Generation of stable cnoidal waves in an erbium doped fiber laser system

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Abstract. In this work an all-fiber laser system able to produce stable cnoidal waves in both limits, i.e. in sinusoidal and soliton limits, is presented. The system is constructed using an erbium highly–doped fiber as an active media and an electrooptic modulator as the core of the control tool. This control tool is able to modulate the cavity losses. The direct modulation allows to control the pulses properties, shape, width and intensity. The proposed system is well described by a three level laser model based on the Statz–de Mars rate equations. Numerical and Experimental results are presented.

1. Introduction

A cnoidal wave, in one of their limits, can be identified as a soliton pulse. The first solitary wave was reported in 1834 [1], nevertheless, it was not until the late 19th century when the first attempt to explanain this phenomenon was done[2]; the subject was left to rest until the 1960's [3]. One rough definition of a soliton is a solitary wave that maintains its shape while it travels through a nonlinear dispersive medium with constant speed. A balancing mechanism between nonlinearity and dispersion is responsible for this phenomenon [4]. Optical solitons can be either spatial [5] or temporal [6, 7]. Hasegawa and Tappert [7] suggested that the balance between self-phase modulation and anomalous dispersion could explain soliton formation in optical fibers. Many exactly solvable models have soliton solutions, including the Korteweg–de Vries, the nonlinear Schrödinger, the Sine-Gordon and the Manakov equations; actually this represents a very active field of mathematical and physical research. The main soliton practical feature, that makes it a good carrier for reliable optical communications, is its capacity to maintain its energy and frequency (shape) while propagating along an optical fiber. However, the soliton production in a laser system its normally expensive [8, 9, 10]. The presented proposal reduces the cost for this kind of pulses. In their other limit, a cnoidal wave is transformed into a sinusoidal wave that can be useful in the study of the nonlinear propagation inside plasmas [11, 12]. In this work we propose an all-fiber system composed by a semiconductor laser acting as a pump source, an erbium doped fiber, that is used as an active medium, and an Electrooptic modulator, placed inside the cavity, as a control tool; the active medium acts also as a saturable absorber for this system. The cavity losses are controlled via the Electrooptic modulator through
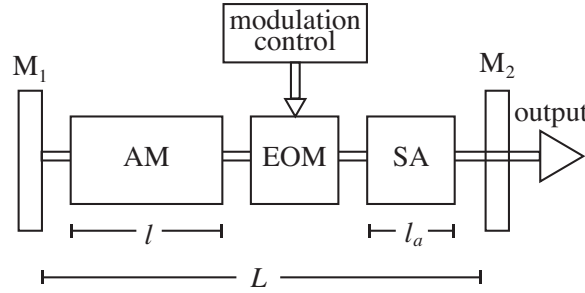
Figure 1. Proposed scheme for cnoidal wave generation. M_1 and M_2 are total reflection and semi-transparent mirrors, AM and SA are active medium and saturable absorber, and EOM is an electrooptic modulator

a functions generator. We demonstrate in this work that such a system can act as a cnoidal wave generator in a wide frequency range at a quite reasonable cost.

2. Theoretical model

The laser system depicted in Fig. 1 can be described by the Statz–de Mars equations [13]. A modified set of Statz–de Mars equations for a three-level laser with a saturable absorber (SA) without modulation control [14, 15] is written as follows:

$$
\begin{aligned}
\frac{dS}{dt} &= \Gamma\nu\sigma NS - \Gamma\nu\frac{l_a}{l}k_a S - \frac{1}{T}S, \\
\frac{dN}{dt} &= -\beta\frac{\sigma}{\hbar w}NS + \frac{N_0 - N}{\tau}, \\
\frac{dk_a}{dt} &= -\frac{2\sigma_a k_a S}{\hbar w} + \frac{k_{0a} - k_a}{\tau_a},
\end{aligned}
\tag{1}
$$

where S is the emitted photon density, N is the population inversion of the active medium, and k_a is the resonant absorption of the saturable absorber. Γ, ν, σ, and T stand, respectively, for cavity filling coefficient, optical frequency, active medium cross-section, and photon lifetime in the cavity, β is the coefficient which accounts for the difference in population inversion coursed by lasing, l and l_a are, respectively, the active medium and the SA lengths, k_{0a} is the linear resonant SA absorption coefficient without lasing, σ_a is the SA cross-section, N_0 is the population inversion in the active medium without radiation, τ and τ_a stand for relaxation time in the active medium and in the SA, respectively, and finally $\hbar w$ is the photon energy.

