Determination of mass of an isolated neutron star using continuous gravitational waves with two frequency modes: an effect of a misalignment angle

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Abstract. A rapidly spinning neutron star (NS) would emit a continuous gravitational wave (GW) detectable by the advanced LIGO, advanced Virgo, KAGRA and proposed third generation detectors such as the Einstein Telescope (ET). Such a GW does not propagate freely, but is affected by the Coulomb-type gravitational field of the NS itself. This effect appears as a phase shift in the GW depending on the NS mass. We have shown that mass of an isolated NS can, in principle, be determined if we could detect the continuous GW with two or more frequency modes. Indeed, our Monte Carlo simulations have demonstrated that mass of a NS with its ellipticity 10^{-6} at 1 kpc is typically measurable with precision of 20% using the ET, if the NS is precessing or has a pinned superfluid core and emits GWs with once and twice the spin frequencies. After briefly explaining our idea and results, this paper concerns with the effect of misalignment angle ("wobble angle" in the case of a precessing NS) on the mass measurement precision.

1. Introduction

The distribution of masses of neutron stars (NSs) gives insights to understand their birth mechanisms and evolution histories. For instance, the mass measurements of the massive pulsar PSR J16142230 [1] and PSR J0348+0432 [2] have great impacts on exotic matter physics and studies of NS interiors.

Currently, mass measurements of NSs are mostly limited to those in binaries [3]. Since possible mutual mass transfers in binaries may change the mass distributions of the component NSs, it is desirable to measure masses of as many isolated NSs as possible.

We have proposed a new method to measure mass of an isolated NS by detecting Coulomb-type phase shifts imprinted in gravitational waves (GWs) by the NS gravitational field [4]. In that paper, we have shown that the mass of a NS at 1 kpc distance and with its ellipticity 10^{-6} is typically measurable with precision of 20% using the third generation GW telescope such as the Einstein Telescope (ET) [5], if the NS is precessing or has a pinned superfluid core and emits GWs with once and twice the spin frequencies.

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In the following sections, we will review the idea of our method, and then comment on our assumptions on misalignment angles in our original paper [4], namely the misalignment angle of the freely-precessing NSs and their effect on mass measurement precision.

2. Method

Let us consider an isolated NS with mass M located at the coordinate origin $r = 0$. We assume that the NS emits GW with multiple frequencies $\omega_\alpha = 2\pi f_\alpha (\alpha = 1, 2, \cdots)$. The two polarization modes of GWs from the NS can be written as

$$\bar{h}_+ = \sum_\alpha A_{+,\alpha} \cos \Psi_\alpha, \quad \bar{h}_\times = \sum_\alpha A_{\times,\alpha} \sin \Psi_\alpha, \tag{1}$$

$$\Psi_\alpha \equiv -\omega_\alpha t + \omega_\alpha r + \Phi_\alpha, \tag{2}$$

$$\Phi_\alpha \equiv 2\omega_\alpha M \ln[2\omega_\alpha r] + \phi_{R\alpha}, \tag{3}$$

where $\phi_{R\alpha}$ denotes the constant reference GW phases. The logarithmic term in Φ_α is the Coulomb phase shift due to the static part of the NS gravitational field [4, 6]. All the searches for GWs from rapidly rotating NSs conducted so far have not taken into account these Coulomb phase shifts.

Our method to measure the mass M of a NS and results are as follows [4]. Suppose that we determine GW phases Φ_α for two or more frequency modes (say, ω_1 and ω_2 with the ratio $K \equiv \omega_2/\omega_1$). If these two modes have such a property that

$$\phi_{R2} - K\phi_{R1} = 0, \tag{4}$$

we can determine the mass from the following combination,

$$\Phi_2 - K\Phi_1 = 2\omega_2 M \ln K \simeq 0.06 \left(\frac{M}{1.4 M_\odot}\right) \left(\frac{f_2}{1\mathrm{kHz}}\right) \left(\frac{\ln K}{\ln 2}\right). \tag{5}$$

The condition (4) is satisfied for a freely-precessing isolated bi-axial NS, which may emit GWs (approximately) at once and twice the spin frequencies ($K = 2$). Another possibility can be found in the model proposed by Jones [7]. He showed that if a tri-axial NS contains a pinned superfluid core and none of the axes of the principal moments of inertia of the solid crust aligns with the spin angular velocity vector of the core, then such a NS emits GWs at once and twice the spin frequencies. Notably, the condition (4) is hold when such a NS is bi-axial (but none of the axes of the principal moments of inertia is aligned with the spin axis).

The GWs from these two models of NSs have (See [4, 8, 9, 10, 11])

$$A_{+,1} = \frac{1}{4} h_0 \sin 2\theta \sin \iota \cos \iota, \qquad A_{\times,1} = \frac{1}{4} h_0 \sin 2\theta \sin \iota, \tag{6}$$

$$A_{+,2} = \frac{1}{2} h_0 \sin^2 \theta \left(1 + \cos^2 \iota\right), \quad A_{\times,2} = h_0 \sin^2 \theta \cos \iota, \tag{7}$$

where the inclination angle ι is defined as the angle between the rotational axis and the line-of-sight, and the misalignment angle θ is defined as the angle between the principal axis and the angular momentum axis. The overall amplitude is $h_0 = 4\varepsilon I \omega_1^2 / r$ where I denotes the star's average moment of inertia and the ellipticity ε quantifies the degree of the non-axisymmetry.

