Relativistic electrons spin states and spin light in dense neutrino fluxes

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Abstract. Relativistic electrons can produce electromagnetic radiation in moving background composed of neutrinos, that is the "*spin light of electron in neutrino flux*" (SLe_ν) [1, 2]. In this paper we further specify the electron quantum states in moving neutrino background by introdusing the electron spin operator that enables one to define the electron wave function in an exact and close form. This justifies our previous studies of SLe_ν in dense neutrino fluxes and derivations of the electron energy spectrum, the radiation rate and power, and also the emitted photon energy. We argue that the SLe_ν can have important consequences in different astrophysical settings.

1. Introduction

In [1, 2] we have considered an electron wave function and energy spectrum in the background of a relativistic neutrino flux, however without exact definition of the electron spin operator. On this basis it has been shown that the electron can produce electromagnetic radiation in moving background composed of neutrinos. We have termed this effect the "spin light of electron in neutrino flux" (SLe_ν) and considered the SLe_ν radiation rate, power and emitted photon energy. The mechanism of the proposed new phenomenon is similar to one of the spin light of neutrino in matter ($SL\nu$) [3, 4, 5, 6]. The latter effect is possible in the case of nonzero neutrino magnetic moments [7] (for a recent status of neutrino electromagnetic properties see a review [8]).

Here bellow we further specify the electron quantum states in moving neutrino background (a moving neutrino flux) by introducing the electron spin operator that enables one to define the electron wave function in an exact and closed form. This justifies our previous studies of SLe_ν in dense neutrino fluxes and derivations of the electron energy spectrum, the radiation rate and power, and also the emitted photon energy. We argue that the discussed SLe_ν can be of interest for astrophysical applications, for supernovae processes in particular.

2. Modified Dirac equation

We consider a beam of electrons moving inside of the neutrino flux composed of three flavors ν_e, ν_μ and ν_τ with number densities n_i (in the laboratory rest frame) moving in the same direction. Following our papers [1, 2], we introduce the average value n of the neutrino number density and the parameter δ_e,

$$n = \frac{n_e + n_\mu + n_\tau}{3}, \qquad \delta_e = \frac{n_\mu + n_\tau - n_e}{n}, \tag{1}$$

and obtain the modified Dirac equation for an electron in the neutrino flux,

$$\{\gamma_\mu p^\mu + \gamma_\mu \frac{c + \delta_e \gamma^5}{2} f^\mu - m\}\Psi(x) = 0, \tag{2}$$

where m and p^μ are the electron mass and momentum, $c = \delta_e - 12\sin^2\theta_W$, $G = \frac{G_F}{\sqrt{2}}$, and G_F is the Fermi constant. The speeds of relativistic neutrinos are $\beta_{\nu_i}^\mu \simeq (1, 0, 0, 1)$, thus the effective neutrino potential in (1) is $f^\mu = G(n, 0, 0, n)$. We suppose here that the neutrino flux propagates along the direction of z axis.

3. Spin operator
Following the analogy between a neutrino motion in a rotating matter and an electron motion in a magnetic field developed in papers [10] and [12], we introduce the matter field tensor T_{ij}

$$T_{ij} = p_i v_j - p_j v_i, \tag{3}$$

and also its dual tensor $\tilde{T}_{ij} = \frac{1}{2}\varepsilon_{ijnm}T^{nm}$, where the effective "vector potential" is the background matter velocity field v_i and p_j is electron energy-momentum vector.

Consider the analogy between this tensor and tensor of electromagnetic field we assume that operator
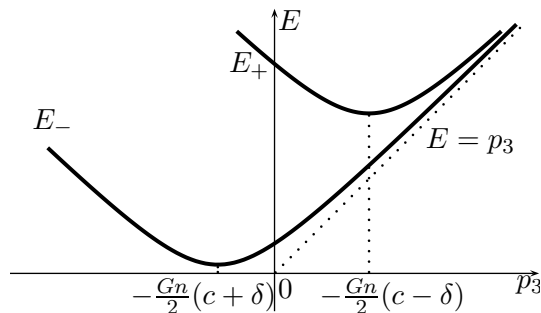
$$T = -\frac{1}{2}\sigma_{ij}\tilde{T}^{ij} = -\frac{1}{4}\varepsilon^{ijnm}\sigma_{ij}T_{nm} \tag{4}$$

commutes with the corresponding Hamiltonian of (2). This fact can be checked by the direct calculations.

4. Exact solution and kinematics
In the considered case of a relativistic neutrino flux ($v \sim 1$) (2) can be solved exactly. By a set of equivalent transformations (2) with account for (4) we get the following equation for the energy spectrum $E(p, s)$:

$$s\delta_e |Ev - \frac{\boldsymbol{p}\boldsymbol{v}}{v}| = \left\{ cE - c\boldsymbol{p}\boldsymbol{v} - \frac{m^2 + \boldsymbol{p}^2 - E^2}{Gn} \right\}. \tag{5}$$

Figure 1. The dependence of the electron energies in two different spin states, $E_+(\boldsymbol{p})$ and $E_-(\boldsymbol{p})$, on electron momentum component p_3.

