

# Dynamics behaviour of an elastic non-ideal (NIS) portal frame, including fractional nonlinearities

J M Balthazar<sup>1</sup>, R M L F Brasil<sup>2</sup>, J L P Felix<sup>3</sup>, A M Tusset<sup>4</sup>, V Picirillo<sup>4</sup>,  
I Iluik<sup>5</sup>, R T Rocha<sup>6</sup>, A Nabarrete<sup>1</sup> and C Oliveira<sup>1,7</sup>

<sup>1</sup> ITA-Aeronautics Technological Institute, Mechanical-Aeronautics Division,  
São José dos Campos, SP, Brazil,

<sup>2</sup> Federal University of ABC, Center for Eng., Modelling and Applied Social  
Sciences,  
Santo Andre, SP, Brazil.

<sup>3</sup> UNIPAMPA - University in Bagé, Alegrete, RS, Brazil.

<sup>4</sup> UTFPR – Federal University of Technology – Paraná, Ponta Grossa, PR,  
Brazil.

<sup>5</sup> USP, Polytechnic School of the University of Sao Paulo, São Paulo, SP,  
Brazil,

<sup>6</sup> UNESP – São Paulo State University, Mechanical Engineering,  
Bauru, SP, Brazil.

<sup>7</sup> UFGD, Federal University of Grande Dourados, Dourados, MS, Brazil.

e-mail :jmbaltha@ita.br

**Abstract.** This paper overviews recent developments on some problems related to elastic structures, such as portal frames, taking into account the full interactions of the vibrating systems, with an energy source of limited power supply (small motors, electro-mechanical shakers). We include a discussion on fractional (rational) damping and stiffness effects on the adopted modelling. This was a plenary lecture, delivered in the event titled: Mechanics of Slender Structures, organized in Northampton, England from 21-22, September 2015.

## 1. Introduction

This paper shows a series of problems related to nonlinear dynamics and control, correlated to Electro-mechanical Systems [1] using an elastic support. Here, we analyzed such systems those fall into three groups: the conventional MACRO, MEMS and NEMS electro-mechanical systems, taking into account the existence of a full interaction between mechanical and electrical field quantities. The aim of this paper is to describe a large number of phenomena caused by the action of vibrations on nonlinear electro-mechanical systems and the energy source supply to it. This paper gives a general description of this class of phenomena which has been discussed in recent years.



We remarked that a large number of publications was based on the assumption that the external excitations are produced by an ideal source of power (IS) with prescribed time history: prescribed magnitude, phase and frequency, or in random problems with prescribed characteristics. In reality, the excitations sources are non-ideal and they have always limited power, limited inertia and their frequencies varies according to the instantaneous state of the vibrating system [2].

We also commented that small motors with limited power are used in laboratory tests and therefore the investigations of mutual interaction of driven and driving subsystems are very important. We will discuss several contributions on the past, nowadays and perspective futures in engineering that may be modelled by non-ideal systems (NIS) with many degrees of freedom. In this lecture, we will present examples of this kind of engineering devices restricted to portal frame foundation.

## **2. Electro mechanical systems**

It is well known that the analyses of the motion of real electro-mechanical systems were carried out by means of mathematical models, which have been always simplified and therefore describe the “exact” behavior with some degree of approximation. The study of problems involving the coupling of several systems was widely explored, in the last years, essentially as function of the change of constructive characteristics on machines and structures.

We noted that a lot of oscillatory (vibrating) phenomena of real systems cannot be explained or solved on the basis of a linear theory and it is important to introduce nonlinear characteristics into the mathematical models of vibrating systems and in particular to electro-mechanical systems. The main difficulty for comparisons with linear systems was mainly due to the absence of validity of the superposition principle. Every nonlinear vibrating system should be solved individually and a special methodology must be developed for each class of problems.

Here, we dealt with a special class of nonlinear systems called non-ideal systems (NIS).

By one hand, we verified that the study of the vibrating systems had been considered a major challenge in theoretical and practical engineering research, when the external excitation was influenced by the response of the system. The motion of a vibrating structure under the action of such energy source is accompanied by the interaction between these non-ideal motors and their supports. In usual approaches, the excitation is considered as ideal (IS); that is, the influence of the motion of the structure on the motor is disregarded. Here, the reciprocal influence of the system on the energy source is considered (NIS).

