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Gravitational-wave detection by dispersion force modulation in nanoscale parametric amplifiers

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Abstract. Two infinite parallel plane slabs separated by a gap alter the zero-point-energy of the matter-electromagnetic field system. Generally speaking, the corresponding interaction depends on the reflection properties of the boundaries, and therefore on the dielectric functions of both the slab and gap materials, on the gap width, and on the absolute temperature of the system. Importantly, it is known experimentally that dispersion forces can be modulated in time. This can be achieved by mechanically varying the gap width so as to introduce parametric oscillations. Much more fundamentally, however, dispersion forces can be altered by acting on the dielectric functions involved as is the case in semiconductors. In the optical analogy, a gravitational wave introduces an additional time dependence of the effective gap dielectric function. These elements, already confirmed by direct experimentation or predicted from the Lifshitz theory, support the design of a novel approach to ground-based nanoscale gravitational wave detection based on parametric amplification driven by dispersion force modulation

1. Introduction

Gravitational wave detection efforts in the past half century have been carried out on macroscopic length scales, such as Weber's original resonant antennas [1, 2] ($\sim 10^0 - 10^1$ m), large baseline interferometric detectors [3] ($\sim 10^3 - 10^4$ m), and, successfully, on the astrophysical scale in the Hulse-Taylor (PSR 1913+16) binary system [4] (semi-major axis $\sim 10^9$ m). Notable efforts have been devoted to exploring in the opposite direction, for instance, seeking out opportunities in atomic systems located in extreme astrophysical environments [5], with devices operating on scales of just a few percent those of interferometers [6], and even in sub-micron particle [7] and atomic [8] traps. Here we expand discussion of gravitational wave sensing by Casimir effect amplification on length scales 10-12 orders of magnitude below those of a LIGO class detector [9].

2. Gravitational waves, electromagnetic radiation propagation, and QED

Following early, pivotal efforts to derive a formal theory of quantized gravity in the presence of electromagnetic fields [10, 11], the first attempts in the "long but somewhat confusing history" [12] of the study of gravitational wave effects on electromagnetic propagation are attributed to Zipoy [13] and, independently, to Winterberg [14]. In the latter, stellar scintillation by a stochastic gravitational wave background of cosmological and astrophysical origin (Ref. [15] and references therein) was considered in a randomly inhomogeneous medium [16].

In order to describe the index of refraction, the effective medium formulation was adopted [14], possibly for the first time in the gravitational wave case. By setting equal to zero the interval

($ds^2 = 0$) for a plane wave of amplitudes $h_+^{\rm TT}(ct-z)$, $h_\times^{\rm TT}(ct-z)$ propagating along the z-axis in the transverse-traceless (TT) gauge, by transforming to spherical polar coordinates (r, θ, ϕ), and solving for the coordinate speed of light $c_n = dr/dt$, the effective index of refraction is, to first order in $h_{+,\times}^{\rm TT}$, $n(\theta,\phi) = \sqrt{\epsilon\mu} \simeq 1 + \frac{1}{2}(h_+^{\rm TT}\cos 2\phi + h_\times^{\rm TT}\sin 2\phi)\sin^2\theta$ [9, 14], where (θ, ϕ) is the optical ray propagation direction, ϵ and μ are the dielectric constant and the magnetic permeability, respectively, and, in the analogy, $\epsilon = \mu$ (Ref. [17] and references therein).

Although such a result has not been questioned, its applications to gravitational wave stochastic refraction and lensing over cosmological distances have been extensively debated [12, 13, 18, 19, 20, 21, 22, 23, 24]. Furthermore, redshift measurements [25], including those derived from spacecraft tracking [26, 27, 28], have also been considered in detail, again within the weak field approximation. More recently, electromagnetic radiation scattering in exact gravitational wave solutions was discussed and extended to colliding plane gravitational wave spacetimes [29].

