Cooling of neutron stars and emissivity of neutrinos by the direct Urca process under influence of a strong magnetic field

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Abstract. Neutron stars are born with high temperatures and during a few seconds suffer rapid cooling by emission of neutrinos. The direct Urca process is the main mechanism to explain this loss of energy. In this work we study the influence of a strong magnetic field on the composition of nuclear matter at high densities and zero temperature. We describe the matter through a relativistic mean-field model with eight light baryons (baryon octet), electrons, muons magnetic field. As output of the numerical calculations, we obtain the relative population for a parametrized magnetic field. We calculate the cooling of neutron stars with different mass and magnetic fields due to direct Urca process

1. Introduction

Strong magnetic fields of magnitudes up to 10^{14} G are suppose to exists at the surface of pulsars. So, it is of interest to study the properties of nuclear matter in the presence of such strong magnetic fields. In this work we study the influence of a strong magnetic field on the composition of nuclear matter at $T = 0$, and investigate the cooling of neutron stars due to the direct Urca process using the resulting equation of state.

The matter at high densities is described using a relativistic mean field (MF) theory which describes correctly the nuclear ground state properties and elastic scattering of nucleons. The Lagrangian that describes this model, with a uniform magnetic field B along the z axis, is given by [1]

$$
\begin{aligned}
\mathcal{L} &= \sum_b \bar{\psi}_b[i\gamma_\mu D^\mu - m_b + g_{\sigma b}\sigma - g_{\omega b}\gamma_\mu\omega^\mu - \tfrac{1}{2}g_{\rho b}\gamma_\mu\boldsymbol{\tau}\cdot\boldsymbol{\rho}^\mu]\psi_b + \tfrac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \tfrac{1}{2}m_\sigma^2\sigma^2 \\
&\quad -U(\sigma) - \tfrac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \tfrac{1}{4}\boldsymbol{\rho_{\mu\nu}}\cdot\boldsymbol{\rho^{\mu\nu}} + \tfrac{1}{2}m_\rho^2\boldsymbol{\rho_\mu}\cdot\boldsymbol{\rho^\mu} \\
&\quad + \sum_{l=e^-,\,\mu^-} \bar{\psi}_l[i\gamma_\mu(\partial^\mu + iq_l A^\mu) - m_l]\psi_l - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu},
\end{aligned}
\tag{1}
$$

where $D^\mu = \partial^\mu + iq_b A^\mu$, $A^0 = 0$, $\vec{A} = (0, xB, 0)$, q_b and q_l are the electric charge of baryons and leptons, ψ_b is the Dirac spinor for baryon b in the octet $\{n, p, \Lambda, \Sigma, \Xi\}$ with mass m_b; m_σ, m_ω,

m_ρ and $g_{\sigma b}$, $g_{\omega b}$, $g_{\rho b}$ are the masses and coupling constants of mesons σ, ω, ρ respectively. The summation in the first line represents the free Lagrangian of baryons together with the interaction between baryons and mesons; The mesonic and electromagnetic field strength tensors are

$$\omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu, \tag{2}$$

$$\rho_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu, \tag{3}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{4}$$

The baryon octet, showing their quantum numbers, charge q_b and isospin projection I_{3b} are shown in Table 1.

Table 1. The baryon octet, showing their quantum numbers, charge q_b, isospin projection I_{3b}.

Species	Mass	q_b	I_{3b}
	(MeV)	(e)	
p	939	1	$1/2$
n	939	0	$-1/2$
Λ	1115	0	0
Σ^+	1190	1	1
Σ^0	1190	0	0
Σ^-	1190	-1	-1
Ξ^0	1315	0	$1/2$
Ξ^-	1315	-1	$-1/2$

The Lagrangian of leptons (electrons and muons) is written in the third line while the scalar self-interactions term is given by

$$U(\sigma) = \frac{1}{3} b\, m_n (g_{\sigma n}\sigma)^3 + \frac{1}{4} c (g_{\sigma n}\sigma)^4, \tag{5}$$

where b and c are constants.