As a direct consequence, in the region of resonance, unstable conditions of motion occur and the form of resonance curve depends on which direction the frequency of the excitation is being altered. We know that the problem of passage through resonance of unbalanced equipment, with an operational speed higher than the lower frequencies of vibration of the supporting structure, has been studied for a long time. However, the machine may not be able to supply the power necessary for this technique, as a large part of its energy is used in shaking the structure and not for increasing the drive speed of the electrical motor. This is well known as Sommerfeld effect (see [2-5], undeserved of others).

Note that as long as the motor drive speed is assumed to be uncoupled from the forced vibrations on the supporting structure, one has a known forcing function of time corresponding to an unlimited or ideal power supply. The introduction of real torque-speed curves for non-ideal motors renders the system nonlinear and capable of multiple steady-state periodic motions whose stability must be assessed. Further complexities may be introduced if the structure itself exhibits nonlinear behavior.

We also found that the right branch of the frequency response curve may be unstable if its negative slope is larger than that of the torque-speed characteristic of the driving motor.

### **2.1. Portal frame foundation**

Some new applications related to portal frame (NIS) structure and their nonlinear vibrations have been investigated by several authors [6-9]. The same problem with two non-ideal sources was studied by [10]. The authors considered a description of a non-linear analysis of a metallic portal frame excited

by a DC motor. In this referenced work, the two columns of the structure were clamped in their bases to a large seismic mass and the motor was attached to the center of the horizontal beam.

In order to allow the possible observation of a non-linear phenomenon, the dimensions of the structure were chosen in such a way that there was a ratio of one-to-two between the first and second natural frequencies for which the modes are anti-symmetrical or a sway mode and symmetrical or a bending mode. For the same reason, all joints connecting the columns to the horizontal beam were welded, in order to reduce the structural damping. Modal saturation, Sommerfeld effect and other interesting phenomena were observed and demonstrated by using analytical, numerical and experimental techniques.

In [11] the authors examined the nonlinear control method based on the saturation phenomenon and of systems coupled with quadratic nonlinear ties applied to a shear-building portal plane frame foundation that supports an unbalanced direct current with limited power supply (NIS).

The authors in [12] presented some research results on the optimization of the efficiency of an impact damper for a NIS. In the work done by [13] it was analyzed the Sommerfeld effect, where a part of the excited system behaves as energy sink was detected.

In [14] the authors considered both active and passive controls to suppress the chaotic behavior of a simple portal frame, under the excitation of an unbalanced DC motor with limited power supply (NIS). The adopted active control strategy consisted of two controls: the nonlinear feedforward in order to keep the controlled system in a desirable orbit, and the feedback control, which may be obtained by considering the state-dependent Riccati equation to bring the system to the desired orbit, using a magneto rheological (MR) damper. To control the electric current of the MR damper the Bouc-Wen mathematical model was used to the MR damper. The passive control was obtained by means of a nonlinear sub-structure whose properties refers to a nonlinear energy sink. Simulations showed the efficiency of both the passive control (energy pumping) and active control strategies in the suppression of the chaotic behavior.

## 2.2. *Base excitation effects.*

Besides excitation due to unbalanced rotating machines, several authors analyze vibrating systems subjected to base excitation. It is known that [6] analyzed a frame under support motion considering linear elastic forces. In [15] it was performed numerical and experimental dynamical analysis of a parametrically excited pendulum. In this case the presence of quasi-periodic nonlinear interactions between the structural frame and the electro-mechanical shakers, characters of the non-ideal phenomenon [5, 24,25,26] in the proposed NIS model. We mention that it was carried out a dynamical analysis of a flexible portal frame under base excitation, which is provided by an electro-mechanical shaker in [17]. The electro-mechanical device considered in this work consists of an electric system magnetically coupled to a mechanical structure as used before by [16]. In this paper, the authors investigated the effectiveness of the linear electromechanical vibration absorber (LEVA) and a nonlinear electromechanical vibration absorber (NEVA) in the vibration attenuation for NIS structures. An analysis of the effects of the parameters of coupling and of nonlinear coefficients with increasing of constant torque of the DC motor is presented.