Despite the need for constant vigilance over the actual physical content of specific effective medium approach applications [30], it is now clear that this strategy can lead to, provide a computational framework for, and ultimately shed light on quantum electrodynamical (QED) processes in curved spacetime, including novel features of interatomic interactions [31, 32]. The nearly resonant interaction between photons and gravitational waves over astrophysical distances was treated within the analogy [33]; those results have been confirmed with the prediction of a dynamical Casimir effect-like photon production in time-dependent dielectric media described by gravitational wave analog metrics equal to the one assumed by Winterberg in the particular case of $h_+^{\rm TT} \to h_+^{\rm TT}\cos[\omega(z-t/c)]$ and $h_\times^{\rm TT} = 0$, within a confined region (see Ref. [34], Eqs. (9)-(11)). Also, gravitational wave detection by phonon production in a BEC cavity has been explored by effective optical medium techniques employing again the same metric (see Ref. [8], Eqs. (3)-(4)).

3. Gravitational waves and the electrodynamical Casimir effect

In the present approach, a plane gravitational wave, described within the effective-medium-analogy and traveling along the z-axis of an oscillating Casimir cavity, introduces a modulation in the optical properties of the gap between two interacting boundaries (Figs.1-2). In the original proposal by this author [9], the effect was explored by neglecting corrections due to the magnetic permeability and by making use of the well-known result for the dispersion force in the limit of two ideal reflectors separated by a gap filled with a material of static dielectric constant $\epsilon_{3,0}$ (Ref. [35], Eq. (4.21). Here we remove that simplification and make use of the generalized Casimir pressure for a medium characterized by $\epsilon_{3,0}$ and $\mu_{3,0} \neq 1$ at $T = 0$ K (Ref. [36], Eq. (90)):

$$P_{\rm Cas}(s) = -\frac{\hbar c\pi^2}{240\, s^4}\sqrt{\frac{\mu_{3,0}}{\epsilon_{3,0}}}\left(\frac{2}{3} + \frac{1}{3}\frac{1}{\epsilon_{3,0}\,\mu_{3,0}}\right) = -\frac{\hbar c\pi^2}{240\, s^4}\left(\frac{2}{3} + \frac{1}{3}\frac{1}{n_0^2}\right), \qquad (1)$$

where s is the gap width, n_0 is the gap medium refraction index, and $\epsilon = \mu$.

Let us consider a detector modeled as a structure (Fig. 1, gray base and side fixed walls) exerting a harmonic restoring force of elastic constant K upon an oscillating plate, also acted upon by the Casimir force due to another facing plate, which can be driven into motion by actuators. In appropriate coordinates, this system is equivalent to a typical elementary gravitational wave bar model (Ref. [38], Fig. 9.2) in which, however, the two masses are unequal and they mutually interact via the Casimir force (Ref. [39], Fig. 3). Notice that, in this particular geometry, no dynamical effect on the oscillator due to the gravitational wave is expected. The result for $n(\theta,\phi)$ by Winterberg [14] given above in Sec. 2 makes it clear that the equivalent medium is not isotropic so that, rigorously speaking, a full treatment of the Casimir force between ideal reflectors separated by a bianisotropic, non-birefringent medium should be employed [29]. Such a calculation is quite intricate even in the case of uniaxial media [40] so here we let the index of refraction enter the above equation as an average, $\overline{n}_0^2 \simeq 1 + \overline{h}_+^{\rm TT}$, appropriately weighted

