

# Study of the $Z_b(10610)$ and $Z_b(10650)$ states through $B\bar{B}^*$ and $B^*\bar{B}^*$ interactions using local hidden gauge approach

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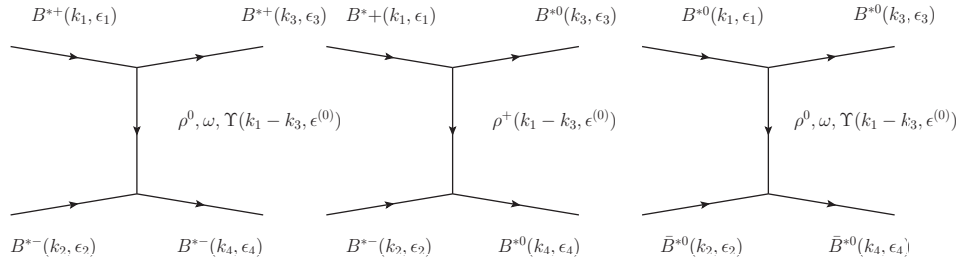
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**Abstract.** We have studied the  $B\bar{B}^*$  and  $B^*\bar{B}^*$  interactions for isospin  $I = 1$  using the Local Hidden gauge approach. Since both interactions via one light meson exchange are not allowed by Okubo-Zweig-Iizuka (OZI) rule, we investigated the contributions for those interactions coming from two pions, interacting and noninteracting among themselves, and also due to the heavy vector meson exchange, in which the OZI rule no longer holds. From the amplitudes calculated by these mechanism, we determine an effective potential which is used as a kernel of the Bethe-Salpeter equation. Our goal is look for poles in the T-matrix in attempt to relate them with the charged  $Z_b(10610)$  and  $Z_b(10650)$  states observed by Belle Collaboration. For the  $B\bar{B}^*$  interaction, we find a loosely bound state with the mass in the range 10587 – 10601 MeV, which is close to the experimental value observed by Belle Collaboration. For the  $B^*\bar{B}^*$  case, we find a cusp at 10650 MeV.

## 1. Introduction

Since the discovery of the  $X(3872)$  by Belle Collaboration [1], many others Collaborations like BaBar, CDF, D0 and BESIII are measuring a series of new structure in the Charmonium mass region. They are called  $XYZ$  Charmoniumlike states. If they were a simple Charmonium, they would decay into a pair of charm mesons, that is, mesons which are made of one quark (anti-quark) charm and a light quark. Instead, they decay into  $J/\psi$  plus pions which is an unusual feature for a simple Charmonium state. Furthermore, properties such as mass and decay modes predicted by the potential models do not fit with that observed by the experiment. All these striking behaviour led to many theoretical and experimental efforts to understand the quark content of the  $XYZ$ . Most of the theoretical work done so far attempts to accommodate them in a exotic picture. By exotic we mean a more complex quark structure beyond the known quark anti-quark (mesons) and three quarks (baryons) configuration like hybrid, meson molecule, glueballs and hadrocharmonium. It is a challenge to understand these new charmoniumlike states as exotic since using the models mentioned above it is relatively simple to reproduce the masses of those states. The same challenges also concern the bottomoniumlike states. Among them, the  $Z_b(10650)$  and  $Z_b(10610)$  are very interesting. They were observed by the





**Figure 1.** Vector exchange diagrams contributing to the process  $B^* \bar{B}^* \rightarrow B^* \bar{B}^*$ .

Belle collaboration in  $\pi^\pm h_b(nP)$  and  $\pi^\pm \Upsilon(mS)$ , with the  $n = 1, 2$  and  $m = 1, 2, 3$ , invariant mass distribution of the  $\Upsilon(5S)$  decay channel [2]. As a result of the measurements, Belle reported:  $M_{Z_b(10610)} = (10608.4 \pm 2.0)$  MeV,  $\Gamma_{Z_b(10610)} = (15.6 \pm 2.5)$  MeV and for  $Z_b(10650)$ ,  $M_{Z_b(10650)} = (10653.2 \pm 1.5)$  MeV and  $\Gamma_{Z_b(10650)} = (14.4 \pm 3.2)$  MeV. The quantum numbers are reported as  $J^P = 1^+$  and positive G parity. Therefore, in this work we use the Lagrangians from the Local Hidden Gauge approach in order to study the interactions  $B\bar{B}^*$  and  $B^*\bar{B}^*$ . Our aim is look into the possibility of creating states from those interactions for  $I = 1$  exchanging mesons. Since the exchange of a light mesons is not allowed by OZI rule, we should circumvent the OZI restriction and investigate the interactions by means of a heavy vector meson exchange and also due to the two pions, interacting and noninteracting among themselves, exchange. In view of this, we follow the procedure done by the authors of Refs. [6, 8] and perform an extension to the bottom sector in order to look on  $B\bar{B}^*$  and  $B^*\bar{B}^*$  interactions.