The authors in [18] considered a flexible portal frame parametrically excited by an electro-dynamical shaker. The dynamical jumps, an important point associated with the proposed system were the energy transfer between the shaker and the FPF. This energy transfer is observed when system behaved periodically but it is more significant in quasi-periodic response.

## 2.3. *Harvesting Energy.*

Recently, much interest has been demonstrated in the concepts of electro-mechanical systems that are able to scavenge or harvest energy from their operating environment. In the process of energy harvesting, the electrical energy is obtained through of conversion of mechanical energy from an ambient vibration source by a type of transducer, as a piezoceramic thin film, which is a material with piezoelectric properties.

In [19] the authors presented a FEM model of a portal frame, which is analyzed by displaying the energy transfer between modes when the structure was excited via the first sway mode. They also showed the saturation of the second mode (vertical symmetrical mode) when the structure was excited by the support motion resonant to this mode, and the energy was transferred to the coupled horizontal sway mode. The computed values were proportional to the power available for energy harvesting in each mode. As a novel feature for this research, we present a situation where the energy introduced in a structural system with a certain configuration was made available for harvesting another vibration mode very different.

In [20] it was presented the extraction of energy from a simple portal frame structure excited via its second (first symmetric) mode. As 2:1 internal resonance is present between that mode and the first (sway) mode, the phenomenon of mode saturation and energy exchange (modal coupling) occurs. Energy pumped into the system through the second (vertical) mode was partially transferred to the horizontal (sway) mode. An evaluation of the energy available for harvesting in each of the considered modes was computed. Here, the nonlinearities present in the piezoelectric material were considered in the piezoelectric coupling mathematical model.

In [21] it was considered the piezoceramic coupled in the column of a portal frame with 2:1 internal resonance may reduce the column vibration and transfer its vibration energy to electrical energy. In this paper we consider a simple portal frame structure with 2:1 internal resonance excited by an electro-dynamical shaker which can simulate a harmonic displacement and a piezoelectric material coupled in a column of the portal frame. We also mention that [22] presented an analysis of a new energy harvester model, based on a (NIS) simple portal frame structure. The horizontal motion of the portal frame was considered under a NIS excitation, and the approximated mathematical model of the system was obtained, considering the coupled oscillators. To model the piezoelectric coupling, the nonlinearities of the piezoelectric material were considered. To check the robustness of the control strategy, an analysis considering uncertainties in the parameters of the model was performed, showing the efficiency of the passive control (energy pumping) in the suppression of the chaotic behavior, as well as the sensitivity of the control system to parametric errors [23]. Also presented a model of energy harvester based on a simple portal frame (NIS). The nonlinearities present in the piezoelectric material are considered in the piezoelectric coupling mathematical model. The system is a bi-stable Duffing oscillator presenting a chaotic behavior. Analyzing the average power variation, and bifurcation diagrams, the value of the control variable that optimizes power or average value that stabilizes the chaotic system in the periodic orbit was determined. The control sensitivity was determined to parametric errors in the damping and stiffness parameters of the portal frame. The proposed passive control technique used a simple pendulum to tune to the vibration of the structure to improve the energy harvesting.