Solving this equation we get the energy spectrum of an electron in the presence of the relativistic neutrino flux (see also [1, 2]):

$$E_s^\varepsilon(\boldsymbol{p}) = \varepsilon\sqrt{m^2 + \boldsymbol{p}_\perp^2 + \left(p_3 + A\right)^2} - A, \tag{6}$$

where $A = \frac{Gn}{2}\left(c - s\delta\right)$, $\delta = |\delta_e|$, p_3 is the electron momentum in the direction of the neutrino flux propagation and $\boldsymbol{p} = (\boldsymbol{p}_\perp, p_3)$ is the total electron momentum. The value $\varepsilon = \pm 1$ corresponds to the positive and negative frequency solutions (for the electron $\varepsilon = +1$).

The energy spectrum (6) also contains the second integer number $s = \pm 1$ that distinguishes two possible electron spin states.

Thus we see that the obtained spectrum branches are classified by both the frequency sign $\varepsilon = \pm 1$ and $s = \pm 1$. The latter quantity corresponds to two neutrino spin states determined by the spin operator (4). Two particular electron energy branches $E_s^\varepsilon(\boldsymbol{p})|_{\varepsilon=+1} = E_s(\boldsymbol{p})$ with $s = \pm 1$ as functions of the momentum $\boldsymbol{p}$ are plotted in Fig. 1. It is clearly seen that two corresponding curves have no common points that means there are no energy states with "undefined" spin. In [2] we show that $E_+(\boldsymbol{p}) > E_-(\boldsymbol{p})$ for any momentum $\boldsymbol{p}$.

The modified Dirac equation 2 can be solved exactly. The wave functions for each spin state can be found in the form $\psi_\pm(\boldsymbol{r}, t) = \frac{1}{L^{\frac{3}{2}} C_\pm} e^{i(-E_\pm t + \boldsymbol{pr})} \tilde{\psi}_\pm$, where

$$\tilde{\psi}_+ = \begin{pmatrix} 0 \\ m \\ p_\perp e^{-i\phi} \\ E_+ - p_3 \end{pmatrix}, \qquad \tilde{\psi}_- = \begin{pmatrix} E_- - p_3 \\ -p_\perp e^{i\phi} \\ m \\ 0 \end{pmatrix}, \tag{7}$$

here L is the normalization length and $C_\pm = \sqrt{m^2 + p_\perp^2 + (E_\pm - p_3)^2}$.

5. Spin light of electron

Using the "method of exact solutions" [10, 11, 12] we consider the radiative transition of an electron with emission of a photon in the presence of the relativistic flux of neutrinos. We term this process the *spin light of electron in dense neutrino flux* (SLe_ν).

The element of S-matrix defining the process amplitude is given by (see also [9]):

$$S_{fi}^{(\lambda)} = -e\sqrt{4\pi} \int d^4x \, \bar{\psi}_f(x)(\boldsymbol{\gamma} \boldsymbol{e}^{(\lambda)*}) \frac{e^{ikx}}{\sqrt{2\omega L^3}} \psi_i(x), \tag{8}$$

where e is the electron charge, $\psi_i(x)$ and $\psi_f(x)$ are the wave functions of the initial and final electrons in the background neutrino flux, $k = (\omega, \boldsymbol{k})$ and $\boldsymbol{e}^{(\lambda)}$ $(\lambda = 1, 2)$ are the momentum and polarization vectors of the emitted photon.

In general case the considered electron can also move in respect to an observer. The electron rest frame in moving neutrino background is defined as one where the electron energy E_+ gets its minimum, $\frac{\partial E_+}{\partial \boldsymbol{p}} = 0$ (see [4, 13, 14, 15]): $p_3 = -\frac{Gn}{2}(c - \delta)$, $\boldsymbol{p}_\perp = 0$. Thus the initial value of the electron momentum third component can be represent as

$$p_3 = -\frac{Gn}{2}(c - \delta) + \tilde{p}_3, \tag{9}$$

where $\tilde{p}_3$ is an "access" of the momentum component over its value in the rest frame where this value gets its minimum. In the following we distinguish two particular cases when
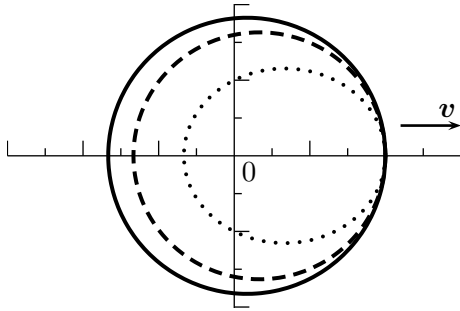
(i) for the nonmoving electron

$$p_3 = -\frac{Gn}{2}(c - \delta), \qquad \boldsymbol{p}_\perp = 0, \tag{10}$$

and

(i) for the relativistic electron moving in the opposite direction to the neutrino flux propagation

$$|\tilde{p}_3| \gg m, \quad |\tilde{p}_3| Gn\delta \ll m^2, \quad \text{and} \quad \tilde{p}_3 < 0. \tag{11}$$

