

Standard Model fermion masses and mixing angles generated in 3HDM

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Abstract. We present a framework to generate the mass hierarchies and mixing angles of the fermionic sector of the Standard Model with two extra Higgs doublets and one right-handed neutrino. The masses of the first and second generation are generated by small quantum effects, explaining the hierarchy with the third generation. The model also generates a natural hierarchy between the first and second generation after the assumption that the Yukawa couplings are of rank 1. All the quark and lepton mixing matrices can also be generated by quantum effects, reproducing the hierarchies of the experimental values. The parameters generated radiatively depend logarithmically on the heavy Higgs masses. Therefore this framework can be reconciled with the stringent limits on flavour violation by postulating a sufficiently large new physics scale.

1. Introduction

The origin of fermion mass hierarchies and mixing angles is one of the main unsolved puzzles in Particle Physics. Furthermore, the improvement in the determination of neutrino mass splittings and mixing angles over the last decade, far from solving this question, shows a scenario with striking differences between neutrino and quark and charged lepton parameters [1, 2]. The first unresolved question is why neutrino masses are much smaller and have a milder hierarchy than quark and charged lepton masses. The second issue is to understand why the entries of the leptonic mixing matrix are all $\mathcal{O}(0.1)$ while the entries of the quark mixing matrix display the strong hierarchy $|V_{ub}|, |V_{cb}| \ll |V_{us}|$.

A possible scenario to explain the smallness of neutrino masses in comparison to the quark and charged lepton masses is known as the see-saw mechanism which consists in introducing heavy Majorana right-handed neutrinos, much heavier than the electroweak symmetry breaking scale [3]. This mechanism predicts that neutrino masses should be much smaller than the quark or charged lepton masses, although the precise value cannot be predicted. Furthermore, due to the high scale of lepton flavour violation, this framework predicts tiny rates for the rare lepton decays [4], in agreement with the stringent experimental bounds. Nevertheless, this scenario tends to generate hierarchies in the neutrino sector which are too large [5]. This issue can be solved by introducing a second Higgs doublet [6, 7].

Here we present the framework to generate masses and mixing angles of quarks and leptons extending the Standard Model by one right-handed neutrino and two extra Higgs doublets.



2. Extending the Standard Model with extra Higgs doublets and one right-handed neutrino.

We present here the extension of the Standard Model with one right-handed neutrino and extra Higgs doublets. We do not impose any symmetry on the model. The flavour dependent part of the Lagrangian in this model is given by

$$-\mathcal{L} = Y_{d,ij}^{(a)} \bar{q}_{Li} d_{Rj} \Phi_a + Y_{u,ij}^{(a)} \bar{q}_{Li} u_{Rj} \tilde{\Phi}_a + Y_{e,ij}^{(a)} \bar{l}_{Li} e_{Rj} \Phi_a + Y_{\nu,ij}^{(a)} \bar{l}_{Li} \nu_{Rj} \tilde{\Phi}_a - \frac{1}{2} M_{M,ij} \bar{\nu}_{Ri}^C \nu_{Rj} + \text{h.c.}, \quad (1)$$

where $i, j = 1, 2, 3$ are flavour indices, $a = 1, \dots, n$ is the Higgs index, where $n - 1$ corresponds to the number of extra Higgs doublets and $\tilde{\Phi}_a = i\tau_2 \Phi_a^*$. We assume that the mass scale of the right-handed neutrinos is much larger than the electroweak symmetry breaking scale and the mass scale of all the extra Higgs states, which we denote collectively by M_H . Therefore, at energies relevant for experiments right-handed neutrinos are decoupled and the leptonic part of the model can be described by the effective Lagrangian

$$-\mathcal{L}^{l,\text{eff}} = Y_{e,ij}^{(a)} \bar{l}_{Li} e_{Rj} \Phi_a + \frac{1}{2} \kappa_{ij}^{(ab)} (\bar{l}_{Li} \tilde{\Phi}_a) (\tilde{\Phi}_b^T l_{Lj}^C) + \text{h.c.} \quad (2)$$

