Theoretical challenges in Double Beta Decay

Sabin Stoica

Horia Hulubei Foundation, P.O. Box MG12, 077125 Bucharest-Magurele, Romania

E-mail: sabin.stoica@unescochair-hhf.ro

Abstract. The study of the double beta decay (DBD), particularly the neutrinoless decay mode, is of great interest for testing the lepton number conservation (LNC) and getting information about neutrino properties, as the neutrinos character (Dirac or Majorana particles?), their absolute mass and hierarchy, etc. [1]-[2]. To make predictions of the DBD lifetimes and put constraints on the neutrino parameters, one needs accurate calculations of the nuclear matrix elements (NME) and phase space factors (PSF) entering the DBD lifetime expressions. In this paper I present recent calculations of these quantities, performed with approaches developed by our group. Then, I compare the theoretical predictions for the two-neutrino (2ν) DBD lifetimes, for the most experimentally interesting nuclei, with the experimental ones, and comment on the reliability of the neutrinoless (0ν) DBD calculations.

1. Introduction

The double beta decay is the nuclear process with the longest lifetime measured so far, which is of great interest, especially to test the LNC and understand the neutrino properties. Moreover, it has a broader potential to give us information on other beyond Standard Model (SM) processes, for example to check various scenarios of occurrence of 0νDBD modes (see [1]-[2] and the references therein).

According to the number and type of the released leptons we may have the following DBD modes: i) $2\nu\beta^-\beta^-$; ii) $0\nu\beta^-\beta^-$; iii) $2\nu\beta^+\beta^+$; iv) $0\nu\beta^+\beta^+$; v) $2\nu EC\beta^+$; vi) $0\nu EC\beta^+$; vii) $2\nu ECEC$ viii) $0\nu ECEC$ ix) $0\nu\beta^-\beta^- M$.

The 2νDBD modes occur with the LNC and are allowed in the original SM formulation. There are already measurements of $2\nu\beta^-\beta^-$ lifetimes for eleven nuclei, with values ranging between 10^{18}-10^{24} yr. These cases serve to check the reliability of the theoretical calculations for NME and PSF involved in these decay modes. On the other hand, no 0νDBD could be confirmed until now, hence we have only lower limits for their experimental lifetimes. Theoretically, one may consider several mechanisms for the occurrence of a 0νDBD, the most investigated being that which assumes the exchange of light left-handed neutrinos between two nucleons in the nucleus. The DBD lifetimes can be expressed as [1]:

$$\left(T_{1/2}^{2\nu}\right)^{-1} = G^{2\nu}(Q_{\beta\beta}, Z)g_A^4 \mid M^{2\nu} \mid^2 \;\; ; \;\; \left(T_{1/2}^{0\nu}\right)^{-1} = G^{0\nu}(Q_{\beta\beta}, Z)g_A^4 \mid M^{0\nu} \mid^2 \left(\langle m_\nu\rangle / m_e\right)^2 \quad (1)$$

where $G^{(0,2)\nu}$ are the PSF, $Q_{\beta\beta}$ is the energy decay, Z is the nuclear charge, m_e is the electron mass and $M^{(2,0)\nu}$ are NME depending on the nuclear structure of the parent and daughter nuclei. $\langle m_\nu\rangle$ is the Majorana light neutrino mass parameter, which can be expressed as a

linear combination of the light neutrino masses and elements of the first row of the Pontecorvo-Maki-Nakagawa-Sakata neutrino matrix [3]. One sees that the PSF and NME are the two key quantities entering the DBD lifetime expressions whose accurate computation is very needed in order to predict DBD lifetimes and/or derive neutrino mass parameters.

In this paper I present up-date calculations of NME and PSF performed by our group. The NME are calculated with a Shell Model (ShM)-based approach developed recently [4] -[5], while the PSF are computed with an approach described in Refs. [6]-[7], but using new, more efficient routines. Based on these calculations and on other ones reported in literature, I compare the theoretical predictions of the 2νDBD lifetimes for the experimentally most interesting nuclei, with the experimental ones, and comment further on the reliability of the 0νDBD calculations.

