

Dynamic behavior of elevator compensating sheave during buffer strike

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Abstract. This paper shows an elevator dynamic model that calculates the compensating sheave motion during a buffer strike. Our equivalent 2-degree-of-freedom vibration model of an elevator system, which consists of a car, a compensating sheave, and compensating ropes, can evaluate the dynamic tension of the compensating ropes caused by a buffer strike. The constraint force, which restricts the upward motion of the compensating sheave, is estimated from the dynamic rope tension. The constraint force is represented by the summation of two vibration modes and is the function of the limited distance of the compensating sheave's upward movement. Our formula, which evaluates the maximum constraint force, shows that a shorter limited distance of the compensating sheave increases the constraint force.

1. Introduction

Most elevators used in skyscrapers include compensating ropes to satisfy the balanced rope tension between the car and the counter-weight. The compensating ropes receive tension from a compensating sheave, which is installed at the bottom of the elevator shaft. Since the compensating sheave is only suspended by the compensating ropes, the sheave can move vertically while the car is traveling.

When the car can't stop at the highest floor during abnormal situations, the counter-weight can reduce its downward speed in 1g deceleration by a buffer strike for safe stops. During buffer strikes of the counter-weight, suspension ropes lose tension, and the car continues to ascend against gravity's force. The car's upward motion also induces the same upward motion against the compensating sheave. However, the sheave can't move beyond a limited distance for safety reasons, and its restricted upward motion induces higher tension to the compensating ropes.

This paper shows an elevator dynamic model that calculates the compensating sheave motion during a buffer strike. First, the transient behavior of the compensating ropes' tension and the compensating sheave's motion during a buffer strike is validated by the multi-body dynamic model of our elevator system. Its simulation model includes the effects of the buffer's reaction force, tension loss, and restrictions on the sheave's upward motion. Second, we used an equivalent 2-degree-of-freedom (2-DOF) vibration model to calculate the rope tension and the sheave motion. Finally, we explain the relation between the upward limit of the sheave and the rope tension by an equivalent 2-DOF model.



2. Multi-body dynamic model

Figure 1 shows the multi-body dynamic model of our elevator system. Since the suspension and compensating ropes are treated as continuous bodies, they are modeled by alternating the intensive mass and spring [1].

When the counter-weight strikes the buffer at the bottom of the elevator shaft, the car jumps and the compensating sheave also moves upward (figure 2). The compensating sheave's upward motion is restricted by a mechanical stopper. For safety reasons, the compensating sheave only rises to the limit of the mechanical stopper.

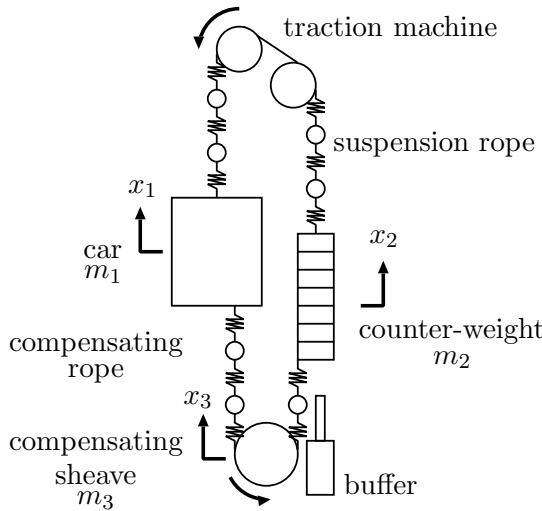


Figure 1. Elevator configuration.

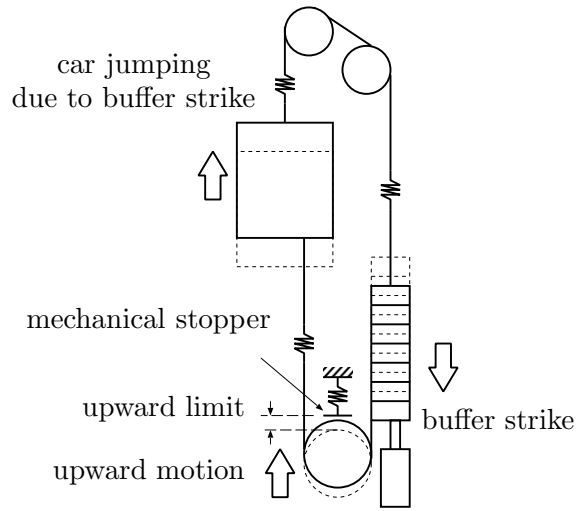


Figure 2. Sheave's upward motion.