## 2. Formalism

### 2.1. The heavy vector meson exchange

In order to study the  $B\bar{B}^*$  and  $B^*\bar{B}^*$  interactions by means of heavy vector exchange, we need Lagrangians which can describe the  $VPP$  and  $VVV$  vertices. They are given by

$$\mathcal{L}_{VPP} = -ig \langle V^\mu [P, \partial_\mu P] \rangle, \quad (1)$$

$$\mathcal{L}_{VVV} = ig \langle (V^\mu \partial_\nu V_\mu - \partial_\nu V_\mu V^\mu) V^\nu \rangle. \quad (2)$$

The coupling  $g$  is given by  $g = M_V/2f_\pi$ , being  $f_\pi = 93$  MeV the pion decay constant, while  $M_V$  is the vector meson mass. In Eqs. (1) and (2), the symbol  $\langle \rangle$  stands for the trace of SU(4). The vector field  $V_\mu$  is represented by the SU(4) matrix, which is parametrised by 16 vector mesons including the 15-plet and singlet of SU(4) [9]. while  $P$  is a matrix containing the 15-plet of the pseudoscalar mesons written in the physical basis in which  $\eta, \eta'$  mixing is taken into account [3],

The channels we are interested in are those with  $B = 0$ ,  $S = 0$  and isospin  $I = 1$ . In the  $B^*\bar{B}^*$  case, they are  $B^*\bar{B}^*$  and  $\rho\Upsilon$ . In the case of  $B\bar{B}^*$  we are only interested in the positive G-parity combination, namely  $(B\bar{B}^* + cc)/\sqrt{2}$  and also  $\eta_b \rho$  and  $\pi \Upsilon$ .

**2.1.1. The  $B\bar{B}^*$  case** For this case, the most important diagrams are depicted in Fig. 1. In order to calculate the vertices of those diagrams we can use the Lagrangians defined earlier. Then, taking into account all particles involved in the exchange, we obtain for the  $I = 1$  combination the following expression

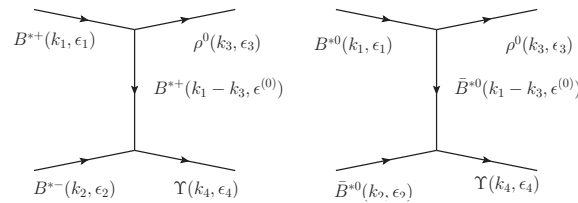
$$t_{B^*\bar{B}^* \rightarrow B^*\bar{B}^*}^{I=1, S=0, 1, 2} = -g^2 + g^2 \left[ \frac{2M_\rho^2 M_\omega^2 + M_\Upsilon^2 (-M_\omega^2 + M_\rho^2)}{4M_\Upsilon^2 M_\omega^2 M_\rho^2} \right] (4M_{B^*}^2 - 3s), \quad (3)$$

where  $s$  stands for the center of mass energy of the  $B^* \bar{B}^*$  system.

Consider now the other channel,  $B^* \bar{B}^* \rightarrow \rho \Upsilon$ . The relevant diagrams are depicted in Fig 2 [9]. The procedure to get the amplitude for this channel is analogous to what we have done earlier. Thus, the amplitude in isospin  $I = 1$  basis for the spin  $S = 0, 2$  states in s-wave, corresponding to all diagrams of Fig. 2 plus the contact term is given by

$$t_{B^* \bar{B}^* \rightarrow \rho \Upsilon}^{I=1, S=0,2} = -2g^2 + g^2 \left[ \frac{2M_{B^*}^2 + M_{\Upsilon}^2 + M_{\rho}^2 - 3s}{M_{B^*}^2} \right]. \quad (4)$$

The interaction in  $S = 1$  vanishes as a consequence of a cancellation of terms where the  $\rho^0$  and  $\Upsilon$  are interchanged in the diagrams. The diagonal  $\rho \Upsilon \rightarrow \rho \Upsilon$  transition is again OZI forbidden and null in this approach.



