem.

1. Introduction

Most modern structural materials have the properties of anisotropy at creep. The anisotropy can arise due to inhomogeneity of the material, which is typical for composites, or after such technological processes as rolling or pressing. Numerical and experimental verification of anisotropic models under complex stress state requires significant time and material expenses. A detailed study of the anisotropic continuum at creep published in recent years can be found in [1, 2, 3, 4]. Mostly, sheet materials have the properties of anisotropy in different directions (along, across, normal to the plate), different resistance in tension and compression, hardening and softening at creep. These features create certain difficulties in solving problems of the shaping parts and their further successful exploitation predictions.

For the alloy V95pcT2 (almost pure alloy V95 treated in mode T2) at $T = 180°\mathrm{C}$ (plate thickness 50 mm) based on experiments in tension and compression a considerable anisotropy and different resistance in tension and compression at creep [5] has been shown. The direction at the angle of 45° to the normal direction of plate was found as the weakest. The results of experiments on dependence of creep at constant stress σ on tension in the longitudinal (along), the transverse and normal directions of plate, compression in the longitudinal direction and tensile samples cut out at 45° to normal direction of plate were presented.

Power dependence for the creep strain rate $\eta = \dot{\varepsilon}^c = B\sigma^n$ approximates the experimental data of the steady state phase with coefficients: $B_\circ = 6.3 \cdot 10^{-31}$ MPa^{-n}s^{-1} for the longitudinal, transverse and normal directions of plate in tension; $B_\triangle = 3.9 \cdot 10^{-30}$ MPa^{-n}s^{-1} for the direction of 45° to the normal direction of plate in tension; $B_\bullet = 2 \cdot 10^{-31}$ MPa^{-n}s^{-1} in the longitudinal direction of plates in compression. The creep exponent $n = 10$ is the same for all directions and is independent of the sign of the applied load.

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The intensity of the process for the longitudinal direction is three times higher for tension then compression. Comparison of experimental results for tensile samples cut out at 45° to the normal direction of plate with the results of experiments in tension for longitudinal, transverse and normal directions of plate shows almost a six-time increase in the intensity of the creep process.

Problems of torsion of arbitrary cross section rods under creep conditions were solved in [6, 7] using the methods of finite element discretization and finite difference approximations for isotropic alloys. The problem of restrained torsion of a rod of rectangular cross-section and its comparison with experimental data is dicussed in [8]. Various approximate methods including the method of combining of the elastic and plastic solutions for isotropic creep medium is considered in [9]. A detailed analysis of torsion of the anisotropic elastic rods is described in [10]. The ideal plasticity ratio of anisotropic medium in torsion is investigated in [11]. Due to the physical nonlinearity the solution of the problem of torsion in anisotropic creep is possible only approximately even for circular cross-section rod.

2. Theory

At an arbitrary stress state the process of creep can be described in a general view

$$\eta_{ij} = \partial\Phi/\partial\sigma_{ij},\ \Phi = T^{n+1}/(n+1),$$

where η_{ij} are the components of creep strain rates tensor, σ_{ij} are the components of stresses tensor, Φ is the scalar potential function of stresses tensor, T is the positively defined quadratic form of stresses, n is a constant of a material [12]. For orthotropic incompressible material in the axes of the coordinates coinciding with the main axes of anisotropy, this quadratic form looks like form used by Hill for the description of anisotropic plasticity and includes six coefficients A_{ij} determined experimentally

$$T(\sigma_{ij}) = (A_{11}(\sigma_{22}-\sigma_{33})^2 + A_{22}(\sigma_{33}-\sigma_{11})^2 + A_{33}(\sigma_{11}-\sigma_{22})^2 + 2A_{12}\sigma_{12}^2 + 2A_{23}\sigma_{23}^2 + 2A_{31}\sigma_{31}^2)^{0.5}.$$

The coefficients of the quadratic form A_{ij} are defined as follows

$$2A_{11} = B_2^m + B_3^m - B_1^m, \quad 2A_{12} = 4B_{12}^m - A_{11}^m - A_{22}^m.$$