Equations (1) can be rewritten in an adimensional form by introducing new parameters and variables defined as $t' = t/\tau$, $G = \tau/T$, $\delta = \tau/\tau_a$, $\rho = 2\sigma_a/\beta\sigma$, $\alpha = \Gamma\nu\sigma TN$, $\alpha_a = -\Gamma\nu Tk_{0a}(l_a/l)$, $n(t') = \Gamma\nu\sigma TN(t')$, $n_a(t') = -\Gamma\nu(l_a/l)Tk_a(t')$, and $m(t') = 2\pi\beta\sigma\tau S(t')/\hbar w$, and including the normalized harmonic modulation control $(1 + cos(\omega t))/2$ to α_a in the third equation which describes the SA:

$$
\begin{aligned}
\frac{dm}{dt'} &= Gm(n + n_a - 1), \\
\frac{dn}{dt'} &= \alpha - n(m + 1), \\
\frac{dn_a}{dt'} &= \delta\alpha_a\frac{1 + cos(\omega t)}{2} - n_a(\rho m + \delta).
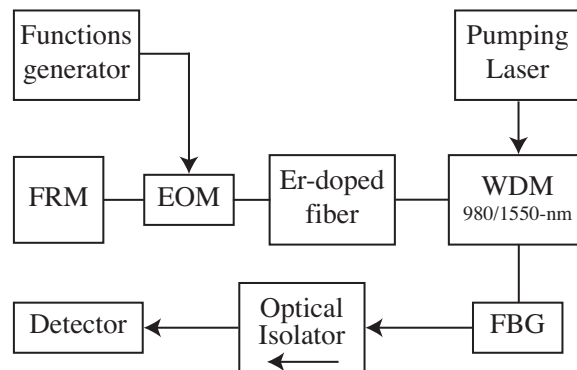\end{aligned}
\tag{2}
$$

Figure 2. Experimental all-fiber set-up for cnoidal wave generation. FRM is a Faraday rotating mirror, EOM stands for electrooptic modulator, WDM represents a wavelength-division multiplexer and FBG is a fiber Bragg grating.

While an external signal injected into an optical cavity has often been used to modify the shape of the output signal [15, 16, 17], in this work it can be injected into the SA through the electrooptic modulator (EOM) in order to modify the system dynamics and generate stable cnoidal waves. In this context, the EOM becomes an active device since the laser output is regulated by it through the cavity losses.

3. Experimental results

For the experiment shown in Fig. 2, we used a 1550-nm erbium-doped fiber laser (EDFL) pumped with a semiconductor laser operating at 980-nm. The active media is given by a section of excited rare-earth erbium-doped fiber (EDF). The resonant cavity has a Fabry-Perot structure. The EDF consists in a 90-cm span of a SCL 110-01 actively doped fiber (with an Er_2O_3 concentration of 2300 ppm), this fiber was pumped by a 980-nm laser diode through a 980/1550-nm wavelength-division multiplexer (WDM) coupler. In order to form the resonator, a Faraday rotating mirror (FRM), and a fiber Bragg grating (FBG) centered at 1550 nm with a 100 pm FWHM, having, respectively, 100 and 88% reflectivities at the laser wavelength were used; the FRM was used in order to avoid polarization mode beating. An electrooptic modulator (EOM), modulated through a function generator, was placed inside the cavity. The fiber output, after passing through the WDM, was recorded with a high-speed photodetector and analyzed with an 8GHz oscilloscope. An opto-isolator was placed at the output in order to avoid any feedback that could cause instabilities in the laser dynamics. In this experiment the pumping-laser current was fixed at 63 mA, which resulted in a pumping power of 15.3 mW corresponding to about 10% above the laser threshold of 14 mW. This value was chosen in order to have a cw-laser operation. The EDFL's relaxation frequency was around 28 kHz due to the applied pumping power. As previously demonstrated [16, 17], depending on the relaxation frequency, certain modulation losses can be applied to the cavity through the EOM to produce cnoidal waves.

Figure 3 shows the obtained cnoidal waves for the presented configuration. The laser's output power registered for the soliton-limit was 2.5 mW with a repetition rate of 8.28MHz, corresponding to 90% of the injected modulation. These pulses are generated when the frequency applied to the EOM is around 9.1 MHz, which corresponds to 325 times the laser's relaxation frequency. The frequency needed to generate cnoidal waves depends on the length of the EDFL; the highest repetition rate is given by the maximum control modulation frequency injected into the EOM and its matching erbium-doped fiber length, meaning that it is limited by
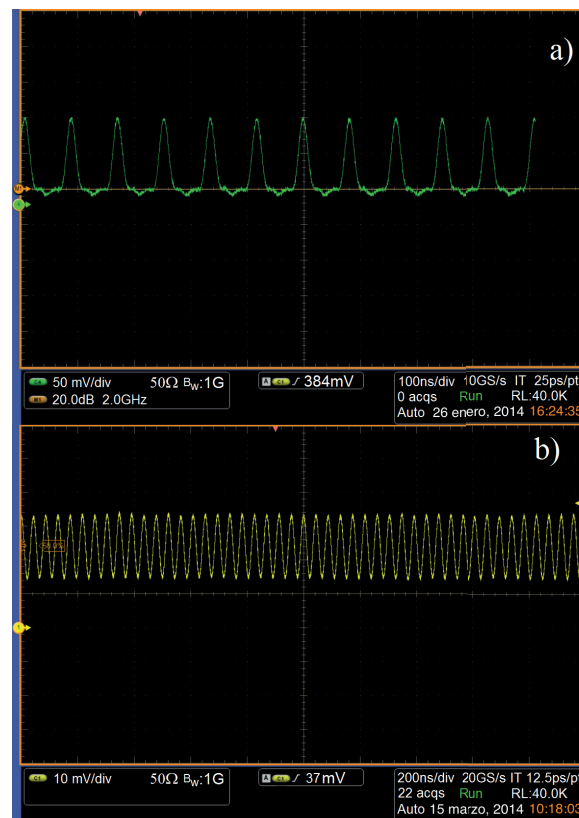
Figure 3. cnoidal wave generation. description.

