Matter confinement in light of the Gribov horizon

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Abstract. The procedure of quantization of Landau-gauge Yang-Mills theories that takes into account the presence of Gribov copies has led to the construction of a scenario of gluon confinement via an infrared effective action, the (refined) Gribov-Zwanziger theory. Here we briefly review and discuss a possible extension of this picture to the matter sector of confining Yang-Mills theories.

1. Introduction

The physical mechanism that drives fundamental degrees of freedom of confining quantum field theories to be absent from the physical spectrum is still a subject of intense research. One possible scenario for gluon confinement has been developed over the last years through the consideration of the effects of Gribov gauge ambiguities in the path integral formulation of Yang-Mills theories [1]. What has been observed in Nature over decades of particle and nuclear physics experiments is, however, the confinement of color, i.e. the absence of colored states in the physical spectrum. In fact, even from the theoretical point of view, there are different examples of confining theories in which colored matter fields also turn out to be confined, whenever the respective gauge bosons are (cf e.g. [2]). For Quantum Chromodynamics (QCD), in particular, this means that the description of gluon confinement alone is not enough for an infrared effective theory of QCD. Up to now, (R)GZ is a consistent and successful formalism to address the nonperturbative dynamics and the confinement of gluons in the infrared regime of Yang-Mills theories. An extension of this scenario to the colored matter sector is thus required. Here we shall briefly review a set of results [3, 4] that represent the first steps in the direction of constructing an extension of (R)GZ that presents a nontrivial matter sector, capable of providing a universal picture of color confinement.

This proceedings contribution is organized as follows. We begin the next section by discussing how Gribov copies may affect matter and argue for the necessity of a non-perturbative transmission between gluon and matter sectors. In section 3 a proposal for a unified model of color confinement is presented. A collection of the results obtained with this proposal is discussed in section 4. Finally, section 5 contains a summary and outlook.

2. How Gribov gauge ambiguities affect matter?

Our aim is to construct an effective action for the infrared regime of Yang-Mills theory coupled to colored matter with (R)GZ theory as a starting point. The ultimate goal would be that this new infrared action could encode an adequate non-perturbative background on top of which perturbative calculations may provide reasonable results for observables and eventually improve our understanding of infrared non-perturbative phenomena, such as confinement.

We shall present a universal picture of matter confinement, that would be valid for any type of field in the adjoint or fundamental representations of the gauge group in question. Let us take, however, the case of QCD, namely $SU(N_c)$ non-Abelian Yang-Mills theory coupled to Dirac fermions, in order to have lattice simulations and experimental observations as a robust guide. Our aim is then:

$$\mathcal{L}_{\rm QCD} \quad \overset{\text{quantum non-pert. effects}}{\Rightarrow} \quad \mathcal{L}_{\rm IRQCD} = \mathcal{L}_{\rm RGZ} + \mathcal{L}_{\rm M}\,, \tag{1}$$

where $\mathcal{L}_{\rm M}$ is the lagrangian associated with the effective dynamics of infrared, confined matter that we would like to obtain. For the action $\mathcal{L}_{\rm IRQCD}$ to represent a good model of infrared QCD, it must be consistent with quark and gluon confinement. Moreover, it is desirable that the model propagators are in line with lattice data, encoding thus part of non-perturbative QCD interactions on a nontrivial background even at tree level. In particular, the quark sector should have dynamical mass generation, compatible with the spontaneous breaking of chiral symmetry in the vacuum of QCD.

Since the construction of the (R)GZ framework is based upon accounting for the effect of Gribov gauge copies in the path integral measure, the first question we have to address is whether and how these ambiguities may affect other colored fields, besides the gauge boson itself. The procedure of gauge fixing, either the perturbative Faddeev-Popov one or the non-perturbative case including the Gribov horizon, is fully implemented in the gauge sector, not affecting the matter sector directly. The influence of Gribov copies on colored matter has thus to be transmitted indirectly, via the gauge-matter interaction.

