

# Magnetic control of waveguide modes of Bragg structures

D A Sylgacheva<sup>1,2</sup>, N E Khokhlov<sup>1,2</sup>, A N Kalish<sup>1,2</sup> and V I Belotelov<sup>1,2</sup>

<sup>1</sup>Lomonosov Moscow State University, Moscow, 119991, Russia

<sup>2</sup>Russian Quantum Center, Skolkovo, Moscow Region, 143025, Russia

E-mail: sylgacheva.darjja@physics.msu.ru

**Abstract.** We present the study of the waveguide modes of one-dimensional magnetic photonic crystals with in-plane-magnetized layers. There is a magneto-optical effect of non-reciprocity for the TM-modes propagating along the layers perpendicularly to the magnetization. Due to the non-reciprocity the phase velocity of the modes changes with magnetization reversal. Comparison of the effect in the non-magnetic photonic crystal with additional magnetic layer on top and a magnetophotonic crystal with altering magnetic layers shows that the effect is greater in the first case due to the higher asymmetry of the claddings of the magnetic layer. This effect is important for the light modulation with external magnetic field in waveguide structures and may be used for design of novel types of the magneto-optical devices, sensors of magnetic field or biosensors.

## 1. Introduction

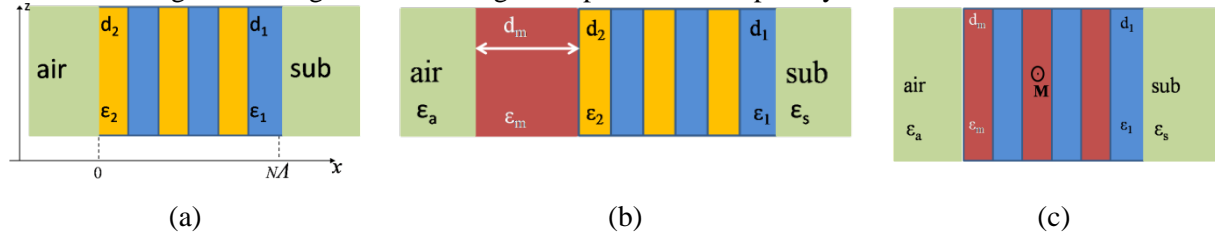
Modern requirements for optical information rates stimulate researchers to discover new methods of light control on submicron scales. Multilayer planar waveguides open new perspectives in this area. Systems of planar waveguides are used in fiber-optic communication WDM system, for example, and other practical applications [1-7]. The combinations of planar waveguides and Bragg structures or 1D photonic crystals (PCs) allow one to obtain the desired transmission spectrum or to specify the necessary eigenmodes dispersion characteristics of the whole structure [5-6]. Also, such structures broaden the applications of waveguides, including optical switchers, waveguide Mach-Zehnder interferometer, etc. [7].

Incorporation of the magnetic media layers into PC represents special interest. In this case, the magnetization of layers modifies the light's polarization and/or phase velocity [8-13] and the resonant enhancement of magneto-optical effects may be observed [15-18]. The authors of Refs. [19-20] investigated the localized states, the so-called optical Tamm states, at the interface between PCs containing magnetic layers and magneto-optical effect enhancement at normal light incidence. However, the case of the light waveguided along the PC's layers is also of great interest. The dispersion of the guided modes in single transversely-magnetized single film depends linearly on the magnetization [21-25]. A similar effect in transversely-magnetized PC's layers is expected. But there may be some special features since the PC's modes are coupled to each other. The topic has not been studied yet to the best of our knowledge.

Here we theoretically investigate the influence of transverse magnetic field on the waveguide TM-modes of a PC containing magnetic layers. Two types of PCs with an additional top layer are considered (figure 1). The structure of type I includes the magnetic layer on top only (figure 1(b)), while the structure of type II is represented as a PC with magnetized layers within its period (figure



1(c)). Here we extend the theory for waveguide modes' dispersion in the case of transverse magnetization of layered periodic structures. The solution of the dispersion equation shows that the transverse magnetization gives rise to magneto-optical non-reciprocity.



**Figure 1.** Schemes of 1D photonic crystals: (a) non-magnetic PC sandwiched between semi-infinite air and semi-infinite substrate (“sub”); (b) non-magnetic PC with magnetic layer on top – investigated structure of type I; (c) magnetic PC with altering magnetic/non-magnetic layers – investigated structure of type II;  $\epsilon_1$ ,  $\epsilon_2$ ,  $\epsilon_m$  are permittivities of layers with thickness  $d_1$ ,  $d_2$ ,  $d_m$  respectively;  $d_1$ ,  $d_2$  are quarter wavelength thicknesses, and so is  $d_m$  for type II. Magnetic field  $\mathbf{B}$  is along y-axis.