The other coefficients and components of creep strain rates are obtained by a cyclic permutation of the indices. Here B_1, B_2, B_3 are the constant one-dimensional creep in three main directions; B_{12}, B_{23}, B_{31} are similar constants in three directions along the axes in the coordinate system obtained by rotating the original coordinate system by 45°; $m = 2/(n+1)$. Then the creep strain rates are

$$\eta_{11} = T^{n-1}\Big((A_{22}+A_{33})\sigma_{11} - A_{33}\sigma_{22} - A_{22}\sigma_{33}\Big), \quad \eta_{12} = 2T^{n-1}A_{12}\sigma_{12}.$$

The dependence $G = T^n$ exists similarly to the isotropic creep, where G is the quadratic form of creep strain rates and coefficients of anisotropy.

In case of the unconstrained torsion of arbitrary cross-section rod the relationships between the creep strain rates and stresses are [9]:

$$\eta_{23} = \theta(W_{,2} + x_1) = 2T^{n-1}A_{23}\sigma_{23}, \quad \eta_{13} = \theta(W_{,1} - x_2) = 2T^{n-1}A_{31}\sigma_{31}, \tag{1}$$

where $T = (2A_{23}\sigma_{23}^2 + 2A_{31}\sigma_{31}^2)^{0.5}$; $W(x_1, x_2)$ is the function corresponding to the displacement of cross-section points (warping) in the direction x_3 along rod in torsion; θ is the rate of torsion angle per unit length. Stresses are expressed through the components of the creep strain rates

$$\sigma_{23} = \frac{\eta_{23}}{2A_{23}}G^{(1-n)/n}, \quad \sigma_{13} = \frac{\eta_{13}}{2A_{31}}G^{(1-n)/n}, \tag{2}$$

where $G = (\eta_{13}^2/(2A_{31}) + \eta_{23}^2/(2A_{23}))^{0.5}$.

The equation of equilibrium for a rod with free ends is

$$\partial\sigma_{31}/\partial x_1 + \partial\sigma_{23}/\partial x_2 = 0,$$

with the boundary condition on the contour of cross section $\sigma_{31}n_1 + \sigma_{23}n_2 = 0$.

From conditions of compatibility of creep strain rates (1) follows that

$$\frac{\partial}{\partial x_2}\Big(2A_{31}T^{n-1}\sigma_{31}\Big) - \frac{\partial}{\partial x_1}\Big(2A_{23}T^{n-1}\sigma_{23}\Big) = -2\theta. \tag{3}$$

The solution of torsion problem can be obtained by converting permitting ratios to the differential equation with respect to function of warping $W(x_1, x_2)$ [6, 7], or with respect to function of stresses $F(x_1, x_2)$, such that $\sigma_{13} = \partial F/\partial x_2, \quad \sigma_{23} = -\partial F/\partial x_1$ [9].

Torque is

$$M = \int\int(\sigma_{23}x_1 - \sigma_{31}x_2)dx_1dx_2 = 2\int\int Fdx_1dx_2.$$

Warping of the cross section is absent in torsion of a solid circular isotropic rod. Since $2A_{31} = 2A_{23} = 3B^m$ and in all directions $B = B_\circ$, the solution of (3) with the boundary condition $F(a) = 0$ on the steady-state stage of creep can be written as [9]

$$F(r) = \frac{3+1/n}{1+1/n}\frac{M}{2\pi a^2}\left(1 - \left(\frac{r}{a}\right)^{1+1/n}\right), \quad \theta = (\sqrt{3})^{n+1}\frac{B}{a}\left(\frac{3+1/n}{2\pi a^3}M\right)^n, \tag{4}$$

where a is the radius of the rod cross section, $r = (x_1^2 + x_2^2)^{1/2}$.