At the scale of the lightest right-handed neutrino mass, the coefficients of the dimension 5 operators $\kappa^{(ab)}$ read

$$\kappa^{(ab)}(M_1) = (Y_{\nu}^{(a)} M_M^{-1} Y_{\nu}^{(b)T})(M_1). \quad (3)$$

Without loss of generality, we chose to work in the basis where only Φ_1 acquires all the vacuum expectation value. Therefore $\langle \Phi_1^0 \rangle = v/\sqrt{2}$, with $v = 246$ GeV, and $\langle \Phi_b^0 \rangle = 0$ for $b = 2, \dots, n$. In this basis, the Dirac masses for the Standard Model fermions are proportional to their Yukawa couplings with Φ_1 . The neutrino mass matrix at the scale of the lightest right-handed neutrino mass can therefore be written as

$$\mathcal{M}_{\nu}(M_1) = \frac{v^2}{2} \kappa^{(11)}(M_1). \quad (4)$$

In view of the large hierarchies in the fermionic mass sector, we assume that all Yukawa couplings are of rank-1 at tree-level. Thus, the Yukawa couplings with Φ_1 can be written in the form

$$Y_u^{(1)}|_{\text{tree}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_u^{(1)} \end{pmatrix}, \quad Y_d^{(1)}|_{\text{tree}} = y_d^{(1)} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \sin \alpha_d^{(1)} \\ 0 & 0 & \cos \alpha_d^{(1)} \end{pmatrix}$$

$$Y_e^{(1)}|_{\text{tree}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_e^{(1)} \end{pmatrix}, \quad Y_{\nu}^{(1)}|_{\text{tree}} = y_{\nu}^{(1)} \begin{pmatrix} 0 \\ \sin \alpha_{\nu}^{(1)} \\ \cos \alpha_{\nu}^{(1)} \end{pmatrix}. \quad (5)$$

With this parametrization at the cut-off scale Λ it is straightforward to calculate at tree-level the masses of each family. The result is:

$$m_t|_{\text{tree}} = \frac{v}{\sqrt{2}} y_u^{(1)}, \quad m_c|_{\text{tree}} = m_u|_{\text{tree}} = 0,$$

$$m_b|_{\text{tree}} = \frac{v}{\sqrt{2}} y_d^{(1)}, \quad m_s|_{\text{tree}} = m_d|_{\text{tree}} = 0,$$

$$m_{\tau}|_{\text{tree}} = \frac{v}{\sqrt{2}} y_e^{(1)}, \quad m_{\mu}|_{\text{tree}} = m_e|_{\text{tree}} = 0,$$

$$m_3|_{\text{tree}} = \frac{v^2}{2} \frac{y_{\nu}^{(1)2}}{M_M}, \quad m_2|_{\text{tree}} = m_1|_{\text{tree}} = 0. \quad (6)$$

Besides, only the 33 element of both the Cabibbo-Kobayashi-Maskawa (CKM) and the Pontecorvo-Maki-Nakagawa-Sakata matrix (PMNS) is defined, since any rotation between the left-handed fermions of the first and second generation leaves the Lagrangian invariant. The value of this term is $V_{CKM33} = \cos \alpha_d^{(1)}$ for the CKM matrix and $V_{PMNS33} = \cos \alpha_\nu^{(1)}$ for the PMNS matrix.

In this basis, the Yukawa couplings to the extra Higgs doublets are general 3×3 matrices for quarks and charged leptons and general dimension 3 vectors for neutrinos.

3. Quantum effects on masses and mixing angles in a 3HDM

We present here the impact of quantum effects on the Yukawa couplings defined in Section 2 in a 3HDM. In contrast to the results in [8, 9], we are able to generate the masses of the first and second generation, m_{x_1} and m_{x_2} respectively, for $x = u, d, e, \nu$.

The one loop corrected couplings approximately read:

$$Y_x^{(1)}|_{1\text{-loop}} \simeq Y_x^{(1)}|_{\text{tree}} + L_x \beta_x^{(1)}, \quad (7)$$

where $L_x = \frac{1}{16\pi^2} \log \frac{\Lambda}{M_H}$ for $x = u, d$, $\frac{1}{16\pi^2} \log \frac{\Lambda}{M_M}$ for $x = e, \nu$ and the β -functions are calculated in [10, 11, 12].

3.1. The Mass Sector

We present in this subsection the results of the radiative generation of fermionic masses in a 3HDM. Assuming $m_{x_2} > m_{x_1}$, the Yukawa couplings of the first and second generation approximately read:

$$y_{x_2}^2 \simeq L_x^2 \left(\beta_{x_{11}}^{(1)2} + \beta_{x_{21}}^{(1)2} + \beta_{x_{12}}^{(1)2} + \beta_{x_{22}}^{(1)2} \right) \quad (8)$$

$$y_{x_1}^2 \simeq L_x^2 \frac{\left(\beta_{x_{11}}^{(1)} \beta_{x_{22}}^{(1)} - \beta_{x_{12}}^{(1)} \beta_{x_{21}}^{(1)} \right)^2}{\left(\beta_{x_{11}}^{(1)2} + \beta_{x_{21}}^{(1)2} + \beta_{x_{12}}^{(1)2} + \beta_{x_{22}}^{(1)2} \right)} \quad (9)$$