Assuming these models, we have performed Monte-Carlo simulations to study the measurement precision of NS mass using Eq. (5) for three-year ET observation [4]. For those simulations, we have randomly selected sets of waveform parameters $(\theta, \iota, \phi_R, \psi, \alpha, \delta)$ where ϕ_R is the reference phase, ψ is the GW polarization phase, and α and δ are the sky position of

the NS. Uniform distributions are assumed for those parameters. The NS moment of inertia I and its ellipticity ε are set to be 10^{45} g·cm^2 and 10^{-6} [12, 13], respectively. We adopted several representative values of r (1 kpc, 10 kpc, and 50 kpc) and f_1 (300 Hz and 500 Hz). Then, We estimate the measurement error in the NS mass for each simulation.

In the ideal case where all the waveform parameters except for ϕ_R and M are known, the measurement error in the GW phase Φ_α is $1/\rho$ where ρ is the signal-to-noise ratio (SNR). Hence, GW detections with $\rho \gtrsim 100$ for both modes may suffice to estimate the NS mass as indicated in Eq. (5). When all the waveform parameters are unknown in advance, the correlations among the parameters degrade the mass measurement precision. To improve the mass measurement precision, we have assumed that the spin frequency, the spin-down rate, the sky position of the NS are known in advance from electromagnetic observations or GW observations by the second generation GW detectors such as the advanced LIGO [14], advanced Virgo [15], and KAGRA [16].

The resulting cumulative distributions of mass measurement precision are plotted in the Fig. 1 of [4]. For example, we found that the mass of the NS with its spin frequency 500 Hz and its ellipticity is typically measurable with an accuracy of 20% using the ET.

3. Misalignment angle

We have assumed a uniform distribution for the misalignment angle θ in our Monte Carlo simulations in the previous paper [4]. In the case of a NS containing a pinned superfluid core, there seems to be neither observational nor theoretical constraint on θ. In fact, a uniform distribution for the misalignment angle θ is assumed in this model [9, 10, 11].

A possible range of the misalignment angle θ in the case of a freely precessing NS is not well-known. Two known examples indicate smaller misalignment angles: $\theta \simeq 3$ degrees (PSR B1828-11) [17] and $\theta \simeq 0.8$ degrees (PSR B1642-03) [18]. On the other hand, the theoretical maximum possible value of θ is estimated by Eq. (24) of [19] as $\theta_{\rm max} \simeq 10$ degrees $(500{\rm Hz}/f_1)(u_{\rm break}/10^{-2})$ where $u_{\rm break}$ is the breaking strain of the NS crust. For the freely precessing magnetars recently discovered in [20, 21], parameter degeneracies prevent from inferring misalignment angles. In any case, one may find it questionable to use a uniform distributions for θ in the case of a precessing NS.

To see how the misalignment angle affects the measurement precision of NS mass in our method, we have conducted 10,000 Monte Carlo simulations for three-year ET observations similar to the previous paper, but for several fixed misalignment angles. In these simulations, the sky position, polarization angle, reference phase, and inclination angles are randomly chosen from uniform distributions. The distance of $r = 1$ kpc and spin frequency of $f = 500$ Hz are assumed. Fisher matrix method is used to estimate measurement errors of the waveform parameters. As in the previous simulations, the frequency, spin-down rate, sky position of each NS are assumed to be known. The results are shown in Fig. 1. This figure indicates that masses of more than 70% of NSs are measurable with a precision of $\Delta M/M \simeq 0.2$ if $10^\circ \leq \theta \leq 80^\circ$. On the other hand, if the misalignment angles are smaller than 10 degrees, as suggested by PSR B1828-11 and PSR B1642-03, even the third generation GW telescope cannot determine the NS mass with sufficient precision (e.g. $\Delta M/M \simeq 0.8$ for 20% of NSs in the case of $\theta = 3^\circ$). This is because the amplitudes for both modes become smaller for the smaller misalignment angle as indicated by Eqs. (6) and (7).

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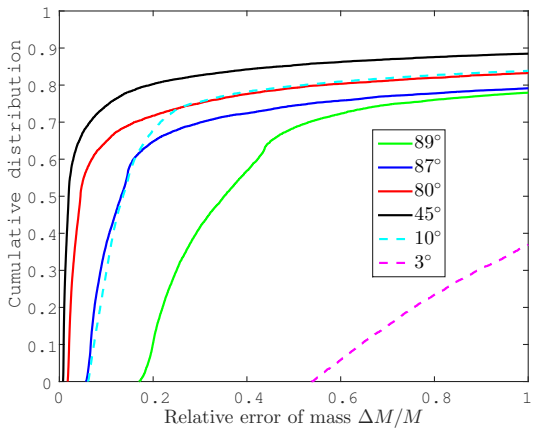

Figure 1. The cumulative distribution functions for relative measurement errors in NS mass $\Delta M_{\rm NS}/M_{\rm NS}$. This figure is obtained by Monte Carlo simulations where the NS sky position, polarization angle, reference GW phase, and inclination angles are randomly chosen from uniform distributions. The distance of $r = 1$ kpc, spin frequency of $f = 500$Hz, and three year observation by the ET are assumed. The solid lines correspond to NSs with the misalignment angles of $\theta = 45°$ (black line), $80°$ (red line), $87°$ (blue line), $89°$ (green line), while the dashed lines correspond to $\theta = 10°$ (cyan line) and $3°$ (magenta line), respectively. While a NS with a pinned superfluid core may have any values of θ, a freely precessing NS may be limited by $\theta < 10$ degrees. Mass measurement precision for a freely precessing NS may be worse than that for a NS with a pinned superfluid core.

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