A new statistical tool to study the geometry of intense vorticity clusters in turbulence

Alberto Vela-Martin[1], Takashi Ishihara[2]

[1]Universidad Politécnica de Madrid, 28040 Madrid, Spain

[2]JST CREST, Graduate School of Engineering, Nagoya University, Nagoya 464-8603, Japan

E-mail: alberto@torroja.dmt.upm.es

Abstract. Recent large-scale direct numerical simulations (DNS) of high-Reynolds number (high-Re) turbulence, suggest that strong micro-scale tube-like vortices form clusters in localized thin regions of space. However, to this date no thorough quantitative and statistical analysis of the geometry of such vortical clusters has been conducted. This study is intended to generate new statistical tools to study the shape and dynamics of these intense vorticity and strain regions. We first propose a new method for locating and analysing the geometrical properties of thresholded vortical clusters contained inside boxes of a given size. Second, we use this new tool to investigate the natural presence of intense shear layers and their relevance as geometrical features of high-Re homogeneous turbulence. This new method is applied to the DNS of homogeneous incompressible turbulence with up to 4096^3 grid points, showing that the geometry of high vorticity regions varies strongly depending on the threshold and on the size of the clusters. In particular for sizes in the inertial range of scales and high thresholds, approximately layer-like structures of vortices are extracted and visualized. Agreement of results with previous observations and known features of turbulence supports the validity of the proposed method to characterize the geometry of intense vorticity and strain regions in high-Re turbulence.

1. Introduction

Vorticity and strain structures are known not to be randomly distributed in turbulence. Rather, they tend to be clustered in localized regions in space, where they are intense [1], and where the local dynamics are believed to be strongly influenced by strong velocity gradients. It has been suggested for some time [2] that these intense clusters tend to be located between large emptier regions in which velocity gradients are relatively weak, and large energy-containing eddies are dominant.

Recent theories [3] point at the possibility that these large empty regions play an important role in the dynamics of intense clusters, as they entail the formation of stable shear layers where intense vorticity is clustered and sheltered in thin layer-like regions of space. As the Taylor micro-scale Reynolds number (Re_λ) is increased, these layers become more relevant due to the inhibition of the Kelvin–Helmholtz mechanism responsible for their unstable behaviour [3]. When Re_λ becomes large, this dynamical effect would imply the prevalence of layers as the relevant geometry for vortex clusters. Visual detection and analysis of individual layers in high-Re_λ DNS, with typical thickness of the order of the Taylor micro-scale, stands as evidence to support this theory [5]. However, to the knowledge of the authors, no extensive statistical research has been conducted to properly identify and characterize these layers, and it is unclear how relevant they are in high-Re_λ turbulence.

This work explores new statistical tools to study the geometrical distribution of intense vorticity and strain regions. We propose a new method for locating and analysing these clusters and their geometry through an extensive and quantitative statistical approach. Using this new tool, we expect to elucidate whether intense shear layers are naturally present in isotropic turbulence, and to what extent they represent a relevant geometric feature of high-Reynolds number turbulence.

2. Method

The method is intended as a reliable way to qualitatively and quantitatively describe the clustering geometry of intense vorticity regions. It is based on randomly probing the vorticity field to find the core of high density regions through an iterative method. The structure of the localized intense clusters is analysed using the inertia tensor of the set of thesholded points that form the cluster. Similar methods have been used to describe the structure of galaxy clusters [4].

The enstrophy field $|\omega|(\vec{x})$ is thresholded, and a field $P(\vec{x})$ is obtained such that,

$$P(\vec{x}) = 1 \text{ if } |\omega|(\vec{x}) \geq \alpha\omega',$$

$$P(\vec{x}) = 0 \text{ if } |\omega|(\vec{x}) < \alpha\omega',$$

where ω' is defined as the root-mean-square of the vorticity $\omega'^2 = \epsilon/\nu$, ϵ is the mean energy dissipation rate per unit mass and ν is the kinematic viscosity. The thresholded field is randomly covered with cubic boxes of size δ^3. Each box, defining a domain B, is centred at $\vec{x}'$, and its centre of gravity $\vec{x}_{cg}$ is computed as

$$\vec{x}_{gc} = \frac{\sum_{\vec{x}\in B} \vec{x}P(\vec{x})}{\sum_{\vec{x}\in B} P(\vec{x})}, \tag{1}$$

where the sum extends to all points within the box. If $\vec{x}_{gc}$ is equal to the box centre $\vec{x}'$, the point is marked as the centre of a local cluster. Otherwise, the centre of the box is shifted to the centre of gravity of the thresholded field, $\vec{x}_{gc} \to \vec{x}'$, and the process is repeated to convergence.