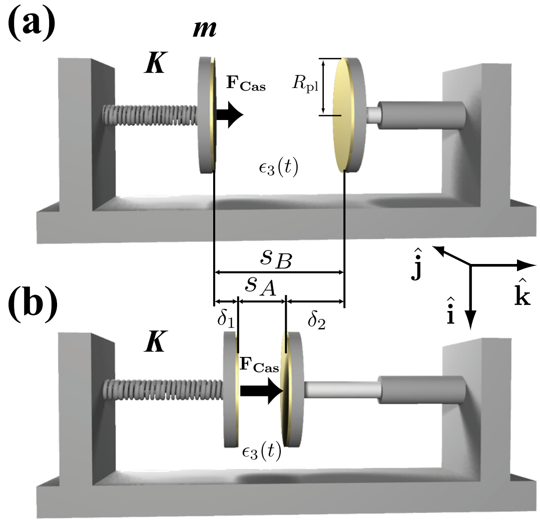

Figure 1. Casimir force parametric amplification by time-dependent gap width. As the right plate is periodically moved to the left by δ_2 from an initial wide separation s_A (a), the equilibrium position of the left plate is displaced by δ_1 and the natural frequency of oscillation is altered as the Casimir force increases due to a decrease of the gap to s_B (not to scale) [37]. A gravitational wave ($\lambda_{GW} \gg s_{A,B}$) propagating along $\hat{\mathbf{k}}$ affects the optical properties of the gap medium indicated as $\epsilon_3 = \mu_3$ (see text).

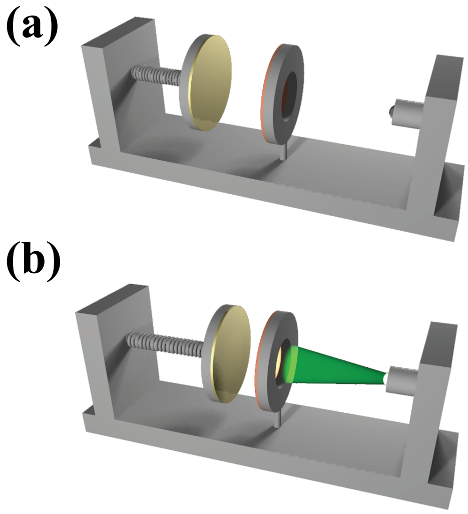

Figure 2. Casimir force parametric amplification by time-dependent illumination. The right plate is a semiconductor that is periodically back-illuminated by radiation capable to alter the free charge carrier density. (a) In the dark state, the opposing oscillating gold coated plate responds to the interaction with an insulator. In the illuminated state (b), the van der Waals forces increases and the position of equilibrium of the left plate changes although the right plate is at rest. (see text).

over all virtual photon angles of incidence on the ideal boundaries (Ref. [14], Eq. (2.17)). If the gravitational wave wavelength, $\lambda_{GW} = 2\pi c/\omega_{GW} \gg s$, we have, assuming $h_\times^{\rm TT} = 0$:

$$P_{\rm Cas}(t) = -\frac{\hbar c \pi^2}{240\, s^4}\left[\frac{2}{3} + \frac{1}{3}\left(1 - \overline{h}_+^{\rm TT}\cos\omega_{GW} t\right)\right] = -\frac{\hbar c \pi^2}{240\, s^4}\left(1 - \frac{1}{3}\overline{h}_+^{\rm TT}\cos\omega_{GW} t\right). \quad (2)$$

Hence the improved treatment shows an effect larger by a factor $\frac{4}{3}$ than found previously [9].