**Figure 2.** Vector exchange diagrams contributing for the  $B^* \bar{B}^* \rightarrow \rho \Upsilon$  channel.

Eqs. (3) and (4) will be used as a kernel of the Bethe-Salpeter equation as we shall discuss it later.

**2.1.2. The  $B^* \bar{B}^*$  case** In this case, the Lagrangians defined in Eqs. (1) and (2) can also be used to provide the vertices of the  $PV \rightarrow PV$  interaction through exchange of a heavy vector. In Refs. [5, 4] these amplitudes were already calculated in s-wave, where the authors were concerned with axial-vector resonances dynamically generated. Besides, in Ref. [6] the same equation for the amplitude is used for the  $D \bar{D}^*$  interaction. In our case, we extend these amplitudes for the  $B \bar{B}^*$  interaction in the isospin  $I = 1$  channel, with the result

$$V_{ij}(s) = -\frac{\vec{\epsilon} \vec{\epsilon}'}{8f_{\pi}^2} \mathcal{C}_{ij} \left[ 3s - (M^2 + m^2 + M'^2 + m'^2) - \frac{1}{s}(M^2 - m^2)(M'^2 - m'^2) \right], \quad (5)$$

where the masses  $M$  ( $M'$ ) and  $m$  ( $m'$ ) in Eq. (5) correspond to the initial (final) vector meson and pseudoscalar meson, respectively. The indices  $i$  and  $j$  represent the initial and final  $VP$  channels  $(B \bar{B}^* + cc)/\sqrt{2}$ ,  $\eta_b \rho$  and  $\pi \Upsilon$ .

The  $\mathcal{C}_{ij}$  are constants that can be displayed in a  $3 \times 3$  matrix, which for the positive  $G$ -parity of the  $B \bar{B}^*$  combination, is

$$\mathcal{C}_{ij} = \begin{pmatrix} -\psi & \sqrt{2}\gamma & \sqrt{2}\gamma \\ \sqrt{2}\gamma & 0 & 0 \\ \sqrt{2}\gamma & 0 & 0 \end{pmatrix}, \quad (6)$$

where  $\gamma = \left(\frac{m_L}{m_H}\right)^2$  and  $\psi = \left(\frac{m_L}{m_{H'}}\right)^2$ . Those factors are defined in this way in order to take into account the suppression due to the exchange of a heavy vector meson. Concerning the parameters  $m_L$ ,  $m_H$  and  $m_{H'}$ , we choose their values in order to have the same order of magnitude of the light and heavy vector meson masses:  $m_L = 800$ ,  $m_H = 5000$  MeV and  $m_{H'} = 9000$  MeV. These masses stand for the  $\rho$  or  $\omega$ ,  $B^*$  and  $\Upsilon$  respectively.

### 3. $T$ -matrix

The amplitudes obtained above provide the kernel of the Bethe-Salpeter equation in coupled channels,

$$T = (1 - VG)^{-1}V, \quad (7)$$

where  $V$  is the potential. In the  $B\bar{B}^*$  case, it is a  $3 \times 3$  matrix given by the Eq. (5) with the  $C_{ij}$  defined by Eq. (6), which is associated with the channels:  $B\bar{B}^*$ ,  $\eta_b\rho$  and  $\pi\Upsilon$ . In the  $B^*\bar{B}^*$  case,  $V$  is a  $2 \times 2$  matrix whose elements are the amplitudes given by Eqs. (3) and (4) corresponding to the channels  $B^*\bar{B}^*$  and  $\rho\Upsilon$ , respectively. In Eq. (7),  $G$  is a diagonal matrix whose elements are associated with the two meson loops,  $G_l$ , for each channel  $l$ . Explicitly, we have