As the upward motion is inhibited by the stopper, the compensating ropes experience high tension due to the car's continuous upward movement. The multi-body dynamic model in figure 1 is represented by the following equation of motion:

$$M\ddot{\mathbf{x}} + C\dot{\mathbf{x}} + K\mathbf{x} = \mathbf{F}, \quad (1)$$

where $\mathbf{x}$ is the vector of state variables. The vector contains the translational motion of car, counter-weight, compensating sheave and divided rope masses. It also contains the rotational motion of traction machine and compensating sheave.

When a rope tension imbalance on the sheave reaches the traction limit, the rope starts slipping. Otherwise, the rope's rotational speed on the sheave is constrained by the sheave speed due to the traction force.

In the case of buffer strike of the counter-weight, the counter-weight motion is constrained by the buffer motion.

These constraint motions of rope slip and buffer strike are evaluated by the following extended equation of motion:

$$\begin{bmatrix} M & E \\ E^T & 0 \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{x}} \\ \Lambda \end{bmatrix} = \begin{bmatrix} \mathbf{F} - C\dot{\mathbf{x}} - K\mathbf{x} \\ f_e \end{bmatrix}, \quad (2)$$

where E and Λ are the constraint matrix and the constraint force vector as Lagrangian multipliers, respectively [2].

3. Equivalent model

3.1. Car and counter-weight motion during sheave's upward motion

The multi-body dynamic model in figure 1 can numerically calculate the transient rope tension during a buffer strike. However, since it remains unclear how the compensating sheave's motion affects the rope tension, we focus on the transient motion of the compensating ropes and the compensating sheave to evaluate it. To simplify the model in figure 1, we assume the following conditions:

- Before the counter-weight strikes the buffer, the car keeps rising at a constant speed and the counter-weight falls at the same constant speed.
- The suspension ropes lose tension when the car jumps due to the counter-weight striking the buffer and continues to climb against gravity.

Time difference t_0 from the counter-weight that strikes the buffer to the suspension ropes that lose tension is given by

$$t_0 = \frac{L_m}{v_m}, \quad v_m = \sqrt{\frac{E_m}{\rho_m}}, \quad (3)$$

where L_m is the suspension rope length of the counter-weight, E_m and ρ_m are Young's modulus and the density of the suspension rope. v_m is the propagation velocity in the rope axial direction.

The car speed is expressed by

$$\ddot{x}_1(t) = \begin{cases} 0 & (t < t_0) \\ -g & (t \geq t_0) \end{cases} \longrightarrow \dot{x}_1(t) = \begin{cases} V_0 & (t < t_0) \\ V_0 - g(t - t_0) & (t \geq t_0) \end{cases}, \quad (4)$$

where V_0 is the initial speed at time t which equals zero.

The counter-weight's speed is reduced by the buffer's resistance. Its average deceleration is $1g$, but the typical deceleration pattern due to a buffer strike is approximated by the following linear function of time t . The counter-weight speed is given by the integration of the acceleration:

$$\ddot{x}_2(t) = \begin{cases} C_0(1 - t/t_2) & (t < t_2) \\ 0 & (t \geq t_2) \end{cases} \longrightarrow \dot{x}_2(t) = \begin{cases} -V_0 + C_0 \left(t - \frac{t^2}{2t_2} \right) & (t < t_2) \\ 0 & (t \geq t_2) \end{cases}, \quad (5)$$

where t_2 is the stop time of the counter-weight and C_0 is the initial deceleration caused by the buffer strike.

3.2. Equation of motion related to car and compensating sheave

An equivalent 2-DOF vibration model of the compensating ropes is shown in figure 3. We assume that the car is in place on the top floor and the counter-weight is in place on the bottom floor. Table 1 defines the parameters.