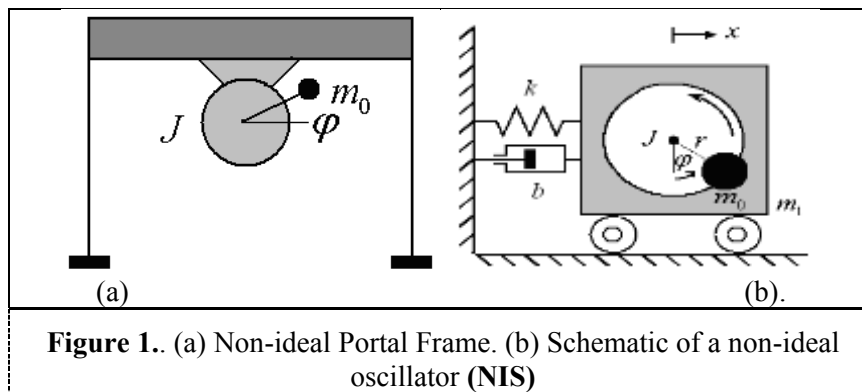
#### 2.4. *NIS Emergent problems.*

In the papers [24] the authors analyzed a proposed mathematical model, by using numerical integrations, taking into account the reciprocal influence between the vertically excitation, due an electro-dynamical shaker (EDS) and the vibration response of a tuning fork beam (TFB). The internal physical components of a shaker consist of an electric component coupled magnetically to a mechanical structural one. The behavior of the shaker type utilized here is similar of the one used by [15] also [27] studied a tuning fork vibratory micro machined gyroscope subjected to base excitation. Here, the investigation was based the behavior for the macro system. The (TFB) is modeled by two inverted pendulums of motion in the opposite directions hung by the same rods of vertical and horizontal motion according to suggestions by [28,29]. Recently, the researcher in [29] investigate the interaction of the dynamics of the electro-shaker with the gyroscope, demonstrated that under certain parameters the system can exhibit complex dynamic behavior such as chaotic motion [30], analyze the influence on the electrical charge, between the cantilever beam and the wide electrode by using numerical methods. For the analysis, the authors used the mathematical model of a Duffing oscillator,

under electrostatic effects, with variable capacitor, investigating the dynamic interaction, between a micro machined rate gyroscope and variable force actuators.

### 3. Rational nonlinearities and numerical simulations results

By other hand, Fractional stiffness and damping is appearing in different contexts in any systems with memory and hysteresis. Such damping is defined by a fractional derivative in contrary to classical viscous damping term with the first order derivative. As the memory of the dynamical system induces extra degree of freedom for the phase space the standard methods of dynamical response analysis and system identification, which relies on the knowledge of system dimensionality cannot be used. Here, we present nowadays results in order to start new researches on the NIS portal frame. The results present here is an extension of the paper [31] using an uncontrolled model of portal frame (Figure 1).



**Figure 1.** (a) Non-ideal Portal Frame. (b) Schematic of a non-ideal oscillator (NIS)

For this work, the parameters of the coupled dynamical system consist of  $(m_0, m_1, k, k_{nl}, c, x_1, \varphi, J, r, d, s)$ , the mass, unbalanced mass, linear stiffness, non-linear stiffness, linear damping, displacement, angular displacement, inertia moment, eccentricity of the unbalanced mass,  $d$  is related to the voltage applied across the armature of the DC motor and  $s$  is a constant for each model of DC motor considered. The resulting mathematical model of the structure is a Duffing-like equation:

$$\begin{aligned} (m_1 + m_0)\ddot{x} + b\dot{x} - k_l x + k_{nl} x^3 &= m_0 r (\ddot{\varphi} \sin(\varphi) + \dot{\varphi}^2 \cos(\varphi)) \\ (J + r^2 m_0)\ddot{\varphi} - r m_0 \ddot{x} \sin(\varphi) &= L(\dot{\varphi}) = d - s \dot{\varphi} \end{aligned} \quad (1)$$

Next, we render Eqs. (1) dimensionless in terms of new variables defined by:  $\tau = \omega t$ ,  $x_1 = \frac{x}{x^*}$ , and  $x_3 = \frac{\varphi}{\varphi^*}$ , where  $x^*$  and  $\varphi^*$  are constant characteristics. Equations (1) can be represented in a state space, in dimensionless form, as:

$$\begin{aligned} x_1' &= x_2 \\ x_2' &= -\alpha x_2 + \beta_1 x_1 - \beta_3 x_1^3 + \delta_1 \sin(\varphi^* x_3) x_4' + \delta_1 \cos(\varphi^* x_3) x_4^2 \\ x_3' &= x_4 \\ x_4' &= \rho_1 \sin(x_3) x_2' - \rho_3 x_4 + \rho_2 \end{aligned} \quad (2)$$

where:  $\alpha = \frac{b}{(m_1 + m_0)\omega}$ ,  $\omega = (k_1/(m_1 + m_0))^{1/2}$ ,  $\beta_1 = \frac{k_l}{(m_1 + m_0)\omega^2}$ ,  $\beta_3 = \frac{k_{nl}x^{*2}}{(m_1 + m_0)\omega^2}$ ,  
 $\delta_1 = \frac{m_0 r \varphi^*}{(m_1 + m_0)x^*}$ ,  $\rho_2 = \frac{d}{(J + r^2 m_0)\omega^2 \varphi^*}$ ,  $\delta_2 = \frac{m_0 r \varphi^{*2}}{(m_1 + m_0)x^*}$ ,  $\rho_1 = \frac{r m_0 x^*}{(J + r^2 m_0)\varphi^*}$ ,  $\rho_3 = \frac{s \omega \varphi^*}{(J + r^2 m_0)\omega^2 \varphi^*}$ .