The dynamic equations of nucleon and mesons (Dirac and Klein-Gordon equations respectively) are obtained from the Euler-Lagrange equations

$$\frac{\partial \mathcal{L}}{\partial \phi(x)} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = 0, \tag{6}$$

where $\phi(x)$ is the corresponding field. The resulting equations of motion in the mean field approximation are given by

$$g_{\omega n}\omega_0 = \left(\frac{g_{\omega n}}{m_\omega}\right)^2 \sum_b \chi_{\omega b}\rho_b, \tag{7}$$

$$g_{\rho n}\rho_{03} = \left(\frac{g_{\rho n}}{m_\rho}\right)^2 \sum_b \chi_{\rho b} I_{3b}\rho_b, \tag{8}$$

$$m_n^* = m_n + \left(\frac{g_{\sigma n}}{m_{\sigma n}}\right)^2 \left[b\, m_n (m_n - m_n^*)^2 + c(m_n - m_n^*)^3 - \sum_b \chi_{\sigma b} n_s \right], \tag{9}$$

where I_{3b} is the 3-component of the isospin of the baryon b, $m_n^* = m_n - \chi_{\sigma b} g_{\sigma n}\sigma$, $\chi_{\sigma b} = g_{\sigma b}/g_{\sigma n}$, $\chi_{\omega b} = g_{\omega b}/g_{\omega n}$, $\chi_{\rho b} = g_{\rho b}/g_{\rho n}$; The scalar density is given by

$$n_s = n_s^{q=0} + n_s^{q \neq 0}, \tag{10}$$

where

$$n_s^{q=0} = \frac{m_b^*}{2\pi^2}\left[\mu_b^* k_b - m_b^{*2}\ln\left(\frac{\mu_b^* + k_b}{m_b^*}\right)\right], \tag{11}$$

$$n_s^{q\neq 0} = \frac{m_b^*|q_b|B}{2\pi^2}\sum_{\nu=0}^{\nu_{\max(b)}} g_\nu \ln\left[\frac{\mu_b^* + k_{b,\nu_b}}{m_{b,\nu_b}^*}\right], \tag{12}$$

with

$$m_{b,\nu_b}^{*2} = m_b^{*2} + 2\nu_b|q_b|B, \tag{13}$$

$$k_{b,\nu_b}^2 = \mu_b^{*2} - m_b^{*2} - 2\nu_b|q_b|B, \tag{14}$$

where q_b and μ_b^* are the electric charge and effective chemical potential of baryon b, k_b and k_{b,ν_b} are the Fermi momentum of neutral and charge baryons respectively. ν_b is the Landau principal quantum number, which can take all possible positive integer values including zero. The upper limit $\nu_{\max(b)}$ is defined by the condition $k_{b,\nu_b}^2 \geq 0$, then

$$\nu_{\max(b)} = \text{int}\left[\frac{\mu_b^{*2} - m_b^{*2}}{2|q_b|B}\right]. \tag{15}$$

The effective chemical potential of baryons are given by

$$\mu_b^* = \mu_b - \chi_{\omega b}g_{\omega n}\omega_0 - \chi_{\rho b}g_{\rho n}I_{3b}\rho_{03}. \tag{16}$$

They are constrained due to the β-equilibrium condition, which reads

$$\mu_b = \mu_n - q_b\mu_e, \tag{17}$$

$$\mu_{e^-} = \mu_{\mu^-}, \tag{18}$$

where μ_n and μ_e are the chemical potentials of neutron and electron respectively. The baryon densities are

$$\rho_b^{q=0} = \frac{k_b^3}{3\pi^2}, \tag{19}$$

$$\rho_b^{q\neq 0} = \frac{|q_b|B}{2\pi^2}\sum_{\nu=0}^{\nu_{\max(b)}} g_\nu k_{b,\nu_b} \tag{20}$$

while the lepton (electrons and muons) densities are

$$\rho_l = \frac{|q_l|B}{2\pi^2}\sum_{\nu=0}^{\nu_{\max(l)}} g_\nu k_{l,\nu_l}, \tag{21}$$

with

$$k_{l,\nu_l}^2 = u_l^2 - m_l^2 - 2\nu_l|q_l|B \tag{22}$$

and

$$\nu_{\max(l)} = \text{int}\left[\frac{\mu_l^2 - m_l^2}{2|q_l|B}\right], \tag{23}$$

where $m_{b,\nu_b}^{*2} = m_b^{*2} + 2\nu_b|q_b|B$, ν is the Landau principal quantum number and the Landau level degeneracy g_ν is 1 for $\nu = 0$ and 2 for $\nu > 0$.

