Comparative study between Glauber-Velasco and the Stochastic Vacuum models for high energy hadronic collision

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Abstract.

The outlines of the Glauber-Velasco (GV) and the Stochastic Vacuum models (SVM) for describing pp elastic scattering are presented. The amplitudes and the cross-sections of both models are investigated and compared at $\sqrt{s} = 7\,\text{TeV}$. The asymptotic energy behaviour for the SVM is also studied.

1. Introduction

For decades, it has been a challenge to provide a suitable description of microscopic high-energy hadronic processes. The pp (and $p\bar{p}$) elastic collisions involve non-perturbative aspects of strong interactions and play a role in the understanding of QCD interactions and hadronic structure. Recently, the interest in this problem has increased due to the high-energy data avaiable from TOTEM [1, 2, 3] and ATLAS [4] experiments.

The understanding of the differential cross-section $d\sigma/dt$ depends directly on the interplay of the imaginary and real parts of the scattering amplitudes. For example, the existence of a dip in the $d\sigma/dt$ is related to the occurrence of a zero in the imaginary part and its height is influenced by the magnitude of the real part. The analytical representation of the amplitudes can only be achieved through models.

The literature provides a large number of phenomenological models. All of them should respect general quantum field theory principles, like analiticity and unitarity. Both properties may be assured in eikonalized models, which is an approach suitable to high energy scattering at small angles. An interesting way to probe these models is compare them with each other and, more importantly, with the data.

In the present work, the Stochastic Vacuum Model (SVM) and the Glauber-Velasco (GV) model were studied and compared at $\sqrt{s} = 7\,\text{TeV}$. The above two models reflect the two complementary aspects of QCD, particle and field. Our main goal is to clarify the relation between Glauber-Velasco model and the Stochastic Vacuum Model in the point of view of QCD. Some predictions for asymptotic energies were also investigated.

2. The Stochastic Vacuum Model

More than two decades ago, E. Ferreira and H. G. Dosch [5, 6] developed the so-called Stochastic Vacuum Model (SVM), a model where the QCD vacuum structure is represented by a stochastic background.

Rather than calculating the quark-quark scattering amplitudes, the basic entities in this model are the Wilson loops. The Wilson loops are gauge-invariant field operators responsible for parallel transport in a closed path. A meson, for instance, can be treated as a quark and an antiquark, following an antiparallel trajectories (see Fig. 1). In this description, it is also necessary to transport color charges within the quark pair to obtain a colorless meson, and this role is played by a Wilson loop.
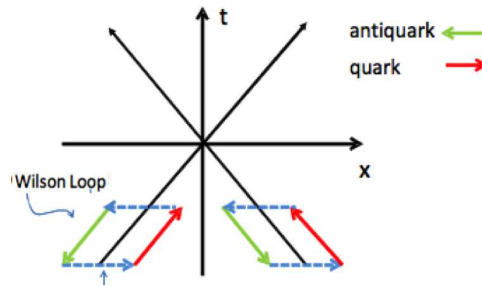

Figure 1. Schematic representation of a meson. Extracted from reference [7].

In order to evaluate the hadron-hadron scattering amplitude, some of the main assumptions of the model can be summarized as:

- the quark in study must be very energetic; thus, when it interacts with the background field it moves on a approximately straight lightlike line and the eikonal approximation is valid;
- the low-frequency contributions to the functional integral can be accounted as a stochastic process (the high-frequency contribution may be considered perturbatively);

The eikonal function is supposed to be linear with the quark-antiquark potential and this potential depends on Wilson expectation value $\langle W(C) \rangle$; only the lowest order on loop expansion $\langle W(C) \rangle$ is kept and it depends explicitly on gluon condensate, the correlation length of the vacuum field fluctuations and string tension.

A very specific feature of the model is the dependence of the cross section on the hadron size even if it is large compared to the correlation length. This effect spoils the quark additivity and is related to the same mechanism that leads to confinement.