After all the boxes converge, a set of local clusters is obtained. It is known that the thresholded enstrophy field contains large empty regions, and this algorithm occasionally detects some clusters that consist of a few isolated points within those empty regions. To avoid very low-density clusters, all boxes with a mean volume fraction $V_B = \sum_{\vec{x}\in B} P(\vec{x})/\delta^3$ lower than the average volume fraction of P over the whole field are discarded. Moreover, since this method might identify some clusters more than once, repeated clusters are also discarded. The number of boxes used in our analysis is such that the total sum of their volume is eight times larger than the volume of the domain. Around 20% of the trials result in repeated clusters, suggesting that this number of boxes is large enough to properly sample the flow. The parameter δ is important, because it implicitly defines the maximum size of the structures contained in the box. It is through this parameter that we control the size of the clusters that we detect.

For each individual box B, the geometry of the cluster can be studied through the inertia tensor of the points within it,

$$\begin{aligned} I_{xx} &= \frac{\sum_{\vec{x}\in B}\{(y-y')^2+(z-z')^2\}P(\vec{x})}{\sum_{\vec{x}\in B} P(\vec{x})} \\ I_{yy} &= \frac{\sum_{\vec{x}\in B}\{(z-z')^2+(x-x')^2\}P(\vec{x})}{\sum_{\vec{x}\in B} P(\vec{x})} \end{aligned}$$

$$
\begin{aligned}
I_{zz} &= \frac{\sum_{\vec{x}\in B}\{(x-x')^2+(y-y')^2\}P(\vec{x})}{\sum_{\vec{x}\in B}P(\vec{x})}\\
I_{xy} &= \frac{\sum_{\vec{x}\in B}(x-x')(y-y')P(\vec{x})}{\sum_{\vec{x}\in B}P(\vec{x})}\\
I_{yz} &= \frac{\sum_{\vec{x}\in B}(y-y')(z-z')P(\vec{x})}{\sum_{\vec{x}\in B}P(\vec{x})}\\
I_{zx} &= \frac{\sum_{\vec{x}\in B}(z-z')(x-x')P(\vec{x})}{\sum_{\vec{x}\in B}P(\vec{x})}
\end{aligned}
$$

where primes denote the coordinates of the centre of each box, which is also the centre of gravity of the distribution of P in the box. From the tensor, the three principal moments of inertial, $I_1 > I_2 > I_3 \geq 0$, can be obtained, as well as their respective inertia directions, $\vec{v}_1, \vec{v}_2, \vec{v}_3$.

These three quantities contain information on the shape and size of the cluster. The ratio $\rho = (I_1 + I_2 + I_3)^{1/2}/\delta$ describes the ratio between the radius of inertia of P and the size of the box, and indicates how well we have achieved our aim of isolating clusters of a given size. The two aspect ratios $\epsilon_1 = I_2/I_1$ and $\epsilon_2 = I_3/I_1$, such that $1 \geq \epsilon_1 \geq \epsilon_2$, define the geometry of the structures. There are three ideal cases: $\epsilon_1 = \epsilon_2 = 1$ represents a perfect sphere; $\epsilon_1 = \epsilon_2 = 0.5$ represents a perfectly thin circular pancake or layer; and $\epsilon_1 = 1$ and $\epsilon_2 = 0$ represents an infinitely thin tube.

3. Dataset

The vorticity fields used in the analysis are obtained from integrating the incompressible Navier–Stokes equations in a triply periodic domain. The dataset contains flow fields from three simulations of forced homogeneous isotropic turbulence at different Re_λ. Further details about the simulations can be found in [6, 7]. For the lower-Reynolds number cases, enough time-decorrelated fields have been used to obtain a proper statistical representation of the flow. For the largest case, only one field is available but, given its high Reynolds number, the small scales are sufficiently represented to have converged statistics. Characteristic parameters of the simulations are presented in Table 1.