For rod which is cut out in the normal direction to plate (axis x_2 coincides with the normal direction of plate) the equation of equilibrium is converted to the form $\partial\sigma_{12}/\partial x_1 + \partial\sigma_{23}/\partial x_3 = 0$. In this case the warping is absent for transversely-isotropic alloy with the reduced resistance to creep strain in the direction 45° to the normal direction of plate, i.e. when

$$B_1 = B_2 = B_3 = B_{31} = B_\circ, \quad B_{12} = B_{23} = B_\Delta. \tag{5}$$

After solving the equation similar to (3) the rate of torsion angle can be written as

$$\theta = (2A_{12})^{(n+1)/2}\frac{1}{a}\left(\frac{3+1/n}{2\pi a^3}M\right)^n. \tag{6}$$

For a rod which is cut out in the longitudinal direction x_3 from transversely-isotropic alloy with constants (5) we can assume that the function of warping approximately has the form [10] $W(x_1, x_2) = Cx_1x_2$, where C is a constant that can be found through a variational principle [9]. In this case

$$\theta = \left(\frac{M}{R}\right)^n, \quad R = 4\int_0^a\int_0^{f(x_2)}\left(\frac{x_1^2(C+1)}{2A_{23}} - \frac{x_2^2(C-1)}{2A_{31}}\right)\left(\frac{x_1^2(C+1)^2}{2A_{23}} + \frac{x_2^2(C-1)^2}{2A_{31}}\right)^k dx_1dx_2, \tag{7}$$

where $f(x_2) = (a^2 - x_2^2)^{1/2}, \quad k = (1-n)/(2n)$. If $C = 0$, then (7) gives a lower estimate of the torsion angle rate.

For numerical simulations of isotropic and anisotropic material the Solid45 element of the ANSYS was tested. The hill option of TB command activates the properties of the material anisotropy in plasticity and creep. The coefficients R_{xy}, R_{yz} are connected with coefficients (5) by the relations $R_{xy} = R_{yz} = \Big(3/(4(B_\Delta/B_\circ)^m - 1)\Big)^{1/2}$, $R_{xx} = R_{yy} = R_{zz} = R_{xz} = 1$. The problem is solved in a geometrically linear statement.

3. Results and discussion

The calculations are performed for material V95pcT2 at a temperature of $T = 180°$ C based on the material properties only in tension with parameters: the radius of the rod $a = 0.02$ m, the length $L = 0.1$ m, torque $M = 3500$ N·m, the Poisson's ratio $\nu = 0.4$, Young's modulus $E = 55000$ MPa, the time of torsion $t = 600$ s.

Table 1 includes a full angle of torsion (radians) calculated analytically ($\varphi = Lt\theta$) and with finite element method (angle estimated by the displacement) without taking into account the elastic angle at $t = 0$ for the rods in the assumption of isotropic and transversely-isotropic alloy.

Table 1. Full angle of torsion.

type of anisotropy/ direction of rod cut	φ / formula	φ Solid45
isotropic	0.18 / (4)	0.20
transversely-isotropic/ longitudinal direction	0.51 / (7)	0.75
transversely-isotropic/ normal direction	1.77 / (6)	1.98

Figure 1 *a* and *b* show the isolines of the stresses σ_{13} and σ_{23} (MPa) at the torsion of isotropic rod.

Figure 2 *a* and *b* show the isolines of the same stresses at the torsion of rod cut out in the longitudinal direction x_3 (coincide with Z) made of an alloy with the reduced resistance to creep strain in the direction 45° to the normal direction of plate x_2 (coincide with Y). Numerical simulations demonstrated that the cross-section warping (displacement in direction Z, m) occurs during the torsion of circular rods (Figure 2 *c*). In experiments, this is visually confirmed by the appearance of spiral lines on the samples [13]. Similar warping occurs at torsion of anisotropic elastic circular rods [10].

Figure 3 *a* and *b* show the diagrams of contour stresses σ_{13} and σ_{23} calculated by an approximate method (2), (7). In this case, maximum stresses (2) are $\sigma_{13} = 239.9$ MPa and $\sigma_{23} = 190.6$ MPa, the torsion angle $\varphi = 0.51$. The work [14] contains a description of the test element Beam189 in ANSYS. The resulting torsion angle φ calculated by the finite element method close to the angle derived from (7) at $C = 0$ equals 0.5, i.e. the warping is absent.

The values and contours of the shear stresses of rod cut out in a normal direction of plate made of transversely-isotropic alloy almost coincide with those demonstrated in Figure 1 *a, b*. In this case, the warping of the cross section is absent, and the lines-notches in torsion remain parallel cross sections on the surface of the samples, as in case of isotropic material [13].