## 2. Method of dispersion calculation

The properties of the waveguide modes in the layers of non-magnetic 1D PC were theoretically studied in [26-32]. According to the developed approach the dispersion law for PC's waveguide modes can be found by constructing a scattering matrix equation and finding the condition for its non-trivial solutions.

We briefly outline the main points of this approach below. Let's consider a PC structure sandwiched between various semi-infinite media (figure 1(a)). The waves propagating forward and backward in the air (with complex amplitudes  $f_l$  and  $b_l$ ) and in the substrate ( $f_r$  and  $b_r$ ) are related to each other through the linear equation with transfer matrix  $\hat{T}$ :

$$\begin{pmatrix} f_r \\ b_r \end{pmatrix} = \hat{T} \begin{pmatrix} f_l \\ b_l \end{pmatrix}. \quad (1)$$

The expression for amplitudes of the waves outgoing from the PC through waves incident on the PC has the form with the scattering matrix  $\hat{S}$ :

$$\begin{pmatrix} f_r \\ b_l \end{pmatrix} = \hat{S} \begin{pmatrix} f_l \\ b_r \end{pmatrix}. \quad (2)$$

The elements of  $\hat{S}$  can be expressed through the elements of  $\hat{T}$ ,  $t_{ij}$ , as follows:

$$\hat{S} = \begin{pmatrix} 1/t_{11} & t_{21}/t_{11} \\ -t_{12}/t_{11} & (t_{11}t_{22} - t_{12}t_{21})/t_{11} \end{pmatrix}^{-1}. \quad (3)$$

The  $\hat{S}$ -matrix is a function of frequency and quasiwavenumber.

To find eigenmodes we assume that waves incident on the PC are absent, i.e.  $f_l = b_r = 0$ . According to equation (2), the dispersion equation is obtained by equating to zero the determinant of the inverse scattering matrix:

$$\det(\hat{S}^{-1}) = 0. \quad (4)$$

According to equation (3),  $\det(\hat{S}^{-1}) = t_{22}/t_{11}$ . Consequently,

$$t_{22} = 0. \quad (5)$$

To take into account the effect of the magnetization of the layers we represent the medium dielectric tensor in a coordinate system where the  $x$ -axis is orthogonal to the layers and modes propagate along the  $z$ -axis:

$$\hat{\varepsilon}_M = \begin{pmatrix} \varepsilon_x & ig_z & -ig_y \\ -ig_z & \varepsilon_y & ig_x \\ ig_y & -ig_x & \varepsilon_z \end{pmatrix}, \quad (6)$$

where  $\mathbf{g}$  is the gyration vector, linear in the magnetization  $\mathbf{M}$  and co-directional with it. Here we assume the mediums are isotropic,  $\varepsilon_x = \varepsilon_y = \varepsilon_z = \varepsilon$ , and  $\mathbf{M}$  is in the transverse configuration, i.e.  $g_x = g_z = 0$ ,  $g_y = g$ .

The PC is formed by two types of layers, “1” and “2”, so:

$$\varepsilon(x) = \begin{cases} \varepsilon_2, & 0 < x < b \\ \varepsilon_1, & b < x < \Lambda \end{cases},$$

$$g(x) = \begin{cases} g_2, & 0 < x < b \\ g_1, & b < x < \Lambda \end{cases}.$$

where  $\Lambda$  is the period of the structure;  $\varepsilon(x) = \varepsilon(x + \Lambda)$  and  $g(x) = g(x + \Lambda)$ . The complex amplitude of the magnetic field inside each homogeneous layer  $\alpha$  ( $\alpha=1,2$ ) of  $n$ -th PC unit cell can be represented as the sum of forward and backward plane waves:

$$H_y(x, z) = [a_n^{(\alpha)} e^{ik_{\alpha x}(x-n\Lambda)} + b_n^{(\alpha)} e^{-ik_{\alpha x}(x-n\Lambda)}] e^{i\beta z},$$

where  $k_{\alpha x} = \sqrt{k_0^2 \varepsilon_{\alpha} - \beta^2}$ ,  $\beta$  is the mode wavenumber, and  $k_0$  is the vacuum wavenumber.