Neutron star matter satisfies the constraints of conservation of baryon number and neutrality of electric charge, which reads

$$\rho = \sum_b \rho_b, \tag{24}$$

$$0 = \sum_b q_b\rho_b + \sum_{l=e^-,\,\mu^-} q_l\rho_l. \tag{25}$$

The energy density due to the matter is given by [2]

$$
\begin{aligned}
\varepsilon_m = {} & \frac{1}{3}b\,m_n(g_{\sigma n}\sigma)^3 + \frac{1}{4}c\,(g_{\sigma n}\sigma)^4 + \frac{1}{2}\left(\frac{m_{\sigma n}}{g_{\sigma n}}\right)^2(g_{\sigma n}\sigma)^2 + \frac{1}{2}\left(\frac{m_{\omega n}}{g_{\omega n}}\right)^2(g_{\omega n}\omega_0)^2 \\
& +\frac{1}{2}\left(\frac{m_{\rho n}}{g_{\rho n}}\right)^2(g_{\rho_n}\rho_{03})^2 + \sum_{b(q=0)}\frac{1}{8\pi^2}[2\mu_b^{*3}k_b - m_b^{*2}\mu_b^*k_b - m_b^{*4}\ln\left\{\frac{\mu_b^*+k_b}{m_b^*}\right\}] \\
& +\frac{|q|B}{4\pi^2}\sum_{b(q\neq 0)}\sum_{\nu_b=0}^{\nu_{\max(b)}} g_\nu[\mu_b^*k_{b,\nu_b} + m_{b,\nu_b}^{*2}\times\ln\left\{\frac{\mu_b^*+k_{b,\nu_b}}{m_{b,\nu_b}^*}\right\}] \\
& +\frac{|q|B}{4\pi^2}\sum_{l=e^-,\,\mu^-}\sum_{\nu_l=0}^{\nu_{\max(l)}} g_\nu\left[\mu_l\,k_{l,\nu_l} + m_{l,\nu_l}^2\ln\left\{\frac{\mu_l+k_{l,\nu_l}}{m_{l,\nu_l}}\right\}\right],
\end{aligned}
\tag{26}
$$

where

$$m_{l,\nu_l}^2 = m_l^2 + 2\nu_l|q_l|B. \tag{27}$$

The matter pressure is given by

$$P_m = \mu_n\rho_b - \varepsilon_m. \tag{28}$$

Next, we add to these the contribution from electromagnetic field tensor, obtaining the total energy density and pressure as

$$\varepsilon = \varepsilon_m + \frac{B^2}{2}, \tag{29}$$

$$P = P_m + \frac{B^2}{2}. \tag{30}$$

The magnetic field is parametrized by

$$B(\rho/\rho_0) = B^{surf} + B_0[1 - \exp\{-\beta(\rho/\rho_0)^\gamma\}], \tag{31}$$

where $B^{surf} = 10^8$G and $B_0 = 10^{19}$G are the magnetic fields at the surface and the center of the star respectively, with the parameters $\beta = 10^{-4}$ and $\gamma = 17$, and ρ_0 is the saturation density.

The system of coupled nonlinear equations (7-9) with constraints (24-25) is solved numerically by iteration. We use the Newton-Raphson method with global search of the solution. We adopt natural units. As output, we obtain the relative population of each specie of particles as a function of the baryon density, and the energy density and pressure with and without magnetic field. The coupling constants are given in Tables 2, 3 e 4.

Table 2. Nucleon-meson coupling constants to compression $K = 300$ MeV and $m^*/m = 0.70$ [1].