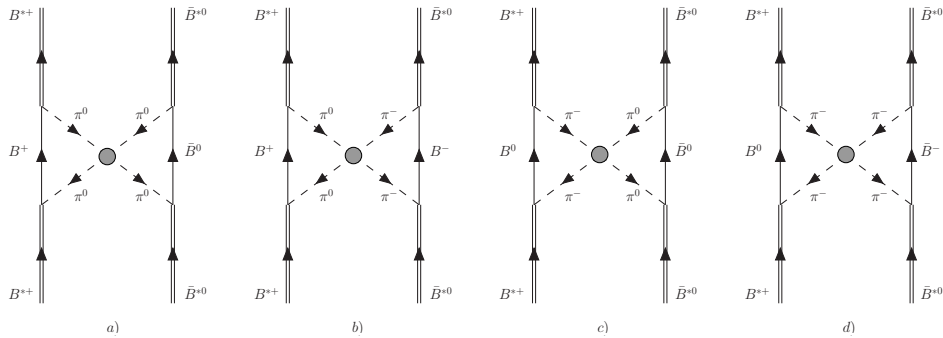
$$G_l = i \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - m^2 + i\epsilon} \frac{1}{(q - P)^2 - M^2 + i\epsilon}, \quad (8)$$

where  $m$  is the mass of the pseudoscalar (in the  $B\bar{B}^*$  case) or vector (in the  $B^*\bar{B}^*$  case), while  $M$  is the vector meson mass involved in the loop in the channel  $l$ . In Eq. (8)  $P$  means the total four-momentum of the mesons. The integral of Eq. (8) is logarithmically divergent and it can be regularized with a cut off in the momentum space or dimensional regularization [6, 8, 9].

#### 3.1. $B\bar{B}^*$ and $B^*\bar{B}^*$ interactions by means of two interacting pion exchange

Following the idea of the authors of Refs. [6, 8], where the authors investigated the  $D\bar{D}^*$  and  $D^*\bar{D}^*$  interactions by means of two interacting pions exchange, we extent these mechanism to the bottom case, studying the  $B\bar{B}^*$  and  $B^*\bar{B}^*$  interactions.

**3.1.1. The  $B\bar{B}^*$  case** For this case, the relevant diagrams contributing to this interaction are depicted in Fig. 3.

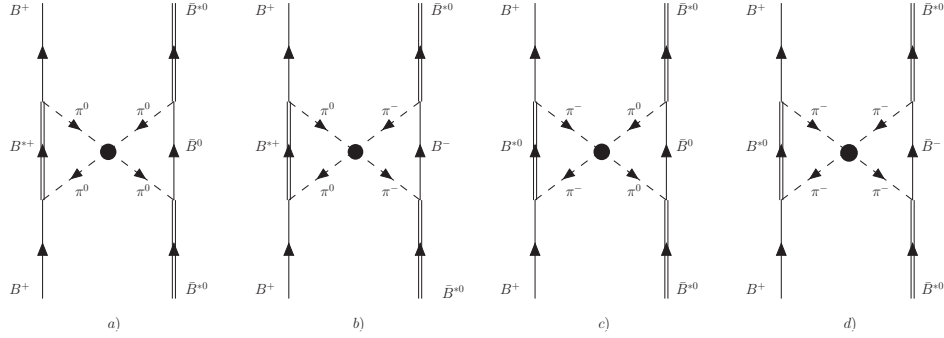


**Figure 3.** Diagrams contributing to the two pions interaction in lowest order in  $I = 1$  for the  $B^*\bar{B}^* \rightarrow B^*\bar{B}^*$  process [9].

Since we are following the approach of the Ref. [8], we can start from the amplitude obtained by its authors and replace the masses of the  $D$  and  $D^*$  mesons by the  $B$  and  $B^*$  mesons, respectively. Therefore, we get

$$t_{B^*\bar{B}^*}^\sigma(\vec{q}) = a^2 \frac{3}{2} \left[ \frac{1}{f^2} \frac{\vec{q}^2 + \frac{m_\pi^2}{2}}{1 - G(-\vec{q}^2) \frac{1}{f^2} (\vec{q}^2 + \frac{m_\pi^2}{2})} \right]. \quad (9)$$

Next, we shall consider the same mechanism for the  $B^*\bar{B}^*$  interaction.



**Figure 4.** Diagrams contributing to the two pion exchange interaction in lowest order for the  $B\bar{B}^* \rightarrow B\bar{B}^*$  process in  $I = 1$  [9].