The continuity conditions for components  $E_z$  and  $H_y$  for  $x=(n-1)\Lambda$  and  $x=(n-1)\Lambda+b$  leads to the matrix equation:

$$\begin{pmatrix} a_{n-1}^{(1)} \\ b_{n-1}^{(1)} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} a_n^{(1)} \\ b_n^{(1)} \end{pmatrix}. \quad (7)$$

In this equation, the elements within the linear approximation in  $\mathbf{g}$  are represented as follows:

$$A = e^{-ik_{1x}a} \left( \cos(k_{2x}b) - i \frac{1}{2} \sin(k_{2x}b) \left[ \frac{k_{1x}\varepsilon_2}{k_{2x}\varepsilon_1} + \frac{k_{2x}\varepsilon_1}{k_{1x}\varepsilon_2} \right] \right),$$

$$B = e^{ik_{1x}a} \left( i \frac{1}{2} \sin(k_{2x}b) \left[ \frac{k_{1x}\varepsilon_2}{k_{2x}\varepsilon_1} - \frac{k_{2x}\varepsilon_1}{k_{1x}\varepsilon_2} + \frac{2i\beta\varepsilon_2}{k_{2x}} \left( \frac{g_2}{\varepsilon_2^2} - \frac{g_1}{\varepsilon_1^2} \right) \right] \right), \quad (8)$$

$$C = e^{-ik_{1x}a} \left( -i \frac{1}{2} \sin(k_{2x}b) \left[ \frac{k_{1x}\varepsilon_2}{k_{2x}\varepsilon_1} - \frac{k_{2x}\varepsilon_1}{k_{1x}\varepsilon_2} - \frac{2i\beta\varepsilon_2}{k_{2x}} \left( \frac{g_2}{\varepsilon_2^2} - \frac{g_1}{\varepsilon_1^2} \right) \right] \right),$$

$$D = e^{ik_{1x}a} \left( \cos(k_{2x}b) + i \frac{1}{2} \sin(k_{2x}b) \left[ \frac{k_{1x}\varepsilon_2}{k_{2x}\varepsilon_1} + \frac{k_{2x}\varepsilon_1}{k_{1x}\varepsilon_2} \right] \right),$$

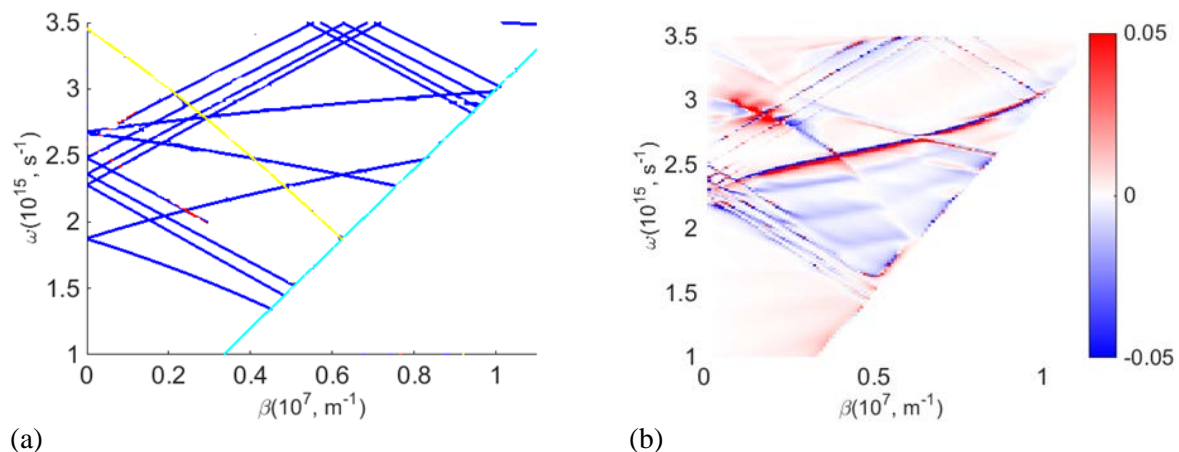
where  $a = \Lambda - b$ . Equation (6) shows the transfer matrix for one PC unit cell. So, the transfer matrix for  $N$  unit cells can be found as:

$$\hat{T} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}^N \quad (9)$$

The number  $N$  of periods of the PC affects the number of guided modes [26-28]. Equations (5), (8) and (9) provide the transcendental dispersion equation which with permittivity tensor (6) gives the opportunity to find magneto-optical features the different PC modes.

The approach is easily generalized to the cases of structures of types I and II, by applying the standard T-matrix formalism for construction of the total transfer matrix for the whole structure.