$(\frac{g_{\sigma n}}{m_\sigma})^2$ (fm^2)	$(\frac{g_{\omega n}}{m_\omega})^2$ (fm^2)	$(\frac{g_{\rho n}}{m_\rho})^2$ (fm^2)	b	c
11.79	7.148	4.410	0.002947	-0.001070

Table 3. Parametrizations used for the hyperon coupling constants [3].

$\chi_{\sigma\Lambda}$	$\chi_{\sigma\Sigma}$	$\chi_{\sigma\Xi}$	$\chi_{\omega\Lambda} = \chi_{\omega\Sigma}$	$\chi_{\omega\Xi}$
0.6106	0.4046	0.3195	2/3	1/3

Table 4. Parametrizations used for the hyperon coupling constants [3].

$\chi_{\rho\Lambda}$	$\chi_{\rho\Sigma}$	$\chi_{\rho\Xi}$
1	1	1
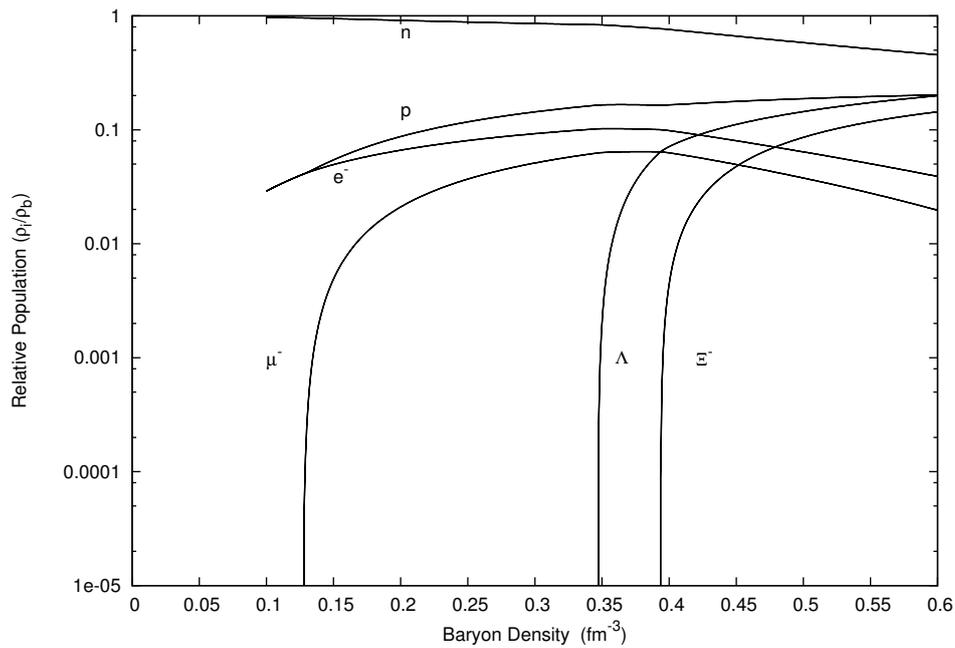
Figure 1. The particle fractions in cold β-equilibrated neutron star without magnetic field.

The Fig 1 and Fig 2 show the results without and with an parametrized magnetic field, respectively. It can be seen that the incorporation of strong magnetic field increases the proton and electron fraction. This effect is important for the neutrino emissivity due to the direct Urca process, having an impact on the cooling of neutron stars.

The direct Urca process is the most powerful mechanism of neutrino emission in the core of neutron stars. Neutron star cooling by Urca process may provide important informations about the interior composition of the star. The reaction is given by