4. Results

When the algorithm is applied to the database mentioned above, it successfully detects a considerable number of unique dense clusters from which the geometric parameters ρ, ϵ_1 and ϵ_2 are extracted. Between 10^3 and 10^7 different acceptable clusters are identified, enough for properly converged statistics.

Table 1. Characteristic parameters of the DNS data. N is the number of Fourier modes per dimension in each simulation, k_{max} the largest resolved wavenumber, η the Kolmogorov viscous scale, and λ the Taylor micro-scale. (N_{fields}) is the number of independent fields used in the statistics. The details of the DNS are described in Ref. [6] for $N = 512, 1024$, and [7] for $N = 4096$.

N	Re_λ	$k_{max}\eta$	L/η	λ/η	N_{fields}
512	284	1.5	193	41.2	20
1024	384	1.5	386	43.9	10
4096	1131	1	2137	66.4	1

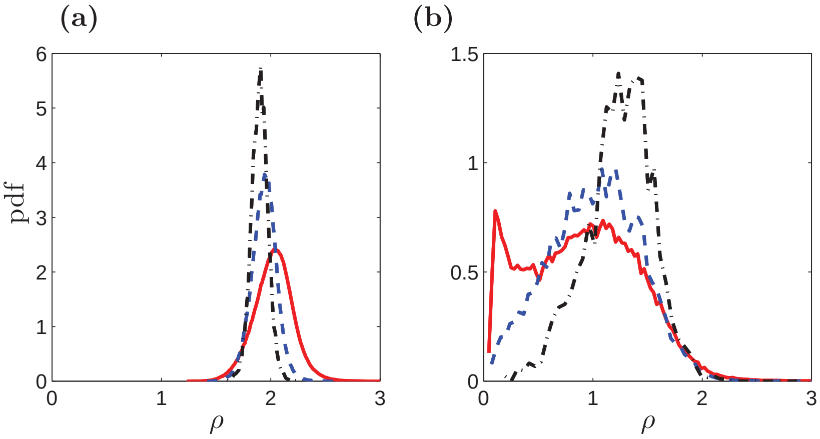

Figure 1. Probability density functions of the size ratio ρ. ——, $\delta = 19\eta$; – – –, $\delta = 70\eta$; –·–·–, $\delta = 136\eta$. (a) $\alpha = 1$. (b) $\alpha = 4$.

Figure 1 shows probability density functions (pdf) of the size ratio ρ. In general, they centre about unity, showing that the boxes identify clusters of roughly the intended size, but it is also clear that the higher thresholds in figure 1(b) contain a sub-population of clusters that are much smaller than the box size.

Figure 2 presents two-dimensional joint pdfs of the aspect ratios (ϵ_1, ϵ_2). Only points inside the triangles drawn in each figure are possible. 'Spheres' are at the top-right corner, 'pancakes' are at the bottom centre, and 'tubes' are at the top left. The geometry of the clusters changes considerably with the threshold and with the clustering size. For low thresholds and/or large box sizes, the clusters are spherical, with $\epsilon_1 \approx \epsilon_2 \approx 1$. This reason is different in each case. If the size δ of the box is much larger than the size of a typical cluster, only the geometry of the far field of intense vorticity is captured, which is distributed roughly homogeneously with respect to the centre of the box. This can be appreciated in the tendency of pdfs to move towards the top-right corner as δ increases. On the other hand, the results for low thresholds represent the geometry of the weak background enstrophy, which is less intermittent and more evenly distributed in space than the intense structures.

The pdfs for small boxes and high α fall near $\epsilon_1 \sim 1$ and $\epsilon_2 \sim 0$, where thin elongated shapes are represented. This geometry is typical of single isolated strong vortical structures, which are known to be representative features of the intense vorticity field in turbulence [2]. That these two well-known features of the flow are identified by our algorithm gives us some assurance about the general soundness of the method.

In between the two limits, larger clusters with increasing α move from isotropic shapes towards intermediate aspect ratios. In this transition, the predominant cluster type moves towards a flatter geometry, which could be interpreted as being closer to layers. Figure 2 shows that this intermediate structures are more common for sizes in the inertial range of scales.