Inspired by that formalism, A. K. Kohara, E. Ferreira and T. Kodama worked on a description of the available pp (and $p\bar{p}$) elastic scattering data from $\sqrt{s} \sim 20$ GeV to 7 TeV [8, 9, 10, 11]. The impact parameter representation (b-space) is defined as the Fourier transform of the amplitudes in t-space. The impact parameter $\vec{b}$ is not an observable, but can give important information about geometrical aspects of the collision. In this sense, the amplitudes were built based of the profiles functions in the b-space and are given by

$$\tilde{T}_K(s, \vec{b}) = \frac{\alpha_K}{2\beta_K} e^{-b^2/4\beta_K} + \lambda_K \, \tilde{\psi}(s, b) \tag{1}$$

with the characteristic shape function

$$\tilde{\psi}_K(s, b) = \frac{2 \, e^{\gamma_K - \sqrt{\gamma_K + b^2/a_0}}}{a_0 \sqrt{\gamma_K + b^2/a_0}} \left[1 - e^{\gamma_K - \sqrt{\gamma_K + b^2/a_0}} \right], \tag{2}$$

where $K = R, I$ labels the real and the imaginary parts of the amplitude; $a_0 = 1.39\,\text{GeV}^{-2}$ is related to the gluon correlation length of the correlation function.

The Fourier transform of Eq. 1 determines the real and imaginary scattering amplitudes in t-space

$$T_K^N(s,t) = \alpha_K(s)\, e^{-\beta_K |t|} + \lambda_K(s)\, \Psi_K(\gamma_K(s), t) + \delta_{K,R}\, R_{ggg}(t). \tag{3}$$

$R_{ggg}(t)$ represents the contribution from three-gluon exchange amplitude. It is responsible for the $|t|^{-8}$ tail behaviour in $d\sigma/dt$. The shape functions in t-space are

$$\tilde{\Psi}_K(\gamma_K(s), t) = 2e^{\gamma_K} \left[\frac{e^{-\gamma_K \sqrt{1 + a_0 |t|}}}{\sqrt{1 + a_0 |t|}} - e^{\gamma_K} \frac{e^{-\gamma_K \sqrt{4 + a_0 |t|}}}{\sqrt{4 + a_0 |t|}} \right]. \tag{4}$$

An interesting feature of this model is the fact that the amplitudes in both b and t-space have simple analytical forms. The amplitudes were built in order to be valid in a large $|t|$ range, accounting for the exponential decrease in the very foward region. For a complete analysis of elastic scattering, the Coulomb phase must be taken into account; thus, the amplitudes are modified as

$$T_R(s,t) = T_R^N(s,t) + \sqrt{\pi}\, F^C(t) \cos(\alpha\Phi)$$
$$T_I(s,t) = T_I^N(s,t) + \sqrt{\pi}\, F^C(t) \sin(\alpha\Phi), \tag{5}$$

where α is the fine-structure constant, $\Phi(s,t)$ is the Coulomb phase and $F^C(t)$ is linked to the proton form factor. With the amplitude choice in Eq. 3, the differential cross-section is

$$\frac{d\sigma(s,t)}{dt} = (\hbar c)^2 \left[T_I^2(s,t) + T_R^2(s,t) \right] \tag{6}$$

and the total cross-section,

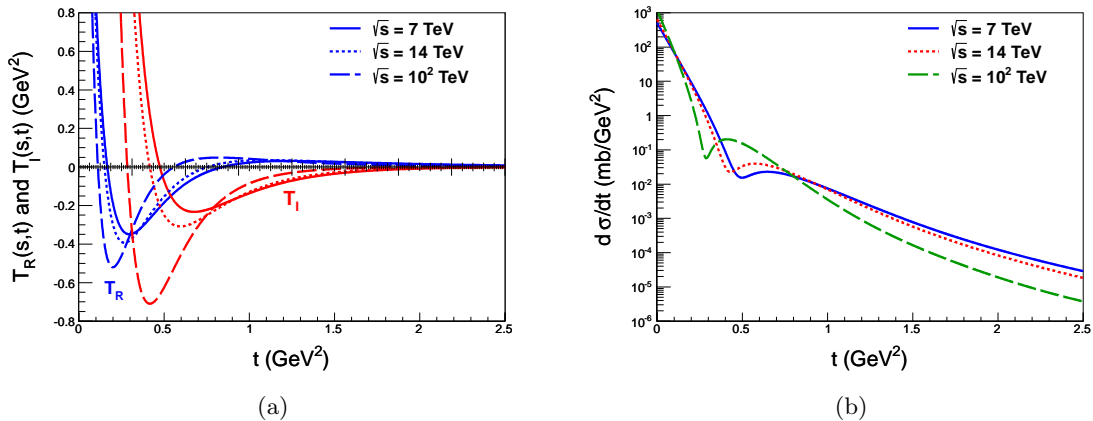$$\sigma_{tot}(s) = (\hbar c)^2 4\sqrt{\pi}\, T_I(s, t=0). \tag{7}$$