One possibility for $\mathcal{L}_{\rm M}$ in Eq.(1) would be to adopt the standard Dirac term and the minimal coupling between quarks and gluons, $\overline{\psi}(\not{D}+m)\psi$. This choice however does not provide a useful effective theory as a (semi)analytical approach to infrared QCD. Quarks would receive solely perturbative corrections originated by non-perturbative gluons (i.e. Gribov, confined gluons) and would, for example, be present in the physical spectrum (perturbatively) predicted by this action. Moreover, chiral invariance would remain untouched, since the only vertex present would be the minimal coupling and all diagrams constructed with it preserve this symmetry (i.e. give corrections $\propto m$). This is clearly not a good model of infrared QCD, being incompatible with lattice data and experiments.

We conclude that, if Gribov ambiguities indeed provide a possible scenario of confinement, then a *non-perturbative* transmission of their effects to the matter sector is needed. Of course, this is not easy to accomplish analytically. We shall discuss in what follows a proposal that has been put forward, obtaining interesting and consistent results.

3. The inverse Faddeev-Popov operator as a mediator of color confinement

A consistent infrared matter effective lagrangian, $\mathcal{L}_{\rm M}$, should therefore encode a non-perturbative transmission of the effects of Gribov copies already encoded in the gauge sector by the RGZ action[1]:

$$\mathcal{L}_{RGZ} \;=\; \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + b^a\partial_\mu A^a_\mu + \bar{c}^a\partial_\mu D^{ab}_\mu c^b + \mathcal{L}_{\ell H} + \mathcal{L}_R\,, \tag{2}$$

where the restriction to the Gribov region $\Omega = \{A^a_\mu | \partial A = 0, \mathcal{M}^{ab} > 0\}$, involving the Faddeev-Popov operator $\mathcal{M}^{ab} = -\partial_\mu(\delta^{ab}\partial_\mu + f^{abc}A^c_\mu) = -\partial_\mu D^{ab}_\mu$, is implemented by the horizon function

[1] We shall always assume Landau gauge, but this formalism can also be extended to other gauges (cf. e.g. [5, 6]). For details and notations, the reader is referred to Ref.[citar physrep] and references therein.

term

$$\mathcal{L}_H = \gamma^4 \int_y A^a_\mu(x)[\mathcal{M}^{-1}(x,y)]^{ab} A^b_\mu(y) - \gamma^4 D(N_c^2-1) \tag{3}$$

whose localized form reads

$$\mathcal{L}_{\ell H} = \bar{\varphi}^{ac}_\mu \mathcal{M}^{ab} \varphi^{bc}_\mu - \bar{\omega}^{ac}_\mu \mathcal{M}^{ab} \omega^{bc}_\mu + \gamma^2 g f^{abc} A^a_\mu (\varphi^{bc}_\mu + \bar{\varphi}^{bc}_\mu) - \gamma^4 D(N_c^2-1)\,. \tag{4}$$

Here $\bar{\varphi}, \varphi, \bar{\omega}, \omega$ are auxiliary localizing fields, that acquire a nonzero dimension 2 condensate, giving rise, together with the A^2 gluon condensate, to a refinement contribution:

$$\mathcal{L}_R = \frac{m^2}{2} A^a_\mu A^a_\mu + M^2(\bar{\varphi}^{ab}_\mu \varphi^{ab}_\mu - \bar{\omega}^{ab}_\mu \omega^{ab}_\mu)\,, \tag{5}$$

where the parameters m, M can in principle be determined self-consistently thorugh a minimization of the respective effective potential.

One very important feature of (R)GZ theories is that the Becchi-Rouet-Stora-Tyutin (BRST) symmetry is profoundly modified in the infrared. The BRST operator s that is known to leave invariant the perturbative Yang-Mills action turns out to be softly broken by the restriction to the Gribov region: $sS_{(R)GZ} = \gamma^2 \Delta \neq 0$[2]. This property was also verified in lattice simulations [7] via the following signature ($\langle s(\ldots)\rangle \neq 0 \Rightarrow$ BRST breaking):

$$\tilde{\mathcal{Q}}(p^2) = \langle \tilde{\mathcal{R}}^{ab}_\mu \mathcal{R}^{ab}_\mu \rangle \stackrel{\text{GZ}}{=} s(\varphi\bar{\omega}) \stackrel{p\to 0}{\sim} \frac{1}{p^4}\,, \tag{6}$$

where

$$\mathcal{R}^{ab}_\mu(x) = g f^{bcd} \int_y A^d_\mu(y)[\mathcal{M}^{-1}(x,y)]^{ac}\,. \tag{7}$$