To reinforce the geometrical intuition of the shapes implied by the previous results, it is useful to relate the principal moments of inertia to those of an ellipsoid with semi-axes $(a > b > c)$. As before, pair-wise aspect ratios can be defined as $\tilde{b} = b/a$ and $\tilde{c} = c/a$, which are univocally related to (ϵ_1, ϵ_2),

$$\tilde{b} = \left(\frac{1 + \epsilon_2 - \epsilon_1}{1 + \epsilon_1 - \epsilon_2} \right)^{1/2}, \quad \tilde{c} = \left(\frac{\epsilon_1 + \epsilon_2 - 1}{1 + \epsilon_1 - \epsilon_2} \right)^{1/2}. \tag{2}$$

For the typical values found in large boxes and high α ($\epsilon_1 \approx 0.8$ and $\epsilon_2 \approx 0.5$), the aspect ratios of the equivalent ellipsoid are $\tilde{b} \approx 0.7$ and $\tilde{c} \approx 0.5$ which, although not strictly a thin layer, at least suggests a tendency of the vorticity clusters towards this type of geometry.

A typical vorticity cluster with this 'layer-like' geometry is shown in figure 3. It is formed by tightly packed parallel vortices, and has $(\epsilon_1, \epsilon_2) = (0.79, 0.48)$ and $(\tilde{b}, \tilde{c}) = (0.73, 0.45)$. The three different views of this structure in figure 3 show that it is relatively wide in the direction normal to $\vec{v}_1$, and confined to a thinner layer in the other two directions. The joint pdfs in figure 2 show that this type of geometry is relatively common for intermediate values of δ and α at the three Re_λ considered. Similar structures have been reported in [5].