$$n \rightarrow p + e^- + \bar{\nu}_e,$$
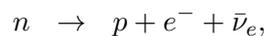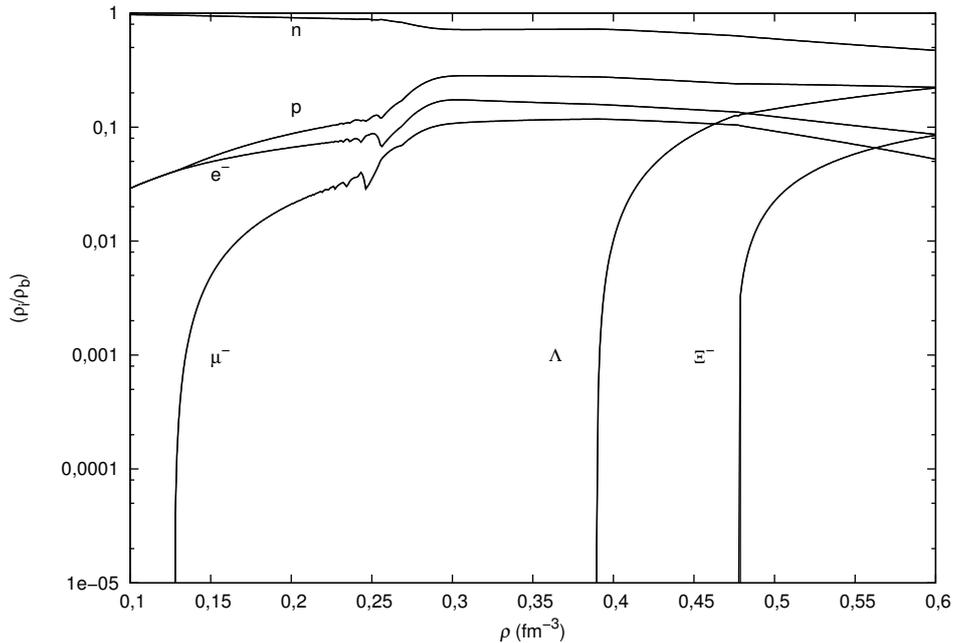
Figure 2. The particle fractions in cold β-equilibrated neutron star for the parametrized magnetic field.

$$p + e^- \quad \to \quad n + \nu_e.$$
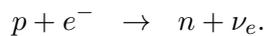
This process may occur if the proton fraction is large enough

$$k_{Fn} \leq k_{Fp} + k_{Fe}, \tag{32}$$

where

$$k_{F\alpha} = (3\pi^2 \rho_\alpha)^{1/3}, \tag{33}$$

in order to conserve momentum in the reaction. As we have showed above, strong magnetic fields lead to an increase of the proton fraction and the cooling of neutron stars is more efficient.

In the Weinberg-Salam theory for weak interactions, the interaction Lagrangian is given by [4]

$$\mathcal{L}_{\text{weak}} = \frac{G_F}{\sqrt{2}} \cos\theta_c l_\mu j^\mu,$$

where G_F is the Fermi weak coupling constant and θ_c is the Cabibbo angle. The Lepton and nucleon charged weak currents are

$$l_\mu = \bar{\psi}_4 \gamma_3 \mu (1 - \gamma_5) \psi_2, \tag{34}$$
$$j^\mu = \bar{\psi}_3 \gamma^\mu (g_V - g_A \gamma_5) \psi_1, \tag{35}$$

g_V and g_A are vector and axial-vector coupling constants and the indices $i = 1 - 4$ refer to the $n, \bar{\nu}_e, p$ and e^-, respectively. The wave functions for neutron and antineutrino are plane wave functions. The wave functions for both protons and electron in the presence of a magnetic field

strong enough that only the ground Landau level is occupied are given by

$$\psi_3(X) = \frac{1}{\sqrt{L_y L_z}} \exp(-iE_3^* t + ik_{3y}y + ik_{3z}z) f_{k_{3y},k_{3z}}^{\nu_3=0}, \tag{36}$$

$$\psi_4(X) = \frac{1}{\sqrt{L_y L_z}} \exp(-iE_4 t + ik_{4y}y + ik_{4z}z) f_{k_{4y},k_{4z}}^{\nu_4=0}, \tag{37}$$

where $f_{k_{3y},k_{3z}}^{\nu_3=0}$ and $f_{k_{4y},k_{4z}}^{\nu_4=0}$ are the 4-component spinor solutions of the corresponding Dirac equation. The only positive energy spinor for protons in the chiral representation is [5]