*3.1.2. The  $B^*\bar{B}^*$  case* The diagrams contributing to this process are illustrated in Fig. 4.

For this case, the only difference to the former case is the appearing of another triangular loop (see the details in Ref. [9]).

### 3.2. Noninteracting two pions exchange

In this case, the contributions to the  $B\bar{B}^*$  and  $B^*\bar{B}^*$  interactions only come from the diagrams (a) and (d) of Figs. 3 and 4, respectively. All the details related to the derivation of the amplitudes  $t_{B\bar{B}^*}^{\pi\pi}$  and  $t_{B^*\bar{B}^*}^{\pi\pi}$  can be found in Ref. [8]. Explicitly, the amplitude  $t_{B^*\bar{B}^*}^{\pi\pi}$  is given by

$$t_{B^*\bar{B}^*}^{\pi\pi} = \frac{5}{4} g_B^4 \frac{A}{15} \int \frac{d^3p}{(2\pi)^3} (4\vec{p}^2 - \frac{\vec{q}^2}{4})^2 F^2 \frac{1}{\omega_1 + \omega_2} \frac{1}{2\omega_1\omega_2} \frac{1}{4E_B^2} \frac{1}{p_1^0 - \omega_1 - E_B + i\epsilon} \\ \times \frac{1}{p_1^0 - \omega_2 - E_B + i\epsilon} \left( 1 + \frac{E_B + \omega_1 + \omega_2 - p_1^0}{p_1^0 - \omega_1 - E_B + i\epsilon} + \frac{E_B + \omega_1 + \omega_2 - p_1^0}{p_1^0 - \omega_2 - E_B + i\epsilon} \right), \quad (10)$$

where  $A = 5$  is associated with spin  $J = 0$ , while  $A = 2$  is related to the  $J = 2$  case,  $\omega_1 = \sqrt{(\vec{p} + \vec{q}/2)^2 + m_\pi^2}$ ,  $\omega_2 = \sqrt{(\vec{p} - \vec{q}/2)^2 + m_\pi^2}$  are the energies of the pions and  $E_B(\vec{p}) = \sqrt{\vec{p}^2 + m_B^2}$  is the energy of the  $B$  meson.  $F(\vec{q})$  is a form factor of the type

$$F = F_1(\vec{p} + \frac{\vec{q}}{2}) F_2(\vec{p} - \frac{\vec{q}}{2}) = \frac{\Lambda^2}{\Lambda^2 + (\vec{p} + \frac{\vec{q}}{2})^2} \frac{\Lambda^2}{\Lambda^2 + (\vec{p} - \frac{\vec{q}}{2})^2}, \quad (11)$$

with  $\Lambda = 700$  GeV, which is used to improve the convergence. On the other hand, for the  $t_{B\bar{B}^*}^{\pi\pi}$  amplitude we have

$$t_{B\bar{B}^*}^{\pi\pi} = -\frac{5}{4} g_B^4 \frac{1}{2} \vec{\epsilon}' \cdot \vec{\epsilon}'' \int \frac{d^3p}{(2\pi)^3} (\vec{p}^2 - \vec{q}^2) \left[ (4\vec{p}^2 - \frac{\vec{q}^2}{4}) - \frac{1}{\vec{q}^2} \left[ (2\vec{p}\vec{q})^2 - \frac{\vec{q}^4}{4} \right] \right] \frac{F^2}{\omega_1 + \omega_2} \frac{1}{2\omega_1\omega_2} \\ \times \frac{1}{2E_B} \frac{1}{2E_V} [\omega_1^2 + \omega_2^2 + \omega_1\omega_2 - (\omega_1 + \omega_2)(2p_1^0 - E_{B^*} - E_B) + (p_1^0 - E_{B^*})(p_1^0 - E_B)] \\ \times \frac{1}{p_1^0 - \omega_1 - E_{B^*} + i\epsilon} \frac{1}{p_1^0 - \omega_1 - E_B + i\epsilon} \frac{1}{p_1^0 - \omega_2 - E_{B^*} + i\epsilon} \frac{1}{p_1^0 - \omega_2 - E_B + i\epsilon}, \quad (12)$$

where  $E_{B^*}(\vec{p}) = \sqrt{\vec{p}^2 + m_{B^*}^2}$  is the energy of the  $B^*$  meson.