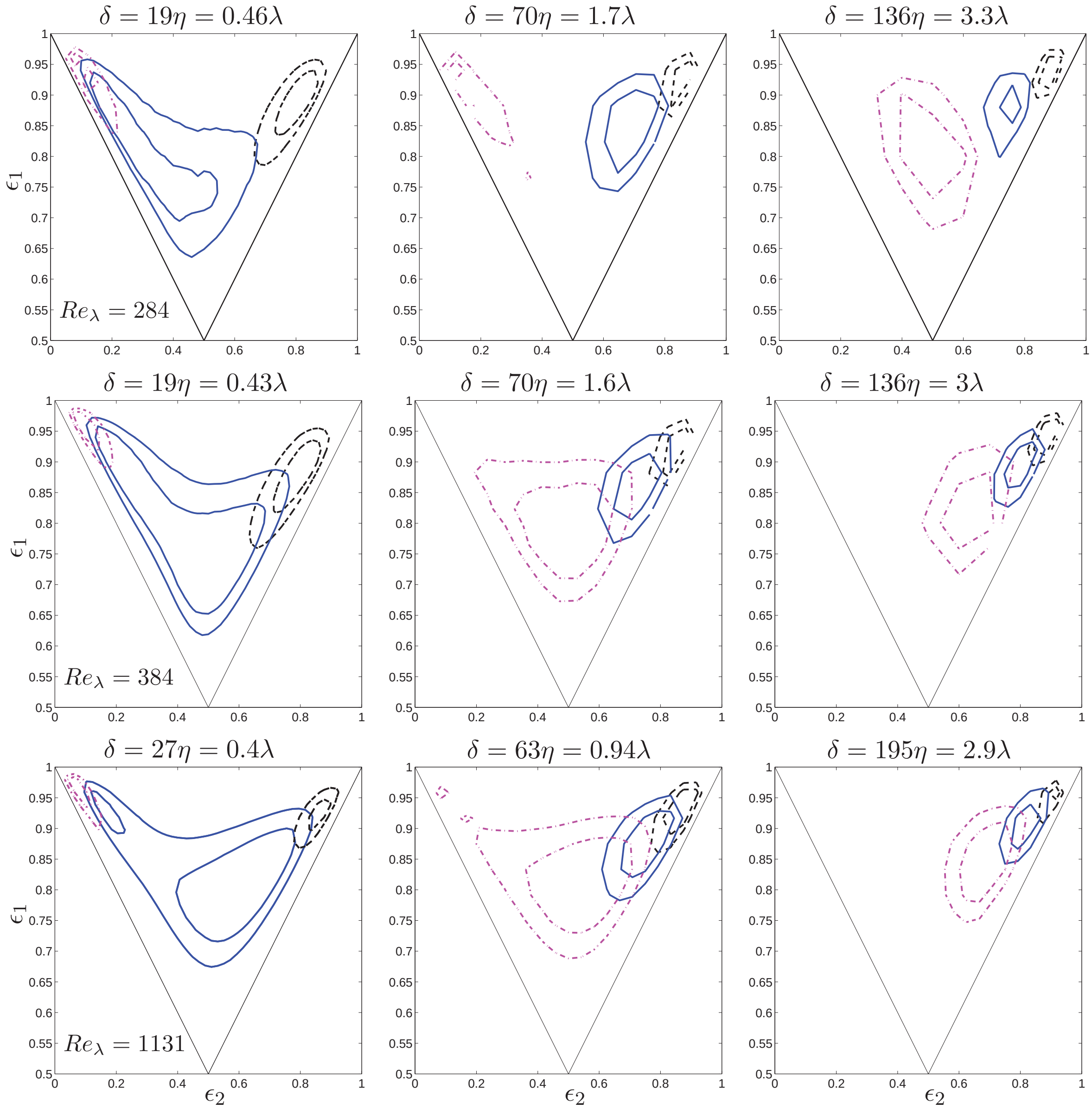

Figure 2. Isocountours at 0.6 and 0.3 of the maximum for the joint pdf of ϵ_1 and ϵ_2. The size of the clustering box is above each figure, in Taylor and Kolmogorov micro-scales. From top to bottom: $Re_\lambda = 284$, $Re_\lambda = 384$ and $Re_\lambda = 1131$. – – –, $\alpha = 1$; ———, $\alpha = 2$; –·–·–, $\alpha = 4$

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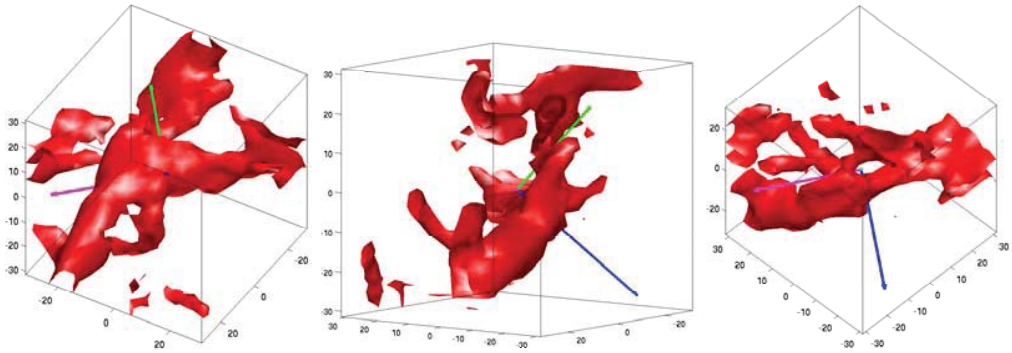

Figure 3. Typical intense vorticity structure at $Re_\lambda = 1131$, box size $\delta = 63\eta$ and threshold $\alpha = 4$. From left to right: view in the direction of the three main eigenvectors, blue $\vec{v}_1$, magenta $\vec{v}_2$ and green $\vec{v}_3$. For this structure: $\epsilon_1 = 0.79$ and $\epsilon_2 = 0.48$. Lengths in Kolmogorov micro-scale units.

5. Conclusions

We have presented a new method for the analysis of the geometry of intense vorticity. Dense clusters of size δ are located via an iterative method applied on thresholded enstrophy fields.

Clusters are analysed by means of the inertia tensor of the thresholded set of high-intensity points, and their geometry is studied considering the three principal inertia moments. An extensive application of this algorithm has been conducted for three different Reynolds numbers, using about 10^7 clusters extracted from our database of isotropic turbulence.

The method is shown to be capable of detecting both locally strong tubular vortices for small clusters, and isotropically scattered vorticity for large ones, validating its ability to locate and characterize the geometry of known vorticity structures. For intermediate sizes within the inertial range of scales, a tendency of these clusters to form approximately layer-like objects is observed. Some of these objects have been extracted from the flow, showing what could be identified as potential shear layers. These results are in accordance with observations by [5], and suggest that thick layer-like vorticity structures might be naturally present in isotropic turbulence.

This analysis is related to the one in ref. [1], and produces similar results in that higher thresholds give rise to vorticity structures that move from sheets to tubes. The two analyses differ in that there was no explicit size control in [1], and that their clusters were geometrically connected, while the present ones are, in principle, general sets of points.

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