### 3.1. Dynamic analysis of a fractional-order

Differential equations may involve Riemann–Liouville differential operators of fractional-order  $q > 0$ , which generally take the form below [32], [33]:

$$D^q x(t) = \frac{1}{\Gamma(\eta - q)} \int_{t_0}^t \frac{x^{(\eta)}(u)}{(t - u)^{q - \eta + 1}} du \quad (3)$$

Where  $\eta$  is the first integer not less than  $q$ . It is easily proved that the definition is the usual derivatives definition when  $q = 1$ . The case  $0 < q < 1$  seems to be particularly important. For simplicity and without loss of generality, in the following, we assume that:  $t_0 = 0$ ,  $0 < q < 1$ .

Considering the force of dissipation  $(-\alpha x_2)$  are fractional order and considering  $x_2 = x_6$  as equation:

$$\begin{aligned} x_1' &= x_2 \\ x_2' &= -\alpha x_6 + \beta_1 x_1 - \beta_3 x_1^3 + \delta_1 \sin(\varphi^* x_3) x_4' + \delta_1 \cos(\varphi^* x_3) x_4^2 \\ x_3' &= x_4 \\ x_4' &= \rho_1 \sin(x_3) x_2' - \rho_3 x_4 + \rho_2 \\ x^{q_1}_5 &= x_6 \\ x^{q_2}_6 &= -\alpha x_6 + \beta_1 x_5 - \beta_3 x_5^3 + \delta_1 \sin(\varphi^* x_3) x_4' + \delta_1 \cos(\varphi^* x_3) x_4^2 \end{aligned} \quad (4)$$

Consider the numerical values:  $\alpha_1 = 0.1$ ,  $\beta_1 = 1$ ,  $\beta_3 = 2$ ,  $\delta_1 = 8.373$ ,  $\rho_1 = 0.05$ ,  $\rho_2 = 100$ ,  $\rho_3 = 200$ ,  $x_1(0) = 0$ ,  $x_2(0) = 0$ ,  $x_3(0) = 0$ ,  $x_4(0) = 0$ ,  $x_5(0) = 0$ ,  $x_6(0) = 0$ . As the force dissipation depends  $x_2 = x_6$  it will be considered  $q_1 = 1$  and  $0.5 \leq q_2 \leq 1$ .