$$f_{k_{3y};k_{3z}}^{\nu_3=0}(x) = N_{\nu_3=0} \begin{pmatrix} E_3^* + k_{3z} \\ 0 \\ -m_3^* \\ 0 \end{pmatrix} I_{\nu_3=0;k_{3y}}, \tag{38}$$

where

$$N_{\nu_3=0} = \frac{1}{[2E_3^*(E_3^* + k_{3z})]^{1/2}}, \quad E_3^* = (k_{3z}^2 + m_3^{*2})^{1/2}, \tag{39}$$

$$I_{\nu_3=0;k_{3y}} = \left(\frac{eB}{\pi}\right)^{1/4} \exp\left[-\frac{1}{2}eB\left(x - \frac{k_{3y}}{eB}\right)^2\right] \frac{1}{\sqrt{\nu_3!}} H_{\nu_3}\left[\sqrt{2eB}\left(x - \frac{k_{3y}}{eB}\right)\right],$$

and are the Hermite polynomial and e is the electron charge.

The emissivity due to the antineutrino emission process in presence of a uniform magnetic field B along z-axis is [**?**]

$$\epsilon_\nu = 2 \int \frac{V d^3 k_1}{(2\pi)^3} \int \frac{V d^3 k_2}{(2\pi)^3} \int_{-qBL_x/2}^{qBL_x/2} \frac{L_y dk_{3y}}{2\pi} \int \frac{L_z dk_{3z}}{2\pi} \int_{-qBL_x/2}^{qBL_x/2} \frac{L_y dk_{4y}}{2\pi}$$

$$\times \int \frac{L_z dk_{4z}}{2\pi} E_2 W_{fi} f_1 [1 - f_3][1 - f_4], \tag{40}$$

where the pre-factor 2 takes into account the neutron spin degeneracy and f_i is the Fermi-Dirac distribution functions. By the Fermi's golden rule, the transition rate per unity volume W_{fi} is

$$W_{fi} = \frac{\langle |M_{fi}|^2 \rangle}{tV}, \tag{41}$$

where t is the time, $V = V_x V_y V_z$ is the normalization volume and the matrix element for the V-A interaction is given by

$$M_{fi} = \frac{G_F}{\sqrt{2}} \int d^4 X \, \bar{\psi}_1(X) \gamma^\mu (g_V - g_A \gamma_5) \psi_3(X) \bar{\psi}_2(X) \gamma_\mu (1 - \gamma_5) \psi_4(X), \tag{42}$$

where $\langle . \rangle$ denotes an averaging over the initial spin of n and a sum over spins of final particles (p, e). Then, the transition rate per unit volume is

$$W_{fi} = \frac{G_F^2}{E_1^* E_2 E_3^* E_4} \frac{1}{V^3 L_y L_z} \exp\left(-\frac{(k_{1x} - k_{2x})^2 + (k_{3y} + k_{4y})^2}{2eB}\right)$$

$$\times [(g_V + g_A)^2 (k_1.k_2)(k_3.k_4) + (g_V - g_A)^2$$

$$\times (k_1.k_4)(k_3.k_2) - (g_V^2 - g_A^2) m^{*2}(k_4.k_2)](2\pi)^3 \delta(E_1^* - E_2 - E_3^* - E_4)$$

$$\times \delta(k_{1y} - k_{2y} - k_{3y} - k_{4y}) \delta(k_{1z} - k_{2z} - k_{3z} - k_{4z}).$$

7

Then, the emissivity is

$$
\begin{aligned}
\epsilon_\nu \;=\; & \frac{457\pi}{5040}G_F^2\cos^2\theta_c(eB)[(g_V+g_A)^2\left(1-\frac{k_{F_3}}{\mu_3^*}\right)+(g_V-g_A)^2\left(1-\frac{k_{F_1}}{\mu_n^*}\cos\theta_{14}\right) \\
& -(g_V^2-g_A^2)\frac{m^{*2}}{\mu_3^*\mu_1^*}\Big]\exp\left[\frac{(k_{F_3}+k_{F_4})^2-k_{F_1}^2}{2eB}\right]\frac{\mu_1^*\mu_3^*\mu_4}{k_{F_3}k_{F_4}}T^6\Theta,
\end{aligned}
$$

$\cos\theta_{14}=(k_{F_1}^2+k_{F_4}^2-k_{F_3}^2)/2k_{F_1}k_{F_4}$, T is the temperature, k_{Fi} is the Fermi momentum and the threshold factor is $\Theta=\theta(k_{F_3}+k_{F_4}-k_{F_1})$, with $\theta(x)=1$ $(x>0)$, $\theta(x)=0$ (otherwise).