2. Formalism

The NME computation is an issue long debated in literature, the most employed methods of calculation being the proton-neutron Quasi Random Phase Approximation (pnQRPA) [8]-[11], ShM [12]-[14], Interacting Boson Approximation [15]-[16], Projected Hartree Fock Bogoliubov (PHFB) [17] and Energy Density Functional [18] methods. The discrepancies between NME values computed with different methods and by different groups are still significant (a factor of 2-3), and are due to both the approximations which are specific of each method and the different nuclear ingredients/parameters used in calculations (they have been largely discussed in the literature [1]-[2]). Here, we refer to the computation of the $0\nu\beta^-\beta^-$ NME performed with a ShM-based approach described in [4]-[5]. The NME can be expressed as:

$$M_\alpha^{0\nu} = \sum_{m,n} \left\langle 0_f^+ \| \tau_{-m}\tau_{-n}O_{mn}^\alpha \| 0_i^+ \right\rangle , \qquad (2)$$

where $\alpha = GT, F, T$ are the contributions associated with the Gamow-Teller (GT), Fermi(F) and Tensor(T) parts of the two-body transition operators O_{mn}^α, and the summation is performed over all the nucleon states. Their explicit expressions can be found, for example, in [1], [14]. The most difficult part is the computation of the radial components of the O_{mn}^α operators. They contain the neutrino potentials and can be defined by integrals over momentum carried by the virtual neutrino exchanged between the two nucleons [10]:

$$H_\alpha(r) = \frac{2R}{\pi} \int_0^\infty j_i(qr) \frac{h_\alpha(q)}{\omega} \frac{1}{\omega + \langle E \rangle} q^2 dq, \qquad (3)$$

where $R = r_0 A^{1/3} fm$ ($r_0 = 1.2 fm$), $\omega = \sqrt{q^2 + m_\nu^2}$ is the neutrino energy and $j_i(qr)$ is the spherical Bessel function (i = 0, 0 and 2 for GT, F and T, respectively). Usually, in calculation one uses the closure approximation, which consists in replacing excitation energies of the states in the intermediate odd-odd nucleus that contributes to the decay, by an average expression $\langle E \rangle$. The expressions of $h_\alpha(\alpha = GT, F, T)$ contain nuclear ingredients as the finite nucleon size (FNS), short range correlations effects and the inclusion of higher order terms in the nuclear currents (HOC) [10], [14]. FNS effect is taken into account through nucleon form factors, G_V and G_A, which depend on the neutrino momentum q and on the axial-vector constants g_V, g_A. In calculations either the quenched ($g_A = 1$) or unquenched ($g_A = 1.25 - 1.273$) values of this constant have been used, while the values of the cut-off parameters are $\Lambda_V = 850 MeV$ and $\Lambda_A = 1086 MeV$ [1]. The SRC effects are included by correcting the single particle wave functions (w. f.) with a correlation function: $f(r) = -c \cdot e^{-ar^2} (1 - br^2)$, that can be parametrized in different ways: the Jastrow prescription with the i) Miller-Spencer (MS) parametrization [19] and the CCM parametrization, derived with realistic ii) CD-Bonn and iii) AV18 NN potentials [10], distinguished by the choice of the the parameters a, b, c. Another method, unitary operator method (UCOM) can also be used to include SRC effects [9]. The tensor (h_T) component [1],

[20] is related to the inclusion of HOC terms. Besides the nuclear effects mentioned above, a number of input parameters as g_A, r_0, (Λ_A, Λ_B) and $< E >$ can play a significant role in the NME calculation.