From inspection of Eqs.(2)–(7), it is easy to see that the Faddeev-Popov operator (or rather its inverse) is omnipresent in the construction of RGZ theories. It is a strictly non-perturbative quantity whose existence is guaranteed inside the Gribov region. Moreover, it is also deeply connected to the breaking of the perturbative BRST symmetry. Even though the restriction of the measure of integration in the path integral to the Gribov region Ω (in which the Faddeev-Popov operator is positive-definite) is fully implemented in the gauge sector of the RGZ framework, it represents a sharp boundary condition on a quantum operator that carries color indices and shall affect all possible correlation functions involving it. In this sense, it is reasonable to assume that this non-perturbative constraint will indeed affect all fields that present color indices in the theory.

This property is an essential part of our proposal for a picture of color confinement emerging from Gribov physics: the inverse Faddeev-Popov operator shall play a central role as a mediator of non-perturbative effects to all colored fields. The general gauge matter infrared action reads:

$$S_{IR} = S_{YM} + \sum_F M_F \mathcal{H}_F\,, \tag{8}$$

where $\mathcal{H}_F$ is a universal term of the form of the Gribov horizon,

$$\mathcal{H}_F = g^2 \int_{x,y} F^i(x)(T^a)^{ij}[\mathcal{M}^{-1}(x,y)]^{ab} F^k(y)(T^b)^{jk}\,, \tag{9}$$

[2] This breaking can be actually seen as a consequence of a non-perturbative modification of the BRST operator [6]: $s \mapsto s_{np} = s + \gamma^2\delta$, with $s_{np}S_{(R)GZ} = 0$ and $s^2_{np} = 0$, which guarantees gauge-parameter independence of the RGZ framework as well as allows for the extension to other gauges.

that communicates confinement to all colored fields F^i in a given representation of $SU(N_c)$, namely $F^i = A^a_\mu, \psi^i, \phi^a, \ldots$ (gauge boson, fermion, adjoint scalar, etc), via infrared parameters for each sector $M_F = (\gamma^2, \Gamma, \Gamma_\phi, \ldots)$.

The gauge boson case is the only one justified by a first-principles construction in terms of the restriction to the Gribov region. There is no such geometrical interpretation for matter fields, yet. As a consequence, in contrast to the Gribov parameter γ^2, that is fixed by a gao equation, the matter horizon parameters $\Gamma, \Gamma_\phi, \ldots$ are free. In practice, however, for most of the applications all of these quantities will be obtained by fitting lattice data for propagators.

4. Matter confinement scenario: a collection of results

Interesting features of the proposal (8) are:

- a local[3] and renormalizable action (cf. e.g. [8]), that allows for predictive power;
- the reduction to the analogous Yang-Mills theory coupled to matter in the ultraviolet regime. In particular, all infrared parameters M_F have renormalization factors that are fully fixed by the original UV theory [9]. They are thus not new independent parameters, but rather compatible with dynamically generated scales;
- a soft breaking of the perturbative BRST operator. This is a prediction of the model that can be tested by lattice simulations using signatures similar to $\langle \mathcal{R}\mathcal{R} \rangle$ discussed above for gluons.

In the following subsections we shall check further the consistency and predictions of the model.

4.1. Propagators for confined matter: comparison to lattice data

First of all, we would like to confront our proposal with lattice simulations of infrared confining gauge theories. For that purpose we choose the two-point functions for matter fields.

The tree-level propagators obtained from the infrared effective action (8) turn out to be compatible with the available lattice data, signaling that the model captures well the non-perturbative background. Indeed, we have explicitly tested this for quarks and adjoint scalars [4]. Figures 1 and 2 display fits of lattice data that have been performed using the analytic form predicted by our matter confinement model, namely:

$$\langle \phi^a(k)\phi^b(-k)\rangle = \delta^{ab} \frac{k^2 + \mu_\phi^2}{k^4 + (\mu_\phi^2 + m_\phi^2)k^2 + 2N_c g^2 \Gamma_\phi^4 + \mu_\phi^2 m_\phi^2} \tag{10}$$