Figure 2. SVM predictions for three different energies. In Fig. (a), real and imaginary amplitudes $T_R(s,t)$ and $T_I(s,t)$ are shown and, in Fig. (b), the differential cross-section $d\sigma/dt$.

The energy-dependence of the amplitudes and the cross-section is given through eight energy-dependent parameters, four for each amplitude, $\alpha_K(s)$, $\beta_K(s)$, $\gamma_K(s)$ and $\lambda_K(s)$ [11] (see Fig. 2). Knowing the energy-dependence of the parameters allows us to estimate what happens in the asymptotic energy limit. The total cross-section increases with the energy and the position
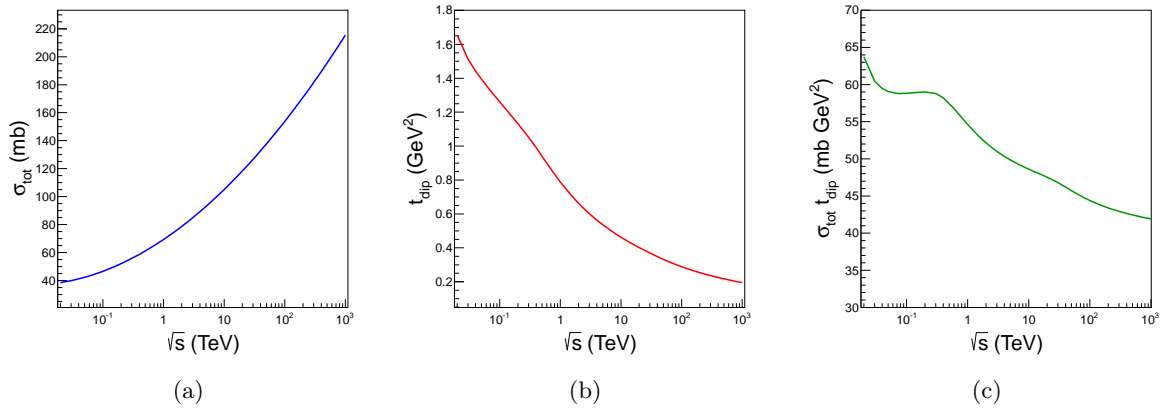
Figure 3. SVM prediction for the energy dependence of the total cross-section σ_{tot} (a), the dip position t_{dip} (b) and the product $\sigma_{tot}\,t_{dip}$ (c).

of the local minimum in the $d\sigma/dt$, $t = t_{dip}$, decreases. It was first noted by T. Csörgo [15] that there is a region where the product $\sigma_{tot}\,t_{dip}$ appears to be constant, but, in the SVM prediction, this seems to be accidental and does not extend for higher energies (see Fig. 3).