Note that the torsion angle obtained using Solid45 is more than 10% greater than the angle calculated analytically. It is related apparently to peculiarities of calculations of shear deformations in the physically nonlinear statement. The elastic torsion angle $\varphi_0 = 7.09 \cdot 10^{-2}$ at the initial time of loading varies for less than 0.01%.

4. Conclusions

The analytical calculations and the finite element method show a significant effect of reduced resistance to creep strain in shear direction: the torsion angle may differ by an order of magnitude

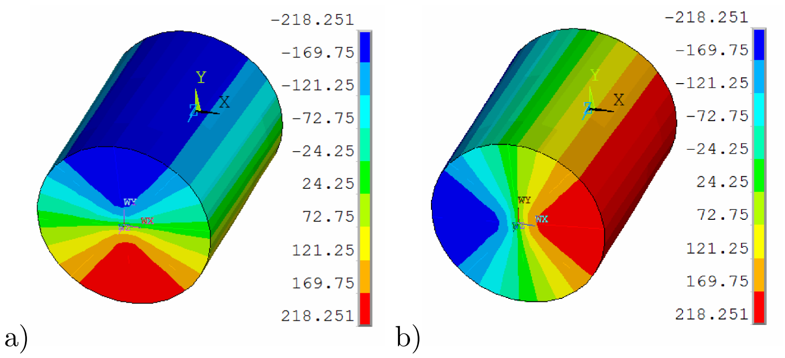

Figure 1. Isolines of the stresses σ_{13} – *a*) and σ_{23} – *b*) at the torsion of the isotropic rod

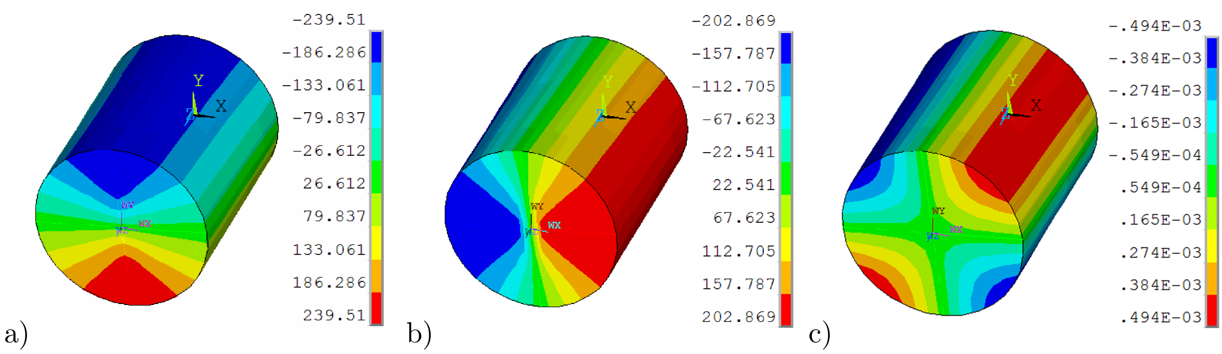

Figure 2. Isolines of the stresses σ_{13} – a), σ_{23} – b) and displacement (warping) – c) at the torsion of the rod cut out in the longitudinal direction x_3 of the transversely-isotropic plate

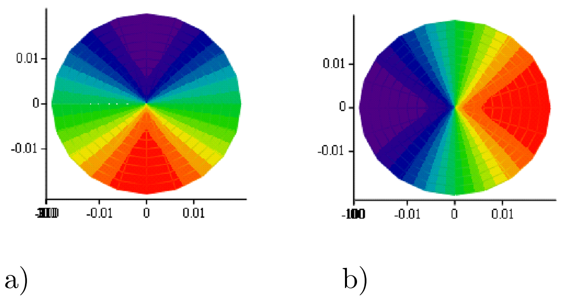

Figure 3. Isolines of the stresses σ_{13} –a) σ_{23} – b) calculated by approximate method (2),(7).

compared with the angle calculated in the isotropic formulation. Thus, the reduced resistance to creep strain in the transverse shear direction may cause reduced stiffness and strength properties of structures that need to be considered in their design and operation.

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