## 4. Results

Once we have obtained all the amplitudes which contribute to the  $B\bar{B}^*$  and  $B^*\bar{B}^*$  interactions, we determine an effective potential to be used as a kernel of the Bethe-Salpeter equation,

as follows: we calculate the integral  $\int d^3q V(|\vec{q}|)$  related to the strength of all the potentials associated with the pions and convert them in a potential of the form

$$V_{eff}\theta(q_{max} - |\vec{q}|)\theta(q_{max} - |\vec{q}'|), \quad (13)$$

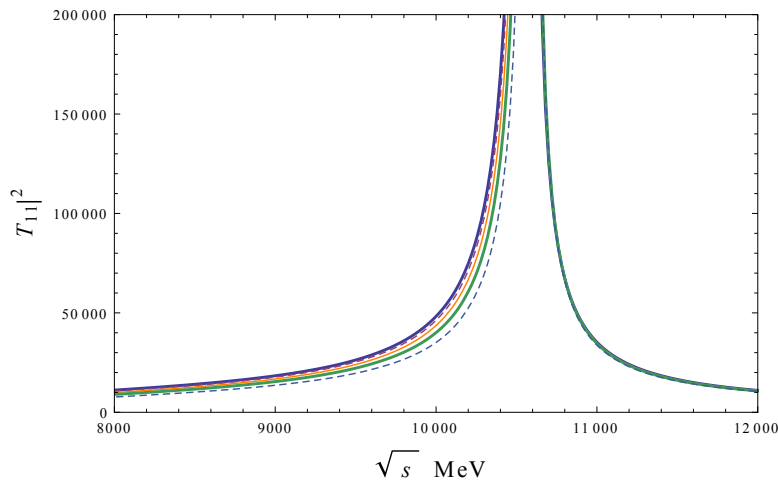
where  $q_{max}$  is the maximum momentum used in the loops in Eq. (8), such that  $\int_{q < q_{max}} d^3q V_{eff}$  is equal to the sum of  $\int d^3q V_i(q)$ . Then, we take as effective potential the sum of the Eq. (13) plus the one coming from the heavy vector exchange. Both have the same form of the Eq. (13) and can be used in the Bethe-Salpeter equation with the same  $G$  loop function regularized with the cut-off  $|\vec{q}_{max}|$ .

On the other hand, the value of the strength depends on the value of the upper limit of the integral  $\int d^3q V(q)$ . For this reason we calculated the effective potential  $V_{eff}$  using values of this limit for the light meson exchange potential varying from 700 to 1100 MeV for both  $B^*\bar{B}^*$  and  $B\bar{B}^*$  interactions. Changing the upper limit in  $\int d^3q V_i(q)$  introduces large uncertainties in the approach concerning the final potential. The strength of the final potential, summing  $V_{eff}$  and the vector exchange, can be a factor 2.4 – 14.5 times the one of the vector exchange alone for the case of  $B^*\bar{B}^*$  with  $J = 0$ , while for  $J = 2$  we find a factor 1.2 – 5.2. For the case of  $B\bar{B}^*$  the factor varies between 30 and 64.

In the following we study the shape of the  $|T_{11}|^2$  which stands for diagonal transitions,  $B\bar{B}^*$  or  $B^*\bar{B}^*$  to themselves, depending on the case.

#### 4.1. The $B\bar{B}^*$ case

For this case, we have calculated the  $T$ -matrix considering the channels:  $B\bar{B}^*$ ,  $\eta_b \rho$  and  $\pi \Upsilon$ . The  $T$ -matrix was evaluated between those channels for the  $\sqrt{s}$  values around 10600 MeV. For the  $G$  loop function, we have use the dimensional regularization formula with the following values for the subtraction constants:  $\alpha_{B\bar{B}^*} = -2.79$ ,  $\alpha_{\eta_b \rho} = -3.56$  and  $\alpha_{\pi \Upsilon} = -3.78$ . This is equivalent to use a cut-off equal to  $|\vec{q}_{max}| = 700$  MeV. In Fig. 5 the shape of  $|T_{11}|^2$ , the component of