The probability of a inelastic interaction decreases smoothly as the impact parameter increases. This behaviour is shown in Fig. 4, where the probability of an inelastic collision $d^2\tilde{\sigma}_{in}/db^2$ is expressed in terms of the impact parameter b and with respect to the scaled variable x

$$x = \frac{b}{\sqrt{2\pi\,\sigma_{tot}(s)}}.$$ (8)

The differential cross-sections $d^2\tilde{\sigma}_{el}/db^2$, $d^2\tilde{\sigma}/db^2$ and $d^2\tilde{\sigma}_{in}/db^2$ are related to the amplitudes in Eq. 1 through

$$d^2\tilde{\sigma}_{el}(s,b)/db^2 = \frac{(\hbar c)^2}{\pi}\,|\tilde{T}(s,b)|^2$$

$$d^2\tilde{\sigma}(s,b)/db^2 = \frac{2}{\sqrt{\pi}}\,(\hbar c)^2\,\tilde{T}_I(s,b)$$

$$d^2\tilde{\sigma}_{in}(s,b)/db^2 = (\hbar c)^2\left(\frac{2}{\sqrt{\pi}}\tilde{T}_I(s,b) - \frac{1}{\pi}|\tilde{T}(s,b)|^2\right).$$ (9)

It is clear that for asymptotic energies, the quantity $d^2\tilde{\sigma}_{in}/db^2$ as a function of x is getting steeper, but does not approach the Heaviside step function. This behaviour is related to the fact that the proton in the SVM does not approach the black disk limit for higher and higher energies.

3. Glauber-Velasco model
The multiple scattering theory was first developed in late 50's by Glauber, and then applied extensively in various areas to calculate high-energy scattering amplitude of composite particles [12].

The proton (or the antiproton) is pictured as a cluster of partons, which pass through each other, and interact through the partonic collisions. The pp scattering amplitude may be written as [13]

$$F(t) = i\int_0^\infty J_0\left(b\sqrt{-t}\right)\left[1 - e^{-\Omega(b)}\right]b\,db,$$ (10)
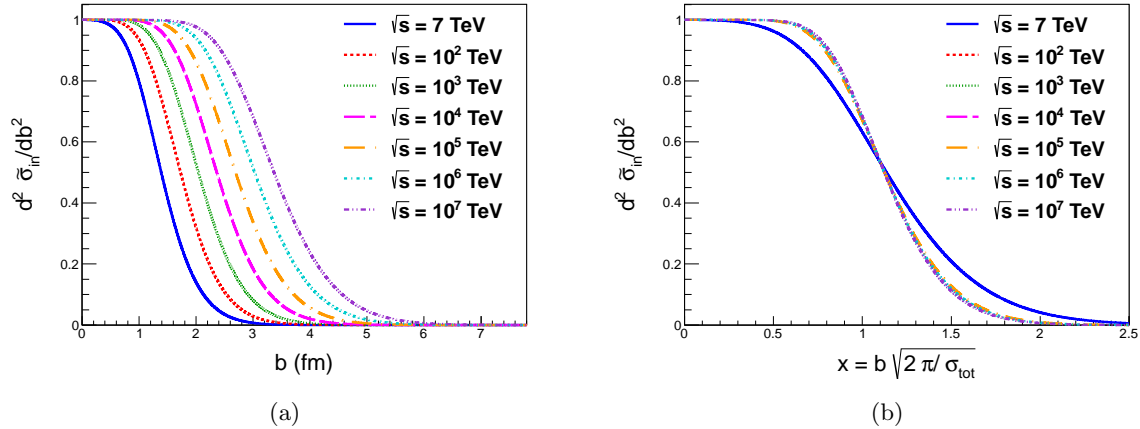
(a)

(b)

Figure 4. Energy evolution of the quantity $d^2\tilde{\sigma}_{in}/db^2$ as a function of the impact parameter b (a) and the variable $x = b/\sqrt{2\pi\,\sigma_{tot}(s)}$ (b).

where $\Omega(b)$ is the opacity function and b is the impact parameter. The elastic differential cross-section is given by $d\sigma_{el}/dt = \pi|F(t)|^2$. Any particular model depends on the choice for $\Omega(b)$, the opacity function (in general, it is complex valued). For two colliding nucleons, that scatter one into another with the parton-parton averaged scattering amplitude $f(t)$, $\Omega(b)$ is given by

$$\Omega(b) = \frac{\kappa}{4\pi}(1-\alpha)\int_0^\infty J_0(b\,q)\,G_{p,E}^2(-t)\frac{f(t)}{f(0)}\,q\,dq. \tag{11}$$