The modes can be excited by means of a diffraction grating deposited on the top of the structure. At this the dispersion curves can be translated to Brillouin zones associated with the diffraction grating, so the modes that appear inside the light cone are excited directly by the incident light. They form the resonant features of the optical response of the structure, and of the magneto-optical response as well. Figure 2 shows the calculated dispersion diagram for the structure covered with metallic grating and the transverse magneto-optical Kerr effect (TMOKE). The latter is a magnetization-induced change in reflectance. The TMOKE was calculated numerically by the Rigorous Coupled Wave Analysis (RCWA) [33]. It exhibits resonant features associated with the excitation of modes. Comparison of figures 2(a) and 2(b) reveals good agreement between calculations by equation (5) and numerical calculations by RCWA.



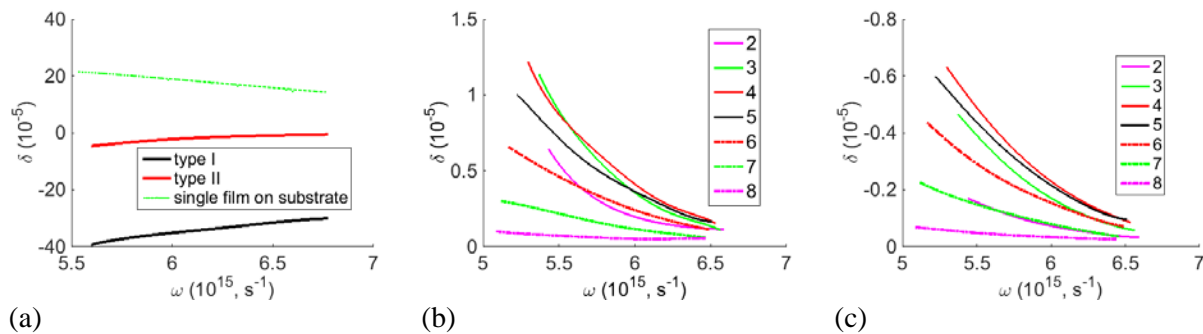
**Figure 2.** (a) Dispersion curves obtained from equation (5) (only the area inside the light cone is shown); (b) the value of the transverse Kerr effect in reflection calculated numerically by RCWA. Structure of type I covered with thin metallic grating is considered.

### 3. The magneto-optical properties of the waveguide modes

The solutions of (5) for structures of types I and II (figure 1 (b,c)) give non-zero effect of the magneto-optical non-reciprocity. It arises due to non-equal wave's phase velocities for opposite magnetization directions and can be characterized by the value:

$$\delta = \frac{\beta(+g) - \beta(-g)}{\beta(+g) + \beta(-g)} \quad (10)$$

The non-zero value  $\delta$  means the non-zero magneto-optical effects for the mode. And the problem for optimal structure configuration for higher  $\delta$  arises. We have calculated  $\delta$  for PCs of types I and II with number of periods  $N=7$  (figure 3).



**Figure 3.** (a) Plots of magneto-optical non-reciprocity value  $\delta$  for waveguide mode “1”, localized mainly inside the top layer for structures of type I (bottom line), type II (middle line) and single magnetic film on substrate (top line); (b, c) the same value  $\delta$  for structures of type I and type II respectively for other modes from the same mode family.

The magnitude of the effect for structure of type I is greater than for the single film (figure 3(a)) due to the greater wave localization inside the magnetic top layer. A more interesting fact is the greater effect in structure of type I than in type II for other modes localized inside the PC's non-top layers. It seems unexpected due to PC of type I is non-magnetic in opposite to magnetic PC of type II. The feature originates due to higher asymmetry of cladding of magnetic in type I rather than in type II. Detailed consideration of transverse magneto-optical Kerr effect and the role of asymmetry of claddings of single magnetic film is presented in [34]. Experimental studies of the transverse magneto-optical Kerr effect for waveguide modes of PC's with a microresonator layer have been performed recently and the resonant effect enhancement was demonstrated for the modes excitations [35]. Another consequence of the asymmetry is the difference in the signs of  $\delta$  in the magneto-optical modes "2-8" in the structures of the type I and II (Figure 3 (b,c)). The situation is similar to that considered in [23].

The magnitude of  $\delta$  decreases at higher frequencies. This can be explained by the increase in the density of modes in this spectral region. The interaction between the modes decreases with the frequency growth. It leads to the convergence of the eigen-frequencies of waveguide modes. Therefore, the configuration of the mode field distribution becomes closer to the configuration of the field of the single symmetric waveguide, for which the value of  $\delta$  is low.

The observed magneto-optical non-reciprocity effect for modes can be revealed in the far-field via the resonant TMOKE that is observed when a structure is covered with diffraction grating, as discussed above (see figure 2). It is the nonreciprocity effect that is the nature of the resonant TMOKE in grating structures [23]. The influence of the mode features on the far-field magneto-optical response provides the opportunity of applying the considered features, i.e., for purposes of magnetic field sensing. The modes are influenced differently by the magnetization of different layers, so the volume sensing of magnetic field becomes possible.