the functions generator. Figure 4 shows the relationship between different erbium-doped fiber lengths and the control frequency needed to produce the cnoidal waves in their bounds. As it can be observed from Fig.4, the relationship between frequency and length becomes almost linear in the soliton limit (solid squares) with two jumps at lengths of 0.15 and 0.9 m. However, at the sinusoidal limit (solid circles) the growth presents jumps and plateaus at different fiber lengths than the soliton-limit. The optical spectrum remains fixed centered at 1552.2-nm. Figure 5 shows the temporal dynamics of the laser output, while the modulation frequency is increased for two different EDFL lengths. It is clear from Figs. 5(b) and 5(d) that the pulses shift from a soliton to a sinusoidal shape, having a strong relationship with the control frequency. Following our results, the cnoidal waves are bounded between two values of the control parameter (this parameter corresponds to the control modulation frequency). In this case, at a low frequency, we can observe that the cnoidal waves are transformed into a *sech* shape and, as the frequency increases, the wave changes into a *cos* shape. Note that the waves produced are asymptotically stable due to the quadratic nonlinearities of the erbium (acts as a saturable absorber and active medium).

4. Conclusions
It is demonstrated that linearly stable cnoidal waves can be produced in an erbium-doped fiber laser assisted by an EOM, which modulates the erbium absorbent centers. Both cnoidal wave limits were achieved, i.e. sinusoidal and soliton limits showing that it is possible to obtain such kind of pulses for different pairs of control frequency and doped fiber length. The dependence between the erbium-doped fiber length and modulation frequency of the EOM
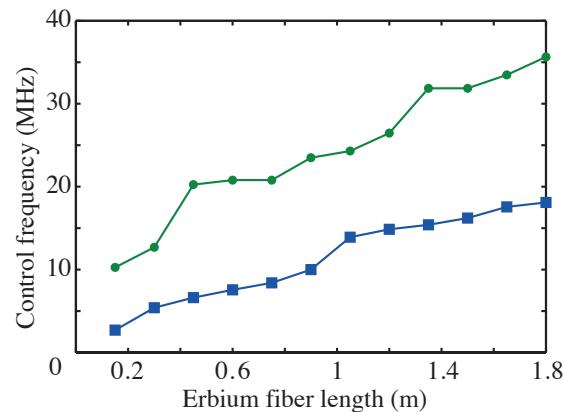
Figure 4. Relationship between fiber length and control frequency. Solid blue squares represent the soliton limit and solid green circles the sinusoidal one.
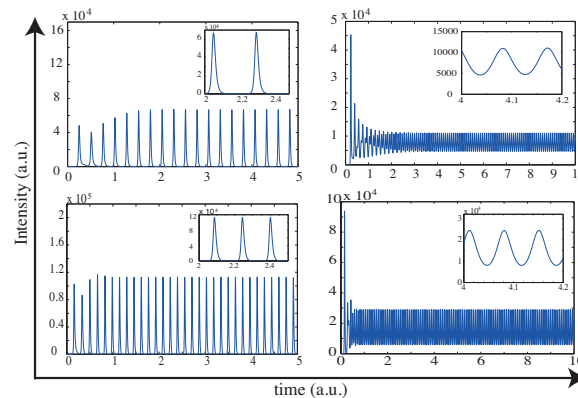

Figure 5. Numerical results for cnoidal waves limits. a) *sech* limit for 0.9m, b) *cos* limit for 0.9m, c) *sech* limit for 1.7m d) *cos* limit for 1.7m.

was experimentally demonstrated. The presented numerical results show the same qualitative behavior as the obtained experimental results. Furthermore, the proposed method is reliable and simple for the creation of soliton-shaped pulses which can be used for optical communication purposes and for sinusoidal-shaped pulses that can be useful in the study of the nonlinear propagation of pulses in different media.

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