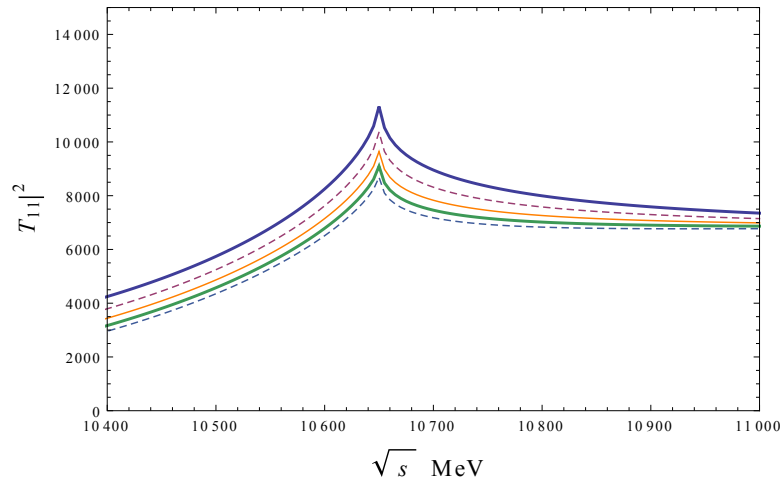
**Figure 5.**  $|T_{11}|^2$  as a function of the  $\sqrt{s}$  center of mass energy for the case of  $B\bar{B}^*$ . Each curve is associated with a value of the integration limit: 700 MeV, 800 MeV, 900 MeV, 1000 MeV, 1100 MeV. The peak moves from right to left as the integration limit increases [9].

the  $T$  matrix that describes the transition  $B\bar{B}^* \rightarrow B\bar{B}^*$ , for different values of the integration limit, is depicted. As can be seen, even choosing values of the limit between 700 and 1100 MeV, the effect on the binding and the width is small. As a result, we find that the position of the

peak moves slightly to higher energies for decreasing values of the upper limit and it is seen in the range of 10587 – 10601 MeV. These values are very close to what was found by the Belle collaboration,  $M_{Z_b(10610)} = (10608.4 \pm 2.0)$  MeV.

#### 4.2. The $B^*\bar{B}^*$ case

For this case, we have two channels:  $B^*\bar{B}^*$  and  $\rho\Upsilon$ . Again, we use the dimensional regularization form of the loop function  $G$  with the subtraction constants  $\alpha_{B^*\bar{B}^*} = -2.79$  and  $\alpha_{\rho\Upsilon} = -3.56$ , corresponding to a cut off value equal to  $|\vec{q}_{max}| = 700$  MeV.



**Figure 6.**  $|T_{11}|^2$  as a function of the  $\sqrt{s}$  center of mass energy for the case of  $B^*\bar{B}^*$  for  $J = 0$ . Each curve is associated with a value of the integration limit: 700 MeV, 800 MeV, 900 MeV, 1000 MeV, 1100 MeV. The peak moves from bottom to top as the integration limit increases [9].

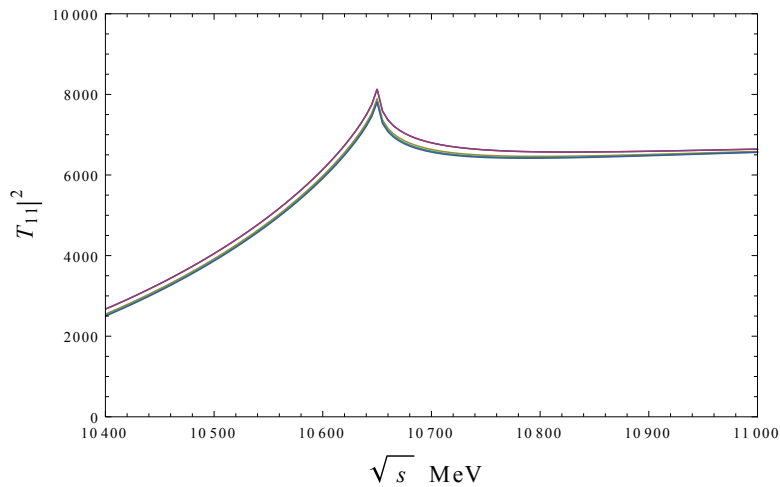
Fig. 6 shows the shape of  $|T_{11}|^2$ , which means the component of the  $T$  matrix that describes the transition from  $B^*\bar{B}^*$  to itself, for different values of the integration limit plotted as a function of the center of mass energy,  $\sqrt{s}$ , of the system. This peak corresponds to spin  $J = 0$ . In Fig 7, we show the shape of  $|T_{11}|^2$  for the  $J = 2$  case, again for different values of the integration limit. It is important to emphasize that, according to Eq. (4), there is no contribution in the transition matrix  $T$  from  $B^*\bar{B}^*$  to  $\rho\Upsilon$  channel for spin  $J = 1$ . In this case,  $B^*\bar{B}^*$  stands as a single channel.