The kinetic energies of car and compensating sheave are given by

$$T_1 = \frac{1}{2}m_1\dot{x}_1^2, \quad T_3 = \frac{1}{2}m_3\dot{x}_3^2. \quad (6)$$

The kinetic energy of the distributed rope mass is given on the assumption of the rope's linear displacement. Therefore, the kinetic energy in any short part of the rope is given by

$$\Delta T_r = \frac{1}{2}\rho dx \left\{ \frac{x}{L} (\dot{x}_1 - \dot{z}) + \dot{z} \right\}^2. \quad (7)$$

The total kinetic energy of the rope is given by integrating the above equation:

$$T_r = \int_0^L \Delta T_r = \frac{1}{2} \frac{m_r}{3} \{ \dot{x}_1^2 + \dot{x}_1 \dot{z} + \dot{z}^2 \}, \quad m_r = \rho L. \quad (8)$$

Table 1. Model parameters.

x_1	car displacement
x_2	counter-weight displacement
x_3	compensating sheave displacement
z	bottom rope displacement
m_1	car weight
m_3	compensating sheave weight
k	stiffness of compensating rope
ρ	line density of compensating rope
L	compensating rope length

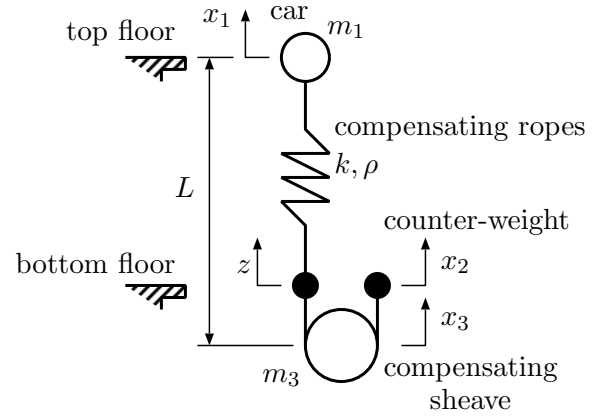


Figure 3. Equivalent 2-DOF model.

The potential energy of compensating rope is given by

$$U = \frac{1}{2}k(x_1 - z)^2. \quad (9)$$

Variable z is the function of x_2 and x_3 :

$$z = 2x_3 - x_2. \quad (10)$$

The equations of motion related to the car and the compensating sheave are given by Lagrange formulation of the total kinetic energy and the potential energy of Eqs. (6), (8) and (9):

$$x_1 : \quad \left(m_1 + \frac{m_r}{3}\right) \ddot{x}_1 + \frac{m_r}{3} \ddot{x}_3 + k(x_1 + x_2 - 2x_3) = \frac{m_r}{6} \ddot{x}_2 - \left(m_1 + \frac{1}{2}m_r\right)g \quad (11)$$

$$x_3 : \quad \frac{m_r}{3} \ddot{x}_1 + \left(m_3 + \frac{4m_r}{3}\right) \ddot{x}_3 - 2k(x_1 + x_2 - 2x_3) = \frac{2m_r}{3} \ddot{x}_2, \quad (12)$$

where the suspension ropes lose tension after the counter-weight strikes the buffer. Eqs. (11) and (12) have coupling effects against the inertia and stiffness terms.

3.3. Constraint force of compensating sheave

The following equation highlights the relationship between x_2 and x_3 by deleting the stiffness term from Eqs. (11) and (12) with substituting $-g$ for $\ddot{x}_1$, related to Eq. (4):

$$\ddot{x}_3 = \frac{m_r}{m_3 + 2m_r} \ddot{x}_2 = \beta \ddot{x}_2. \quad (13)$$

If total rope weight m_r is much heavier than compensating sheave m_3 , β is close to 0.5. Therefore the compensating sheave's acceleration $\ddot{x}_3$ is half of counter-weight $\ddot{x}_2$.

From Eq. (13), we can estimate time t_3 at which the compensating sheave reaches the limit of the upward motion.

We introduce a new variable to evaluate the rope elongation: $y = x_1 + x_2 - 2x_3$. After time t_3 of the upward limit of the compensating sheave, the equation of motion related to the car in Eq. (11) is modified, because the compensating sheave's acceleration $\ddot{x}_3$ equals zero:

$$\left(m_1 + \frac{m_r}{3}\right) \ddot{y} + ky = \left(m_1 + \frac{m_r}{2}\right) (\ddot{x}_2 - g). \quad (14)$$