The PSF were recently calculated using exact electron Dirac w.f., taking into account the FNS and electron screening effects [21], and discrepancies were found in comparison with previous calculations, especially for heavier nuclei. We also recalculated them within a similar formalism but developing new routines and using a Coulomb potential derived from a realistic proton density distribution in the daughter nucleus [6]-[7], and we confirmed the discrepancies. We also found that the numerical precision in the PSF computation is important, especially for cases with small $Q_{\beta\beta}$ and for identifying properly the electron bound states in the case of EC DBD modes. Hence, we improve the precision of our routines and develop a new numerical procedure for the identification of electron bound states. For the 2ν- and 0ν-$\beta^-\beta^-$ decays, the PSF formulas read:

$$G_{2\nu}^{\beta\beta}(0^+ \rightarrow 0^+) = C_{2\nu} \int_{m_e c^2}^{Q^{\beta\beta}+m_e c^2} d\epsilon_1 \int_{m_e c^2}^{Q^{\beta\beta}+2m_e c^2-\epsilon_1} d\epsilon_2 \int_0^{Q^{\beta\beta}+2mc_e^2-\epsilon_1-\epsilon_2} d\omega_1 f_{11}^{(0)} w_{2\nu}$$

$$\times (\langle K_N \rangle^2 + \langle L_N \rangle^2 + \langle K_N \rangle \langle L_N \rangle)$$

$$G_{0\nu}^{\beta\beta}(0^+ \rightarrow 0^+) = C_{0\nu} \int_{m_e c^2}^{Q^{\beta\beta}+m_e c^2} f_{11}^{(0)} w_{0\nu} d\epsilon_1 \tag{4}$$

where $C_{(0,2)\nu}$, $< K_N >$, $< L_N >$ are expressions that depend on electron and neutrino energies and $w_{(0,2)\nu}$ depend on the electron and neutrino energies and momenta. f_{11}^0 are the electron phase factors and are expressed in terms of the radial components of the electron w.f., solutions of the Dirac equations. Their explicit expressions can be found in Refs. [6], [21].

3. Numerical results and discussions

The inclusion of different nuclear effects and input parameters in the NME computation, may result in important differences between their calculated values. In order to better understand these differences, that also helps in interpreting the results reported in literature by different groups, we did a study on the influence of these effects and parameters on the NME values. The results are presented in Table 1.

We also studied the changes in the NME values produced by using different parameters: g_A, r_0 and $\langle E \rangle$, $\Lambda_{V,A}$. The conclusions can be briefly described as follows. The inclusion of the FNS, SRC and HOC substantially reduces the NME values. i) SRC inclusion: J-MS prescription decreases significantly the NME value ($\sim 40\%$) as compared with the softer CCM prescriptions($\sim 20\%$); ii) inclusion of HOC brings a correction of (15-20)%, while the tensor contribution is of (4-9)%; iii) FNS effect influences up to 12% the NME values; iv) the dependence of NME values on the NN effective interaction can be significant; for the case we studied (^{48}Ca) the difference is up to 17%. Further, the influence of the input nuclear parameters can be important: g_A quenched/unquenched (10-14)%; r_0 (1.1fm or 1.2fm) $\sim 7\%$; cut-off constants Λ_A, Λ_V) $\sim 8\%$, while the changes of the "closure" energy values ($< E >$) have a negligible effect on the NME values. In Table 2, I present up-date values of the PSF ($G^{(0,2)\nu}$), NME ($M^{(0,2)\nu}$) and lifetimes ($T^{(0,2)\nu}$). The PSF are calculated with our code, while the $M^{(0,2)\nu}$ and the experimental $T^{(0,2)\nu}$ are taken from the references indicated. I compare the theoretical $T^{2\nu}$ calculated with PSF computed with our code and NME values selected from literature, with the experimental ones. One can see the good power of prediction of the theoretical calculations, when using the ShM calculations for the NME. A particular case is ^{48}Ca, where the theoretical

Table 1. The NME obtained with inclusion of different nuclear effects. 'b' denotes the $M^{0\nu}$ value obtained without any effect included, while 'F', 'H', 'S' and 'total' denote the values when FNS, HOC, SRC and all effects, are included. The set of the three values a), b), c) are obtained with SRC prescriptions: Jastrow with MS parametrization, CCM-AV18, CCM-CD-Bonn, respectively. In calculations we used g_A=1.25, $r_0 = 1.2 fm$, $\Lambda_V = 850 MeV$, $\Lambda_A = 1086 MeV$.