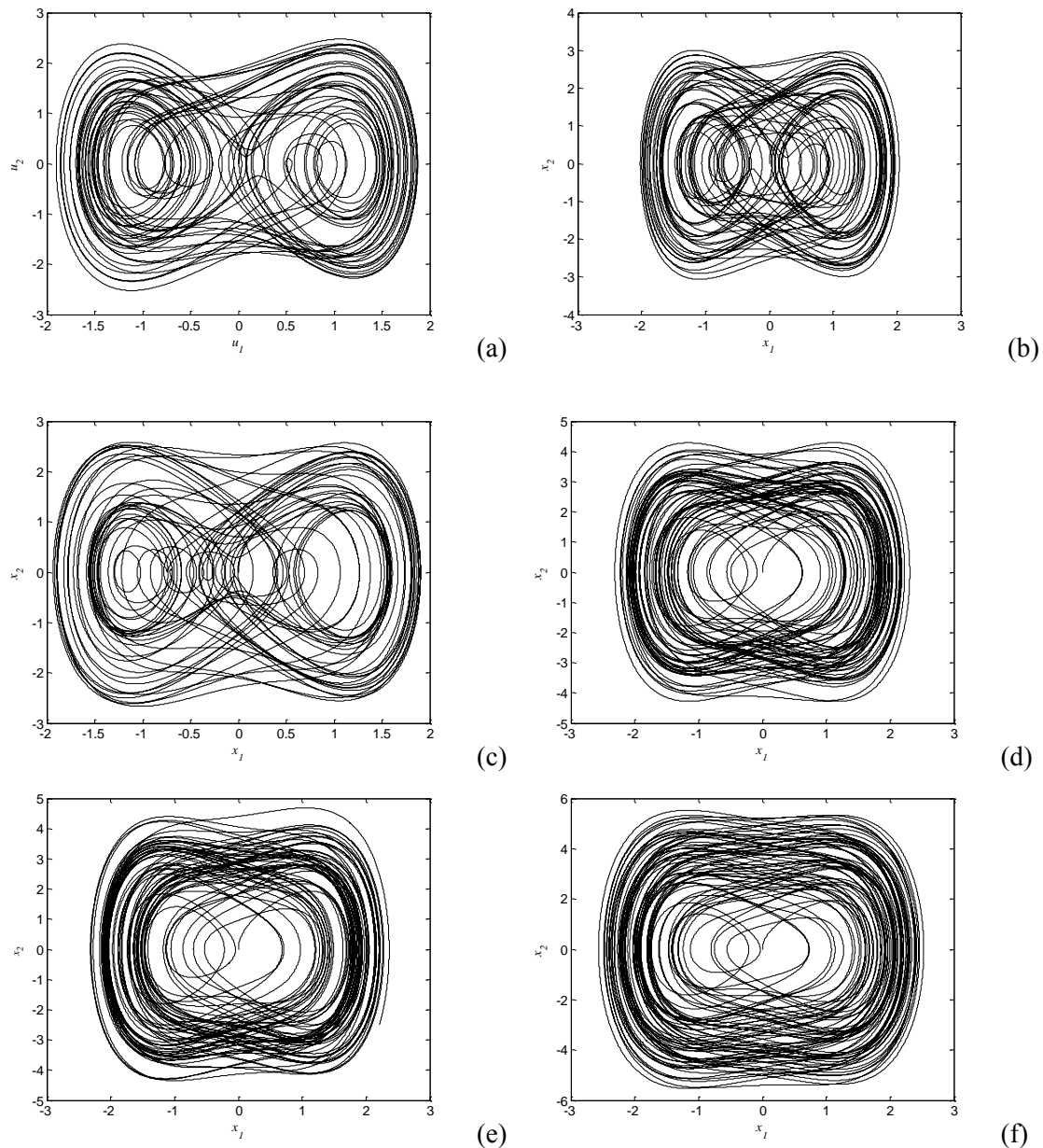


Figure 2. Phase portraits for dissipation force in fractional order. (a)  $q_1=1$  and  $q_2=1$ . (b)  $q_1=1$  and  $q_2=0.9$ . (c)  $q_1=1$  and  $q_2=0.8$ . (d)  $q_1=1$  and  $q_2=0.7$ . (e)  $q_1=1$  and  $q_2=0.6$ . (f)  $q_1=1$  and  $q_2=0.5$ .

As can be seen in Figure 2, the power dissipation is of the order of fractional order derivative will influence the damping. As can be seen in figure 2 the variation of the derivative order generated increase in displacement  $x_1$  and  $x_2$ . Since for  $q_1=1$  and  $q_2=0.5$  (figure 2f) was the largest displacement amplitude, indicating reduction of power dissipation.

We can also observe that the variation  $q_2$  influence the behavior of the chaotic system. To check which variation of chaotic behavior is considered the 0-1 test. The 0-1 test for chaos takes as input a time series of measurements and returns a single scalar value of either 0 for periodic attractors or 1 for chaotic attractors [31,34]. According to [35] value of  $(K_c)$  can be obtained from:

$$k_c = \frac{\text{cov}(X, M(c))}{(\text{var}(X)\text{var}(M(c)))^{1/2}} \quad (5)$$

where vectors  $X=[1, 2, \dots, n_{max}]$ , and  $M(c)=[M(1, c), \dots, M(l, n_{max})]$ ,  $c \in (0, \pi)$  is a fixed frequency chosen arbitrarily, and:

$$M(n, c) = \lim_{n \rightarrow \infty} \frac{1}{N} \sum_{j=1}^N \left[ (p(j+n) - p(j))^2 + (q(j+n) - q(j))^2 \right] \quad (6)$$

$$p(i) = \sum_{j=0}^i \frac{(x_j - \bar{x})}{\sigma_x} \cos(jc) \quad (7)$$

$$q(i) = \sum_{j=0}^i \frac{(x_j - \bar{x})}{\sigma_x} \sin(jc) \quad (8)$$

where:  $\bar{x}$  and  $\sigma_x$  is the mean value and square deviation of examined  $x_i$  series, and  $N$  is the length of the sampled points in the displacement time series.