From these figures we noted that it is always found a structure for the peak of  $|T_{11}|^2$ , which corresponds clearly to a cusp. Whether to call this a resonant state or not it is a question of criterion. We should however note that the  $a_0(980)$  appears in the experiments (or in the theories) [10, 11] as a cusp and is universally accepted as a resonance. Our findings, obtained a cusp for the  $|T_{11}|^2$  amplitude in this case, would come to support the claims of the former works [12, 13].

## 5. Conclusions

We have studied the  $B\bar{B}^*$  and  $B^*\bar{B}^*$  interactions for  $I = 1$  using the Lagrangians from the Hidden Local Symmetry approach. We stressed that from the point of view of the light meson exchange, those interactions are not allowed by the OZI rule. Therefore, we investigated the same interactions through the exchange of heavy meson and also due to the two pions, interacting and noninteracting among themselves, exchange. Our goal was to look for poles in the T-matrix, solving the Bethe-Salpeter equation using an effective potential obtained from the last meson





**Figure 7.**  $|T_{11}|^2$  as a function of the  $\sqrt{s}$  center of mass energy for the case of  $B^*\bar{B}^*$  for  $J = 2$ . Each curve is associated with a value of the integration limit: 700 MeV, 800 MeV, 900 MeV, 1000 MeV, 1100 MeV. The peak moves slightly from bottom to top as the integration limit increases [9].

exchange mechanism, in order to relate them with the charged states  $Z_b(10610)$  and  $Z_b(10650)$  observed by Belle Collaboration. As a result, for the  $B\bar{B}^*$  case, we found a bound state with mass is in the range 10587 – 10601 MeV which is very close to the experimental mass value of the  $Z_b(10610)$  state. For the  $B^*\bar{B}^*$  interaction, we have found a cusp at 10650 MeV for the  $J = 0$  and also  $J = 2$  spin cases.

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### References

- [1] S. K. Choi *et al.* [Belle Collaboration], Phys. Rev. Lett. **91**, 262001 (2003) [hep-ex/0309032].
- [2] A. Bondar *et al.* [Belle Collaboration], Phys. Rev. Lett. **108**, 122001 (2012) [arXiv:1110.2251 [hep-ex]].
- [3] D. Gamermann, E. Oset and B. S. Zou, Eur. Phys. J. A **41**, 85 (2009).
- [4] D. Gamermann and E. Oset, Eur. Phys. J. A **33**, 119 (2007).
- [5] L. Roca, E. Oset and J. Singh, Phys. Rev. D **72**, 014002 (2005).
- [6] F. Aceti, M. Bayar, E. Oset, A. M. Torres, K. P. Khemchandani, F. S. Navarra and M. Nielsen, Phys. Rev. D **90**, 016003 (2014).
- [7] E. Oset, H. Toki, M. Mizobe and T. T. Takahashi, Prog. Theor. Phys. **103**, 351 (2000).
- [8] F. Aceti, M. Bayar, J. M. Dias and E. Oset, Eur. Phys. J. A **50**, 103 (2014).
- [9] J. M. Dias, F. Aceti and E. Oset, Phys. Rev. D **91**, 076001 (2015).
- [10] P. Rubin *et al.* [CLEO Collaboration], Phys. Rev. Lett. **93**, 111801 (2004) [hep-ex/0405011].
- [11] J. A. Oller, E. Oset and J. R. Pelaez, Phys. Rev. D **59**, 074001 (1999) [Erratum-ibid. D **60**, 099906 (1999)] [Erratum-ibid. D **75**, 099903 (2007)] [hep-ph/9804209].
- [12] D. V. Bugg, Europhys. Lett. **96**, 11002 (2011) [arXiv:1105.5492 [hep-ph]].
- [13] E. S. Swanson, arXiv:1409.3291 [hep-ph].