	M_b	M_{b+F}	M_{b+H}	M_{b+F+H}	M_{b+S}	M_{b+S+F}	M_{b+S+H}	$M_{total}^{0\nu}$
^{48}Ca	-1.166	-0.959	-0.923	-0.773	(a)-0.731	-0.680	-0.542	-0.508
					(b)-1.023	-0.930	-0.800	-0.733
					(c)-1.153	-1.008	-0.914	-0.809
$^{48}Ca^*$	1.351	1.116	1.102	0.928	(a) 0.856	0.798	0.670	0.628
					(b) 1.188	1.082	0.962	0.884
					(c) 1.337	1.171	1.092	0.969
^{76}Ge	4.168	3.615	3.497	3.066	(a) 3.025	2.889	2.499	2.378
					(b) 3.807	3.557	3.187	2.979
					(c) 4.153	3.762	3.489	3.177
^{82}Se	-3.779	-3.305	-3.140	-2.780	(a)-2.779	-2.665	-2.275	-2.176
					(b)-3.467	-3.256	-2.876	-2.703
					(c)-3.770	-3.438	-3.137	-2.878

lifetime ($3.46 \times 10^{19} yr$) has been predicted before the experimental measurement ($4.4 \times 10^{19} yr$). This makes us confident in the ShM calculations also for the 0ν NME at least in this case. Also, we notice that for ^{82}Se, after adjusting the theoretical occupation numbers to experiment, the 0ν NME computed with ShM and pnQRPA get closer each other, so one gets more confidence to constrain the $< m_\nu >$ parameter for this isotope, as well. In the next future, there is the hope, the differences between ShM and pnQRPA calculations for NME get closer, for the other isotopes, too.

4. Conclusions

The NME and PSF are the two key quantities in the theoretical study of the DBD whose accurate computation is very need to extract information about neutrino properties. In this paper I present up-date calculations of these quantities. The NME are calculated with a ShM code developed recently [4] -[5]. I showed that the nuclear ingredients and input parameters used in computation can give significant differences in the NME values, hence it is important to understand their effect for interpreting the NME calculated values. The PSF are computed with an approach described in Refs. [6]-[7], but using new, more efficient routines. Based on these calculations and on other ones reported in literature, I compare the theoretical predictions of the 2νDBD lifetimes for the experimentally most interesting nuclei, with the experimental ones. I noticed the good power of prediction of the theoretical calculations, when computing the NME with ShM codes. This give us confidence also for 0νDBD calculations, to constrain the $< m_\nu >$ parameter, particularly in the cases of ^{48}Ca and ^{82}Se nuclei.

5. References

[1] Vergados J, Ejiri H and Simkovic F 2012 *Rep. Prog. Phys.* **75** 106301
[2] Vogel P 2012 *J.Phys. G: Nucl. Part. Phys.* **39** 124002

Table 2. Values of the PSF:$G^{0\nu,2\nu}$, NME: $M^{2\nu,0\nu}$ and lifetimes $T_{1/2}^{2\nu,0\nu}$. For $M^{2\nu,0\nu}$ we take values reported in Refs.: 1. [4], [13], [14]; 2. [15], [18]; 3. [10], [15], [18], [22]; 4. [10], [15], [17].