As for the supernova environment $\frac{Gn}{m} \sim 10^{-8}$, the electron momentum in this case should be within the range $1 \ll \frac{|\tilde{p}_3|}{m} \ll 10^8$.

Figure 2. The angular dependence of the photon energy for different neutrino flux densities $\frac{m}{Gn\delta} = 1$ (dotted), $\frac{m}{Gn\delta} = 4$ (dashed) and $\frac{m}{Gn\delta} = 10$ (solid), $\boldsymbol{v}$ indicates the direction of the neutrino flux.

From the energy-momentum conservation for the emitted photon energy in the case of the nonmoving initial electron we obtain [1, 2]

$$\omega = \frac{m}{1 - \cos\theta + \frac{m}{Gn\delta}}, \qquad (12)$$

whereas for the case of the relativistic initial electron we get

$$\omega = \frac{2Gn\delta}{1 + \cos\theta + \frac{1}{2}\frac{m^2}{\tilde{p}_3^2}}, \qquad (13)$$

where θ is the angle between the direction of the SLe_ν and neutrino flux.

From (12) and (13) it follows that in general the emission is possible in all directions. We get for the photon energy

$$\omega = Gn\delta \qquad (14)$$

in case of nonmoving electron when a realistic condition is fulfilled $Gn\delta \ll m$.

Thus, in the nonmoving case for an initial charged particle with rather large mass or for the case of the background environment with enough small density the emitted photon energy does not depend on the direction of radiation and is determined only by the density of the environment.

It is interesting to compare the energy spectrum of the SLe_ν in the case of the relativistic electron and the electron at rest. Taking into account that in the case of nonmoving electrons $\omega = Gn\delta$, for the photons energy ratio (in the case of electron motion against the neutrino flux propagation, $\theta = \pi$) we get

$$\frac{\omega(|\tilde{p}_3| \gg m)}{\omega(|\tilde{p}_3| \ll m)} = 4\frac{\tilde{p}_3^2}{m^2} \gg 1. \qquad (15)$$

Figure 3. The photon energy angular dependence for a fixed neutrino flux density $\frac{m}{Gn\delta} = 10$ and different electron momenta: $\frac{\tilde{p}_3}{m} = 0.1$ (dotted), $\frac{\tilde{p}_3}{m} = 1$ (dashed) and $\frac{\tilde{p}_3}{m} = 3$ (solid), $\boldsymbol{v}$ indicates the direction of the neutrino flux.
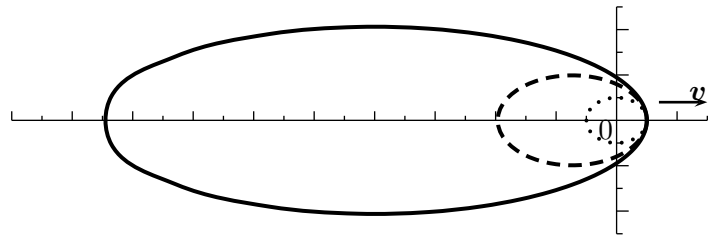
It follows [1, 2] that there is a reasonable increase of the emitted photon energy in case of the relativistic motion of the emitters (the electrons) with respect to nonmoving case.

Using expressions for the amplitude (8) and the wave functions of the initial and final electrons (7), and also for the emitted photon energy (12) and (13) we get (see also [1, 2]) for the SLe_ν total rate and power in the two discussed cases

(i) for the nonmoving electron

$$\Gamma = \frac{4}{3}e^2 m \Big(\frac{Gn\delta}{m}\Big)^3, \quad I = \frac{4}{3}e^2 m^2 \Big(\frac{Gn\delta}{m}\Big)^4, \qquad (16)$$

and

(i) for the relativistic electron

$$\Gamma = \frac{16}{3}e^2 m \Big(\frac{Gn\delta}{m}\Big)^3 \Big(\frac{|\tilde{p}_3|}{m}\Big)^2, \quad I = 16e^2 m^2 \Big(\frac{Gn\delta}{m}\Big)^4 \Big(\frac{|\tilde{p}_3|}{m}\Big)^4. \tag{17}$$

Comparing the corresponding characteristics (16) (17) of the SLe_ν obtained for two different cases we get that

$$\frac{\Gamma(|\tilde{p}_3| \gg m)}{\Gamma(|\tilde{p}_3| \ll m)} = 4\Big(\frac{|\tilde{p}_3|}{m}\Big)^2, \quad \frac{I(|\tilde{p}_3| \gg m)}{I(|\tilde{p}_3| \ll m)} = 12\Big(\frac{|\tilde{p}_3|}{m}\Big)^4.$$

For the case of relativistic electrons $\frac{|\tilde{p}_3|}{m} \gg 1$. Thus there is a reasonable amplification of the SLe_ν rate and power by the relativistic motion of the initial radiating electron.