For a value of  $K_c \cong 0$  indicates a non-chaotic data set while a value of  $K_c \cong 1$  indicates a chaotic data set. In table 1 shows the  $K_c$  variation for the derivate order  $q_1 = 1$  and  $0.5 \leq q_2 \leq 1$ .

Table1: Variation of the chaotic behavior for derivative order variation

q1	q2	Kc
1	1	0.9427
1	0.9	0.8571
1	0.8	0.9038
1	0.7	0.9277
1	0.6	0.8453
1	0.5	0.9562

As can be seen in Table 1, the variation  $q_2$  changes the chaotic behaviour of the system. Reducing for  $1 < q_2 \leq 0.6$  and increasing to  $q_1 = 0.5$ . Demonstrating the influence of the order of the derivative ( $x^{q-}$ ) in the system damping.

Considering the force of stiffness and dissipation  $(-\alpha x_2 + \beta_1 x_1 - \beta_3 x_1^3)$  are fractional order and considering  $x_1 = x_5$  and  $x_2 = x_6$  as equation:

$$\begin{aligned}
 x_1' &= x_2 \\
 x_2' &= -\alpha x_6 + \beta_1 x_5 - \beta_3 x_5^3 + \delta_1 \sin(\varphi^* x_3) x_4' + \delta_1 \cos(\varphi^* x_3) x_4^2 \\
 x_3' &= x_4 \\
 x_4' &= \rho_1 \sin(x_3) x_2' - \rho_3 x_4 + \rho_2 \\
 x^{q_1}_5 &= x_6 \\
 x^{q_2}_6 &= -\alpha x_6 + \beta_1 x_5 - \beta_3 x_5^3 + \delta_1 \sin(\varphi^* x_3) x_4' + \delta_1 \cos(\varphi^* x_3) x_4^2
 \end{aligned} \quad (9)$$



Consider the numerical values:  $\alpha_1 = 0.1$ ,  $\beta_1 = 1$ ,  $\beta_3 = 2$ ,  $\delta_1 = 8.373$ ,  $\rho_1 = 0.05$ ,  $\rho_2 = 100$ ,  $\rho_3 = 200$ ,  $x_1(0) = 0$ ,  $x_2(0) = 0$ ,  $x_3(0) = 0$ ,  $x_4(0) = 0$ ,  $x_5(0) = 0$ ,  $x_6(0) = 0$ ,  $q_1 = q_2$ , and  $0.5 \leq q_2 \leq 1$ .