where $\beta_{x_{ij}}^{(1)}$ corresponds to the ij entry of the $\beta_x^{(1)}$ matrix. Both Yukawa couplings contain a loop suppression factor L_x with respect to the third generation of masses, leading to a natural hierarchy with respect to the latter. Nevertheless, both y_{x_2} and y_{x_1} are generated at one loop, hence it is necessary to impose an extra condition to generate a hierarchy between them. The condition to obtain a mass pattern compatible with experimental results deduced from Eq. (8) and (9) is given by the inequality:

$$\left(\beta_{x_{11}}^{(1)} \beta_{x_{22}}^{(1)} - \beta_{x_{12}}^{(1)} \beta_{x_{21}}^{(1)} \right) < \left(\beta_{x_{11}}^{(1)2} + \beta_{x_{21}}^{(1)2} + \beta_{x_{12}}^{(1)2} + \beta_{x_{22}}^{(1)2} \right) \quad (10)$$

Fulfilling this condition, the hierarchy between each generation in the fermion sector can be reproduced.

It is important to remark that radiatively generated masses depend logarithmically on the heavy

Higgs mass, while the dependence of different flavour violating observables is suppressed by two or even four powers of the latter, depending on the observable. Therefore, by postulating a very large value for the heavy Higgs mass, the predicted rates for flavour violating processes will be within the experimental ranges. While the direct production of the heavy states is far beyond the reach of present and foreseeable collider experiments, the new states generate deviations in flavour observables from the Standard Model values that might be at the reach of future experiments.

3.2. The Mixing Sector

Above the Λ scale, the mixing angles for Standard Model fermions cannot be determined. A consequence of breaking the mass degeneracy between first and second generation of each family of fermions is that the matrices that diagonalize the Yukawa coupling become unambiguously fixed. The unitary mixing can therefore be determined at lower energies which, for the left handed sector approximately reads:

$$U_{L_x}(M_x) \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha_x^{(1)} & -\sin \alpha_x^{(1)} \\ 0 & \sin \alpha_x^{(1)} & \cos \alpha_x^{(1)} \end{pmatrix} \begin{pmatrix} \cos \zeta_{x_L} & \sin \zeta_{x_L} & 0 \\ -\sin \zeta_{x_L} & \cos \zeta_{x_L} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (11)$$

where $M_x = M_H$ for the quark sector, $M_x = M_M$ for the leptonic sector and $\alpha_{u,e}^{(1)} = 0$. The mixing angle ζ_{x_L} is a function of the entries of the $\beta_x^{(1)}$ matrices.

The corresponding CKM and PMNS matrices can be calculated from the mixing matrices as follows:

$$V_{CKM} = U_{L_u}^\dagger U_{L_d} \quad V_{PMNS} = U_{L_e}^T U_{L_\nu} \quad (12)$$

Experimentally it is known that $|V_{ub}|^2 + |V_{cb}|^2 \ll 1$, hence the assumption $\alpha_d^{(1)} \simeq 0$ is necessary to generate a hierarchical CKM matrix. To obtain an anarchical PMNS matrix $\alpha_\nu^{(1)}$ must remain larger than 0. Therefore, in this framework the different pattern of mixing angles in the quark and lepton sectors is related to the amount of misalignment of the tree level Yukawa couplings to the Higgs Φ_1 at a Λ cut-off scale, parametrized by one single angle $\alpha_{d,\nu}^{(1)}$.

4. Conclusions

The hierarchies between fermionic masses of each generation, as well as the hierarchies among the quark mixing angles, strongly suggest the existence of a dynamical mechanism to generate this pattern. We have shown that, starting with rank-1 Yukawa matrices at a cut-off scale, the radiative corrections in a 3HDM can generate the masses of the first and second generation of fermions with their corresponding hierarchies. The pattern of mixing angles in this model accommodates a hierarchical structure in the CKM matrix in contrast with a leptonic mixing matrix with all entries $\mathcal{O}(0.1)$. Such a structure of mixing can be reproduced by changing the misalignment between the rank-1 Yukawa couplings that generate Dirac masses at tree level. This scheme can reproduce the measured values even in the decoupling limit of the heavy Higgs, therefore the strong constraints on the extra Higgs doublets from flavour observables can be easily avoided if the heavy Higgs mass is sufficiently large. On the other hand, if the new physics scale is low enough, new phenomena could be observed in experiments at the intensity frontier, opening possibilities to test this mechanism.

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