2. Results

Figure 3 shows the cooling due to the direct Urca process of neutron stars with 1.4 and 1.6 solar masses (continuous and dashed lines, respectively) for the cases $B=0$ (blue line) and $B(\rho/\rho_0)$ (red line). We can see that the cooling is more intense with the increase in the mass of the star and for the case $B(\rho/\rho_0)$. Theses differences may be attributed to the increase of proton and electron fractions with the mass and magnetic field of the star and to the phase space modifications.
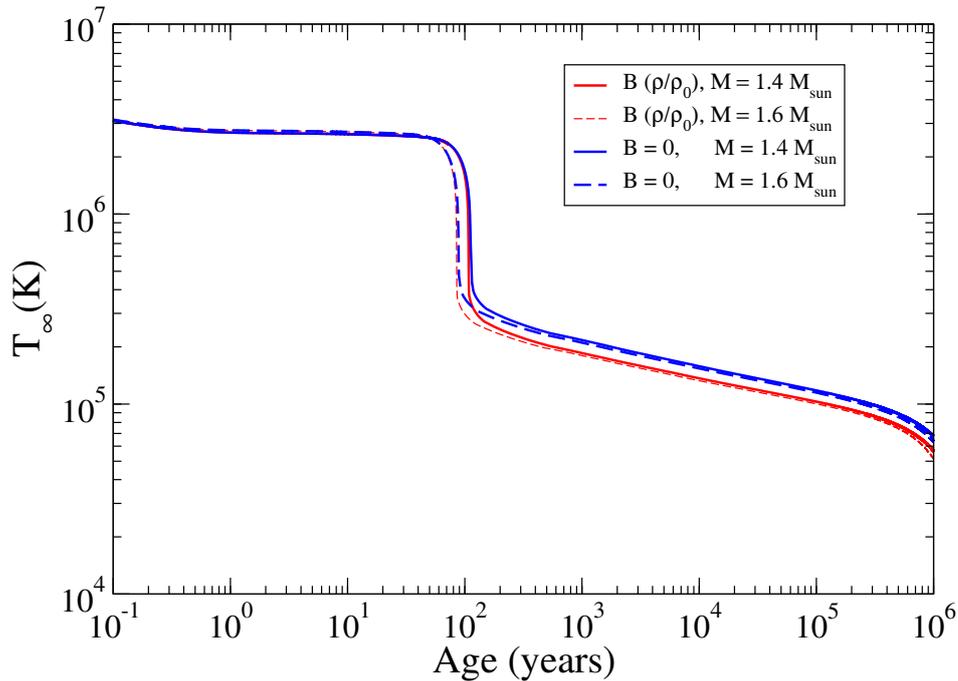
Figure 3. Cooling of a neutron star mass with 1.4 and 1.6 solar masses (continuous and dashed lines, respectively) for the cases $B=0$ (blue line) and $B(\rho/\rho_0)=B^{surf}+B_0[1-\exp\{-\beta(\rho/\rho_0)^\gamma\}]$ (red line).

[1] Glendenning N K 2000 *Compact Stars* (New York: Springer).
[2] Chakrabarty S, Bandyopadhyay D and Pal S 1997 *Phys. Rev. Letter* **78** 2898.
[3] Chiapparini M, Bracco M E, Delfino A, Malheiro M, Menezes D P and Providência C 2009 *Nuclear. Phys. A* **826** 178.

[4] Bandyopadhyay D, Chakrabarty S, Dey P and Pal S 1998 *Phys. Rev.* D **58** 121301.
[5] Kobayashi M and Sakamoto M 1983 *Progress of Theoretical Phys.* **70** 5.