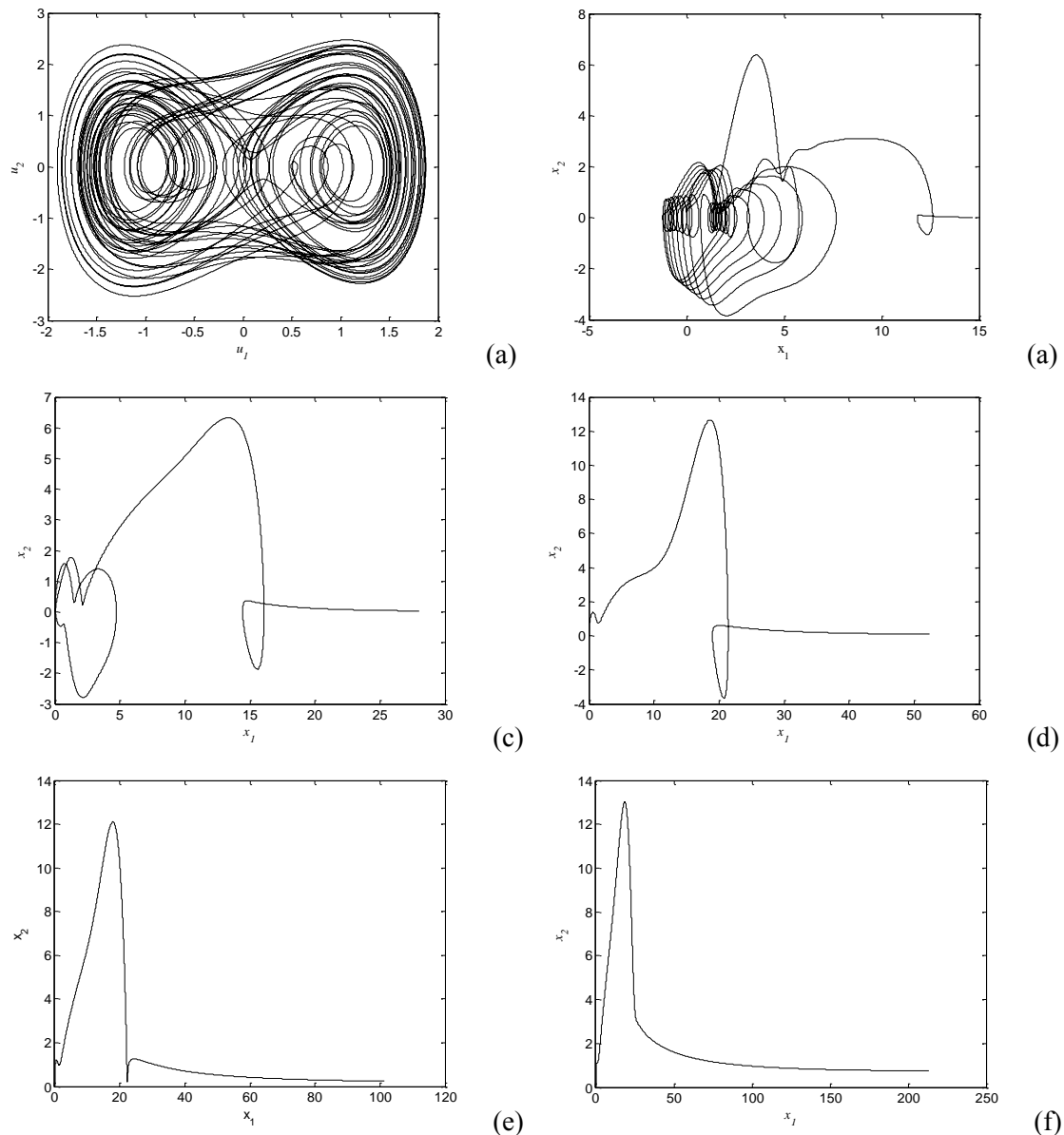


Figure 3. Phase portraits for dissipation force in fractional order. (a)  $q_1=1$  and  $q_2=1$ . (b)  $q_1=0.9$  and  $q_2=0.9$ . (c)  $q_1=0.8$  and  $q_2=0.8$ . (d)  $q_1=0.7$  and  $q_2=0.7$ . (e)  $q_1=0.6$  and  $q_2=0.6$ . (f)  $q_1=0.5$  and  $q_2=0.5$ .

As can be seen in figure 3, in the case of fractional-order stiffness the displacement of the structure amplifies so as to compromise the physical integrity of the structure, whereas the amplitude increases as the order reduced. The results show that for damping in fractional order, the displacement of the structure is chaotic, while in case of the structure stiffness fractional order that the displacement of the structure is amplified according is reduced to the order of the derivative ( $q$ ).

#### 4. Conclusions

We presented an analysis of a dynamical coupling between energy sources and structural response that must not be ignored in real engineering problems, since real motors have limited output power, taking into account a portal frame structure support. The used NIS mathematical model was represented with two coupled nonlinear differential equations, including rational nonlinearities.

The main results obtained showed that: As can be seen in figure 3, in the case of fractional-order stiffness is the system becomes unstable, whereas the amplitude increases as the order  $q_2$  reduced. And the damping is the fractional change order chaotic behavior of the system, but keep the system stable, while in the case of the system stiffness is unstable. Futures works will consider control strategies in order to overcome the main problems appoint here as well an investigation of the dynamics, including chaos, periodic motions, multiperiodic and quasiperiodic motions and an analysis of basins of attraction, considering the dynamical integrity [36].

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