Rope elongation $y(t)$ is given by

$$y(t) = C_1 \cos \omega(t - t_3) + C_2 \sin \omega(t - t_3) + \gamma(\ddot{x}_2 - g) \quad (15)$$

$$\omega = \sqrt{\frac{k}{m_1 + m_r/3}}, \quad \gamma = \frac{1}{k} \left(m_1 + \frac{m_r}{2} \right), \quad C_1 = -\gamma(\ddot{x}_2(t_3) - g), \quad C_2 = \frac{1}{\omega} \{-\gamma\ddot{x}_2(t_3) + \dot{y}(t_3)\},$$

where rope elongation speed $\dot{y}$ at time t_3 is given by

$$\dot{y}(t_3) = \dot{x}_1(t_3) + \dot{x}_2(t_3) - 2\dot{x}_3(t_3) = \dot{x}_1(t_3) + \dot{x}_2(t_3) \quad (\because \dot{x}_3(t_3) = 0). \quad (16)$$

$\dot{x}_1(t_3)$ and $\dot{x}_2(t_3)$ are calculated by Eqs. (4) and (5).

The equation of motion related to the compensating sheave in Eq. (12) is modified using y :

$$\frac{m_r}{3}\ddot{y} + (m_3 + 2m_r)\ddot{x}_3 - m_r\ddot{x}_2 - 2ky = 0. \quad (17)$$

When the compensating sheave reaches the limit of the upward motion, $\ddot{x}_3$ is zero at time t_3 and the compensating sheave's upward motion is restricted. Therefore the equation of motion in Eq. (17) includes constraint force λ to satisfy the constraint equation:

$$\begin{cases} \frac{m_r}{3}\ddot{y} + (m_3 + 2m_r)\ddot{x}_3 - m_r\ddot{x}_2 - 2ky = -\lambda \\ \ddot{x}_3 = 0 \end{cases} \longrightarrow -\lambda = \frac{m_r}{3}\ddot{y} - m_r\ddot{x}_2 - 2ky. \quad (18)$$

Substituting Eqs. (14) and (15) for Eq. (18),

$$\lambda(t) = -(2m_1 + m_r) \{-\omega^2 (C_1 \cos \omega(t - t_3) + C_2 \sin \omega(t - t_3)) + g\} + 2(m_1 + m_r)\ddot{x}_2(t). \quad (19)$$

Constraint force λ in Eq. (19) is twice the compensating ropes' tension, which includes the rope's inertia force.

When the mechanical stopper inhibits the compensating sheave from rising, the compensating ropes receive an impact force that axially propagates the rope vibration:

$$F(t) = F_s \{1 - \cos \omega_r(t - t_3)\} \quad , \quad F_s = (m_3 + 2m_r)\ddot{x}_3(t_3), \quad (20)$$

where impact force F_s corresponds to the second term in Eq. (17) and ω_r is the eigen frequency of the rope axial vibration:

$$\omega_r = \frac{\pi}{L} \sqrt{\frac{EA}{\rho}}, \quad (21)$$

where E and A are Young's modulus and the cross section area of the compensating rope, respectively.

Finally, maximum force F_{max} , which restricts the compensation sheave's upward movement, is given by the summation of Eqs. (19) and (20). As the frequency of impact force $F(t)$ is much higher than the frequency of $\lambda(t)$, the maximum force F_{max} occurs at the peak time t_f of $F(t)$:

$$C_f(t) = \lambda(t) + F(t) \longrightarrow F_{max} = C_f(t_f) = \lambda(t_f) + F(t_f), \quad (22)$$

where $t_f = t_3 + \frac{(2n-1)\pi}{\omega_r}$, $n = 1, 2, \dots$.

4. Simulation results

Next we compare the formulas in Section 3 with the numerical simulation by the multi-body dynamic model in Section 2. The simulation model in Eq. (2) consists of 46 degrees of freedom and it is solved by Runge-Kutta method. Figure 4 shows the time response of the acceleration against the car and the counter-weight. The thin lines correspond to Eqs. (4) and (5).

The upward motion of the compensating sheave is calculated by Eq. (13) (figure 5). As the displacement is normalized by the upward limit, the vertical axis in figure 5 is dimensionless.

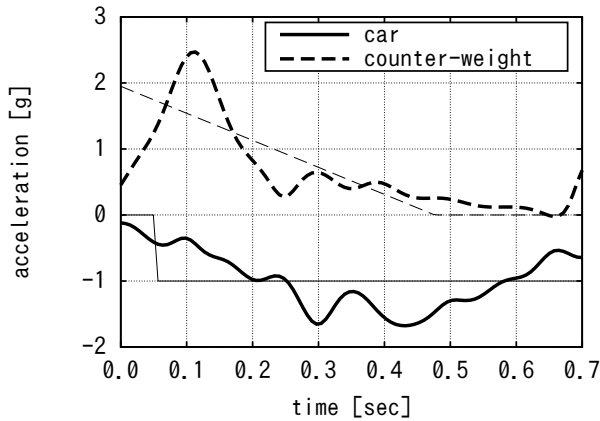


Figure 4. Time response of acceleration.