The parton-parton scattering amplitude depends on the parametrization. The parametrization investigated was [15]

$$\frac{f(t)}{f(0)} = \frac{e^{i(b_1|t|+b_2 t^2)}}{\sqrt{1+a|t|}}, \tag{12}$$

where the parameters b_1, b_2 e a can be extrated from the data fit. In total, there are five free parameters in pp cross section data fit (a_1, a_2, b, κ and α). The $G_{p,E}(t)$ proton form adopted was a four pole parametrization [16]

$$G_{p,E}(q^2) = \sum_{i=1}^{n=4}\frac{a_i^E(m_i^E)^2}{(m_i^E)^2 + q^2}, \quad \sum_{i=1}^{n=4}a_i^E = 1, \quad G_{p,E}(0) = 1 \tag{13}$$

with the parameters a_i e m_i kept fixed in the pp elastic cross section fit.

This model has been successfully applied to pp scattering data in TeV domain. Here, we have not performed a data fit, but only reproduced the model with the parameters of [14] for $\sqrt{s} = 7\,\text{TeV}$ and we have not understood the energy dependence yet. In Fig. 5, the amplitudes and the cross section for $\sqrt{s} = 7\,\text{TeV}$ for GV and SVM are plotted. Both models are in good agreement with the TOTEM 7 TeV data [3].

4. Final Comments
The GV and SVM models are in good agreement with the data. In order to better interpret the relation between them, it is fundamental to clarify the energy dependece of the Glauber-Velasco model. The SVM and the GV reveils complementary aspects of QCD, particle and field. As the
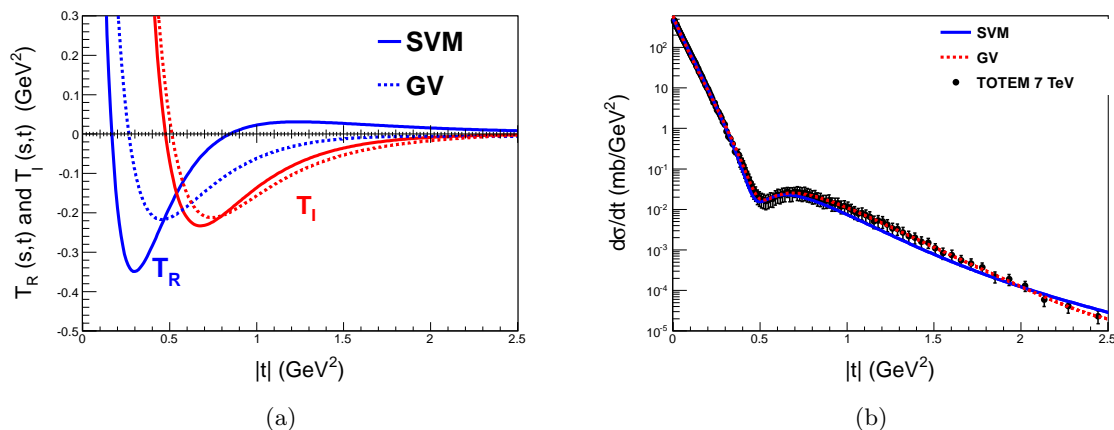
Figure 5. In Fig. (a), real and imaginary amplitudes $T_R(s,t)$ and $T_I(s,t)$ for the SVM and GV for $\sqrt{s} = 7\,\text{TeV}$. In Fig. (b), the differential cross-section $d\sigma/dt$ TOTEM 7 TeV data [3] compared to SVM and GV results.

collision energy increases, the parton-parton interaction range and rhe number of partons are expected to increase as well, which explains the enhacement in the elastic cross section at high energies. As showed in Fig. 3, this enhancement is expected in the SVM.

In the SVM, the proton does not behave as a black disk for asymptotic energies. This behaviour is related to the vacuum property of QCD and the mechanism of confinement and shoud be more profoundly studied. It will be interesting to compare these two models with other models used to describe pp elastic collisions.

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