Nucleus	$G^{2\nu}$ $[yr]^{-1}$	$M^{2\nu}$	$T_{1/2}^{2\nu}$(th) $[yr]$	$T_{1/2}^{2\nu}$(exp) $[yr]$[23]	$G^{0\nu}$ $[yr]^{-1}$	$M^{0\nu}$	$T_{1/2}^{0\nu}$ $[yr]$
^{48}Ca	1.554×10^{-17}	0.0276^1	3.46×10^{19}	4.4×10^{19}	2.465×10^{-14}	0.82^1 2.2^2	$> 1.3 \times 10^{22}$[24]
^{76}Ge	4.647×10^{-20}	0.0613^1	2.34×10^{21}	1.65×10^{21}	2.372×10^{-15}	3.19^1 5.39^3	$> 2.1 \times 10^{25}$[25]
^{82}Se	1.573×10^{-18}	0.0634^1	6.48×10^{19}	0.92×10^{20}	1.014×10^{-14}	2.99^1 4.34^3	$> 3.6 \times 10^{23}$[24]
^{96}Zr	6.744×10^{-18}			2.3×10^{19}	2.048×10^{-14}	2.37^4	$> 9.2 \times 10^{21}$[24]
^{100}Mo	3.231×10^{-18}			7.1×10^{18}	1.584×10^{-14}	4.42^4	$> 1.1 \times 10^{24}$[24]
^{128}Te	2.149×10^{-22}	0.0245^1	3.17×10^{24}	2.0×10^{24}	5.783×10^{-16}	1.73^1 4.47^3	
^{130}Te	1.442×10^{-18}	0.0179^1	8.88×10^{20}	6.9×10^{20}	1.424×10^{-14}	2.02^1 4.04^4	$> 2.8 \times 10^{24}$[26]
^{136}Xe	1.332×10^{-18}	0.0118^1	2.04×10^{21}	2.19×10^{21}	1.454×10^{-14}	1.85^1 3.39^3	$> 1.1 \times 10^{25}$[27]
^{150}Nd	3.540×10^{-17}			8.2×10^{18}	6.194×10^{-14}	2.59^4	$> 1.8 \times 10^{22}$[24]

[3] King S and Luhn C 2013 *Rep. Prog. Phys.* **76** 056201
[4] Neacsu A, Stoica S and Horoi M 2012 *Phys. Rev.* C **86** 067304
[5] Neacsu A and Stoica S 2014 *J. Phys. G: Nucl. Part. Phys.* **41** 015201
[6] Stoica S and Mirea M 2013 *Phys. Rev.* C **88** 037303
[7] Pahomi T, Neacsu A, Mirea M and Stoica S 2014 *Rom. Rep. Phys.* **66** 370
[8] Rodin V, Faessler A, Simkovic F and Vogel P 2003 *Phys. Rev.* C **68** 044302; *Nucl. Phys.* A **793**
[9] Kortelainen M and Suhonen J 2007 *Phys. Rev.* C **75** 051303; *Phys. Rev.* C **76** 024315
[10] Simkovic F, Faessler A, Muther H, Rodin V and Stauf M 2009 *Phys. Rev.* C **79** 055501
[11] Stoica S and Klapdor-Kleingrothaus H 2001 *Nucl. Phys.* A **694** 269
[12] Caurier E. Zuker A.P. Poves A. and Martinez-Pinedo G. 1994 *Phys. Rev.* C **50** 225; Retamosa J, Caurier E and Nowacki F 1995 *Phys. Rev.* C **51** 371
[13] Menendez J, Poves A, Caurier E and Nowacki F 2009 *Nucl. Phys.* A **818** 139
[14] Horoi M, Stoica S and Brown B 2007 *Phys Rev C* **75** 034303; Horoi M and Stoica S 2010 *Phys. Rev.* C **81** 024321
[15] Barea J and Iachello F 2009 *Phys. Rev.* C **79** 044301
[16] Barea J, Kotila J and Iachello F 2013 *Phys. Rev. C* **87** 014315
[17] Rath P *et al* 2013 *Phys. Rev.* C **88** 064322
[18] Rodriguez T and Martinez-Pinedo G 2010 *Phys. Rev. Lett.* **105** 25250
[19] Miller G and Spencer J 1976 *Ann. Phys. NY* **100** 562
[20] Simkovic F, Pantis G, Vergados J and Faessler A 1999 *Phys. Rev.* C **60** 055502
[21] Kotila J and Iachello F 2012 *Phys. Rev.* C **85** 034316
[22] Civitarese O and Suhonen J 2009 *J. Phys: Conf. Series* **173** 012012; Suhonen J and Civitarese O 2012 J.Phys. G: Nucl. Part. Phys. **39** 124005
[23] Barabash A 2015 *Nucl. Phys. A* **935** 52
[24] Barabash A and Brudanin V 2011 *Phys. At. Nucl.* **74** 312
[25] Macolino C *et al* (GERDA Collaboration) 2014 *Mod. Phys. Lett.* A **29** 1430001
[26] Andreotti E 2011 *Astropart. Phys.* **34** 822
[27] Gando A it et al 2013 *Phys. Rev. Lett.* **110** 062502