4. Orders of magnitude

In the archetypal Casimir effect parallel plane geometry, for a plate of radius $R_{\rm pl} \simeq 0.56$ cm (plate area, $A_{\rm pl} = 1$ cm^2), gap width $s_0 = 10$ nm, and $\overline{h}_+^{\rm TT} = 10^{-20}$, the amplitude of the time-dependent force from Eq. (2) is $\Delta F_{\rm Cas,0} = \frac{1}{720}(\hbar c \pi^2 s^4) A_{\rm pl}\overline{h}_+^{\rm TT} \simeq 4.33 \times 10^4$ yN. Omitting for brevity consideration of parallelism issues [41], this quantity is orders of magnitude above the lowest limit for ultrasensitive force detection, which has rapidly decreased from $\simeq 170$ yN [42] to values as low as $\simeq 5$ yN [43]. From the dynamical standpoint, assuming the elastically bound plate to be a flexible membrane of thickness $s_M = 10\ \mu$m and volume density $\rho_M = 8\times10^3$ kg/m^3 (membrane mass, $M_{\rm membr} = 8$ mg), the acceleration is $a_{M,0} = \Delta F_{\rm Cas,0}/M_{\rm membr} \simeq 5 \times 10^{-15}$ m/s^2. This is the same order of magnitude as the forcing tidal acceleration of a bar antenna of length $L_{\rm bar} = 1$ m, $a_{\rm bar} = \frac{1}{2} L_{\rm bar}\omega_{GW}^2 \overline{h}_+^{\rm TT}$ [38]. Driving the harmonic oscillator of Fig. 1 with a term equal to $a_M(t) = a_{M,0}\cos\omega_{GW} t$ at the oscillator resonant frequency, $\omega_{GW} = \omega_0$, with $\omega_{GW} = 10^3$ s^{-1} and a quality factor $Q = 10^5$, yields a steady state amplitude response $x_{\rm Cas,res} = (a_{M,0}/\omega_{GW}^2)Q \simeq 0.5$ fm, to be compared with $x_{\rm bar,res} = \frac{1}{2}L_{\rm bar}\overline{h}_+^{\rm TT} Q \simeq 0.5$ fm for a macroscopic bar. A challenge in this model relates to the mass of the membrane, which, since $a_{M,0} \propto M_{\rm membr}^{-1}$, implies thermal requirements with $T \ll 1$ K unless $s < 10$ nm and $Q > 10^6$.

We also mention here that different geometries show promise for detection with this approach. For instance, partially extruded inner cores of multiwalled carbon nanotubes (MWCNTs), retracted under the effect of unretarded van der Waals forces, experience some of the largest accelerations possible outside exotic astrophysical objects. Even for non-ideal reflectors, the forcing term from Eq. (2) is expected to be magnified by several orders of magnitude due to the interlayer separation $s \simeq 0.34$ nm. The goal in this configuration is to modify the ‘truly non-linear’ (in the Mickens sense) potential and drive the inner core at resonant frequencies in the desired range so as to take advantage of the nearly frictionless motion in such systems [44, 45].

5. Modulated Casimir force-driven parametric amplifier

The potential advantages of parametric amplification by Casimir force modulation have already been discussed in detail leading to gain estimates as high as $\approx 5 \times 10^3$ [37]. This novel approach is based on the familiar phenomenon of mechanical parametric resonance [46] employed in “a child’s swing” and previously shown to yield thermomechanical noise squeezing by electrostatic capacitive pumping [47]. As schematically shown in Fig. 1, Casimir force-enabled parametric amplification can be achieved by mechanical actuation of one of the interacting surfaces so as to induce a periodic shift of the natural frequency of the system [37]. Here we propose that such a strategy can be further improved if, instead of employing mechanical displacement, the Casimir force is modulated by back-illumination of a *fixed* semiconducting surface (Fig. 2). The dependence of dispersion forces in semiconductors on illumination was first suggested and experimentally demonstrated over thirty years ago [48, 49] although some unexplained discrepancies with the Lifshitz theory remained. Much later, a successful verification of this fascinating phenomenon was reported in a different experiment based on the atomic force microscope (AFM) [50]. The present author has carried out calculations based on analytical descriptions of available optical property data for gold and undoped crystal silicon used in the latter experiment and confirmed that, for instance, with $s = 200$ nm and Ar laser ($\lambda = 514$ nm) flux levels ~ 10 W/cm^2 [50] relative dispersion force modulation levels comparable to those computed for mechanical displacement can be obtained [37]. The advantage of this implementation is that both non-linearities due to gap width modulation and induced mechanical vibrations are completely eliminated, and that, with an appropriate use of semiconductors and dopants, higher frequencies can become accessible. The possibility to extend dispersion force modulation strategies to drive MWCNT core oscillations was also illustrated by the present author [44, 45, 51].

In closing, we have shown that gravitational waves traveling through a Casimir cavity produce forces well above the sensitivity limit of present-day experimentation and that sensor response can be parametrically amplified by means of illumination-driven Casimir force modulation.

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