6. Effect of plasma

The electromagnetic wave propagation in the background environment is influenced by the plasma effects. For the $SL\nu$ in matter these effects have been discussed in details in [5, 6, 16]. In [1, 2] we have shown that the effect of nonzero emitted photon mass (the plasmon mass m_γ) in the case of SLe_ν is not important, $\frac{m_\gamma}{Gn\delta} \ll 1$. The Debye screening of electromagnetic waves (another possible plasma effect) could be important for the SLe_ν radiation propagation if electron number density $N_e < 10^{35}\, cm^{-3}$. However, the electron matter with $N_e \sim 10^{19}\, cm^{-3}$ considered here is quite transparent for the SLe_ν.

Conclusion and phenomenology

It is interesting to apply the considered SLe_ν of nonmoving and relativistic electrons in dense neutrino fluxes to an environment peculiar for supernovae phenomena. As it is shown in [17], one can estimate the effective neutrino matter density to be $n \sim 10^{35}\, cm^{-3}$, thus the characteristic parameter $\frac{Gn}{m} \sim 10^{-8}$. The surrounding interstellar medium can contain regions with reasonably high electron density relativistically moving towards the neutrino flux [18, 19]. Under these conditions, the spin light can be emitted by the relativistic electrons in the quantum transition from the energy states E_+ to the states E_-.

From Eq. (12) it follows that for nonmoving initial electron the SLe_ν photon energy is

$$\omega \sim 1\ eV. \tag{18}$$

From (16) we also find for the SLe_ν rate and power

$$\Gamma \sim 10^{-19}\, eV \sim 10^{-4}\, s^{-1}, \qquad I \sim 10^{-7} eV s^{-1}. \tag{19}$$

The corresponding characteristic time of the SLe_ν process is rather big, $\tau \sim 10^4 s$. It means that SLe_ν from a single electron is hardly observable.

From (13) and (17) for the relativistic electrons characterized by $\frac{|\tilde{p}_3|}{m} = 10^7$ we get the following estimations for the SLe_ν photon energy, rate and power, respectively,

$$\omega \sim 10^{14}\ eV, \quad \Gamma \sim 10^{10}\, s^{-1}, \quad I \sim 10^{21} eV s^{-1}. \tag{20}$$

The electron number density at the distance $R = 10\ km$ from the star center can be of order $N_e \sim 10^{19}\, cm^{-3}$. Thus, the amount of SLe_ν flashes per second from $1\ cm^3$ of the electron matter under the influence of a dense neutrino flux is $N \sim 10^{28}\, cm^{-3}\, s^{-1}$. For the energy release of $1\ cm^3$ per one second we get

$$\frac{\delta E}{\delta t \delta V} = I N_e \sim 10^{40}\ eV\ cm^{-3}\ s^{-1}. \tag{21}$$

Now let us also estimate the efficiency of the energy transfer from the total neutrino flux to the electromagnetic radiation due to the proposed SLe_ν mechanism. The total neutrino energy in the neutrino flux (characterized by $n \sim 10^{35}\, cm^{-3}$ and $\langle E \rangle \sim 10^7\, eV$) is

$$\frac{\delta E_\nu}{\delta V} \sim \langle E \rangle n \sim 10^{42}\ eV\ cm^{-3}. \tag{22}$$

It follows that each second a considerable part of neutrino flux energy transforms into gamma-rays by the SLe_ν mechanism. The performed studies illustrates an increase of the efficiency of such energy transfer mechanism in the case when the emitting electrons are moving with relativistic speed against the neutrino flux propagation in comparison with the case of nonmoving initial electrons.

In conclusion, we further develop description of an electron propagation in an environment formed by a dense relativistic neutrino flux and obtain classification of the electron quantum states on the basis of the propose new spin operator. This justifies our previous studies of in dense neutrino fluxes and derivations of the electron energy spectrum, the radiation rate and power, and also the emitted photon energy. We predict that the effect of the spin light of electron in dense relativistic neutrino fluxes SLe_ν can have important consequences in astrophysics, and for the supernova process in particular.

Acknowledgments

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