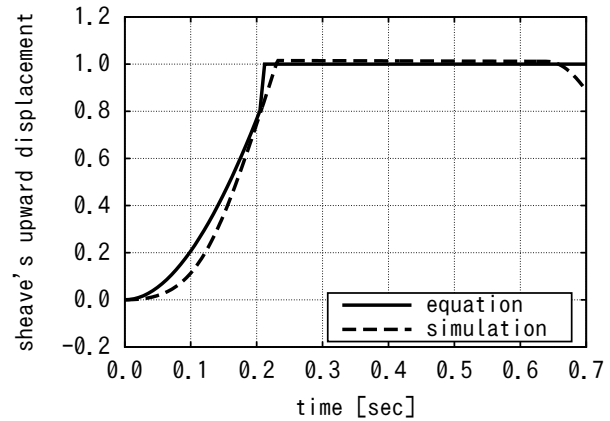


Figure 5. Normalized sheave's upward motion.

The time response of the constraint force in Eqs. (19) and (22) is shown in figure 6. The constraint force is normalized by the maximum force of the simulation result, therefore the vertical axis is represented as a dimensionless value. The solid line corresponds to Eq. (22) and the thin dotted line represents Eq. (19).

As shown in figure 5, time t_3 for reaching the upward limit is close to the simulation result. The transient constraint force in Eq. (22) also resembles the simulation result in figure 6. Therefore we conclude that the formula in Eq. (22) is appropriate to evaluate the maximum constraint force.

To investigate the effect of the upward limit against the constraint force, the upward limit is shifted $\pm 50\text{mm}$ in Eq. (22). Figure 7 shows that a shorter upward limit causes a larger constraint force.

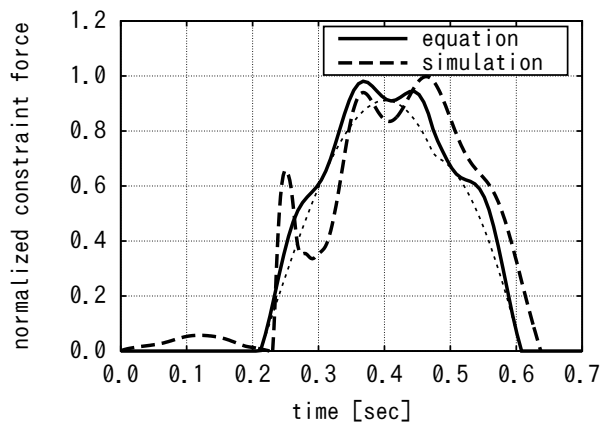


Figure 6. Constraint force.

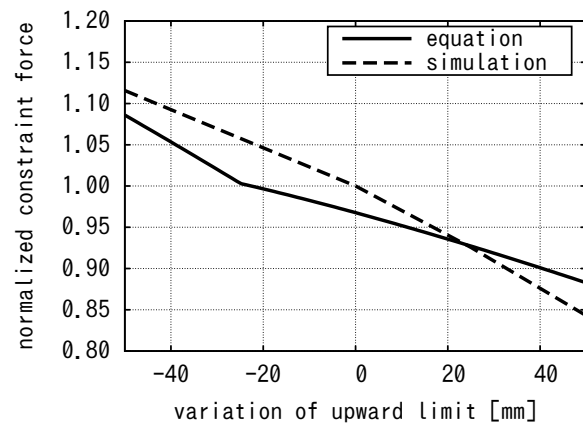


Figure 7. Effect of upward limit.

5. Conclusions

We evaluated the constraint force against a compensating sheave with an equivalent 2-DOF vibration model and validated our formula based on the 2-DOF model by numerical simulation. Our formula suggests that a shorter upward limit causes a larger constraint force that is represented by the summation of two vibration modes.

References

- [1] Watanabe S, Okawa T, Nakazawa D and Fukui D 2013 *Proc. of MoSS 2013*
- [2] Watanabe S, Yumura T, Hayashi Y and Takigawa Y 2002 *Proc. of ACMD'02*