#### 4. Conclusions

We extended the theory for the dispersion of waveguide modes in layered dielectric structures with transversely-magnetized layers. It is shown that the dispersion equation contains additional terms proportional to the material gyration. The solutions show the non-zero magneto-optical non-reciprocity. The effect is 2-2.5 times higher for the non-magnetic PC with an additional magnetic top layer as compared to PC with magnetic layers. The effect on the guided mode localized inside the top magnetic layer of a non-magnetic PC is two times greater than for a single magnetic film. Thus, the magneto-optical properties of waveguide modes are largely determined by the claddings, their arrangement and configuration. The considered effect of non-reciprocity is observed in experiment as resonances of transverse magneto-optical Kerr effect. The considered magneto-optical effect's features may be used for design of different magneto-optical devices, sensors of magnetic field or biosensors.

## Acknowledgments

The work is supported by the Russian Science Foundation (project No. 14-32-00010).

## References

- [1] Freeman R 2002 *Fiber Optic Systems for Telecommunications* (Hoboken, New Jersey: Wiley)
- [2] Unger H G 1977 *Planar optical waveguides and fibers* (Oxford: Clarendon Press.)
- [3] Sklyarov O K 2001 *Fiber optic networks and communication systems* (Moscow: SOLON-Press) (Series Library Engineer)
- [4] Agrawal G P 2012 *Fiber-optic communication systems* (Hoboken, New Jersey: Wiley)
- [5] Salib M *et al* 2004 *Intel Technology J.* **8** 143
- [6] Zheltikov A M 2004 *Phys. Usp.* **47** 1205
- [7] Tanaka Y *et al* 2005 *Phys. Lett.* **86** 081108
- [8] Berzhansky V N *et al* 2012 *J. Magn. Soc. Jpn.* **36** 42
- [9] Inoue M *et al* 2006 *J. Mater. Chem.* **16** 678
- [10] Inoue M *et al* 2006 *J. Phys. D* **39** R151
- [11] Fedyanin A A *et al* 2003 *J. Magn. Magn. Mat.* **96** 258
- [12] Fedyanin A A *et al* 2004 *J. Magn. Magn. Mat.* **282** 256
- [13] Kahl S and Grishin A M 2004 *Appl. Phys. Lett.* **84** 1438
- [14] Chakravarty A *et al* 2011 *Phys. Rev. B* **84** 094202
- [15] Inoue M *et al* 2008 *Iieice Trans. Electron.* **e91–c** 1630
- [16] Gaiyan B *et al* 2012 *Optical Materials* **35** 252
- [17] Vasiliev M *et al* 2008 *IEEE Trans. Magn.* **44** 323
- [18] Zvezdin A K and Belotelov V I 2004 *Eur. Phys. J. B* **37** 479
- [19] Goto T *et al* 2009 *Phys. Rev. B* **79** 125103
- [20] Vinogradov A P *et al* 2010 *Phys. Usp.* **53** 243
- [21] Kalish A N *et al* 2014 *Laser Phys.* **24** 094006
- [22] Prokhorov A M *et al* 1984 *Sov. Phys. Usp.* **27** 339
- [23] Belotelov V I *et al* 2010 *J. Exp. Theor. Phys.* **137** 932
- [24] Kreilkamp L E *et al* 2013 *Phys. Rev. X* **3** 041019
- [25] Ferreira-Vila E *et al* 2013 *Opt. Express* **21** 4917
- [26] Yeh P and Yariv A 1977 *J. Opt. Soc. Amer.* **67** 423
- [27] Yariv A and Yeh P 1984 *Optical Waves in Crystals: Propagation and Control of Laser Radiation* (New York: Wiley)
- [28] Yeh P and Yariv A 1976 *Optics Communications* **19** 427
- [29] Inoue M *et al* 2006 *J. Mater. Chem.* **16** 678
- [30] Johnson S G *et al* 2001 *Opt. Express* **9** 748
- [31] Xu Y *et al* 2003 *Opt. Express* **11** 1039
- [32] Mizrahi A and Schachter L 2004 *Opt. Express* **12** 3156
- [33] Li L 2003 *J. Opt. A: Pure Appl. Opt.* **5** 345
- [34] Zvezdin A K and Kotov V A 1997 *Modern Magneto optic and Magneto optical Materials* (Philadelphia: IOP)
- [35] Khokhlov N E *et al* 2015 *J. Phys. D: Appl. Phys.* **48** 095001