for scalars in the adjoint representation of the gauge group. Here μ_ϕ is a refinement scale in the scalar sector (i.e. a condensate of dimension 2), whereas m_ϕ is the scalar mass. For Dirac fermions, the propagator reads:

$$\langle \psi^i(k)\bar{\psi}^j(-k)\rangle = \delta^{ij} \frac{i\not{k} + \mathcal{A}(k^2)}{k^2 + \mathcal{A}^2(k^2)} \tag{11}$$

where the quark mass function $\mathcal{A}(k^2)$ depends on the current quark mass m_ψ and the refinement condensate mass μ_ψ:

$$\mathcal{A}(k^2) = m_\psi + \frac{g^2 \Gamma_\psi C_F}{k^2 + \mu_\psi^2}\,. \tag{12}$$

[3] The localization of the matter horizon (9) may be undertaken in analogy to the localization of the Gribov one, presented above in the (R)GZ action. This introduces new localizing auxiliary fields that may also give rise to a refinement in the matter sector due to dimension 2 condensates.

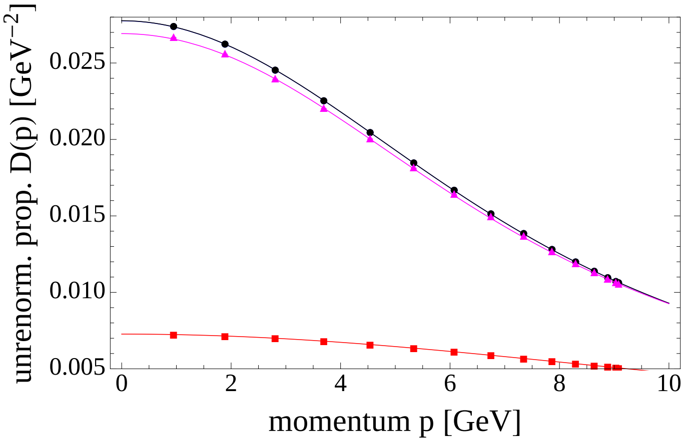

Figure 1. Unrenormalized propagator for different bare masses of the scalar field: $m_{bare} = 0$(top, black), 1 and 10 GeV (bottom, red). The points are preliminary and unpublished lattice data from quenched simulations [11] (for lattice cutoff $a^{-1} = 4.54$ GeV, $N = 30$ and $\beta = 2.698$; cf. also [10] for more details on the lattice setup and measurements) and the curves are the corresponding fits, whose parameter values can be found in Table 1 of [4].

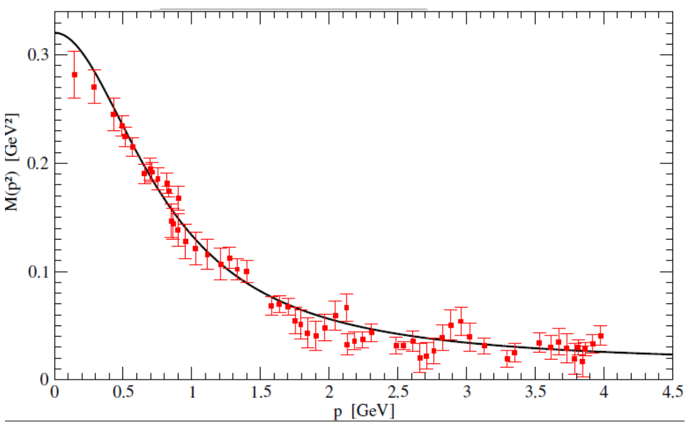

Figure 2. Lattice quark mass function [12] with its fit $\mathcal{A}(p^2)$. Figure extracted from [3]; fit obtained by O. Oliveira [13].

4.2. Absence of colored fields from spectrum: OS positivity violation

A matter confinement scenario should not display asymptotic states of matter colored fundamental fields. The way this appears in our formulation is in the form of a violation of Osterwalder-Schrader's axiom of reflection positivity.

The particle interpretation of the excitation of a given field relies on the existence of a Källen-Lehmann representation with a positive spectral function. This is, however, not achievable when reflection positivity is violated, so that a field with this property will be absent from the physical spectrum predicted by the theory.

This feature has been consistently verified for gluons in the RGZ framework. The case of matter fields displaying a non-Abelian charge was investigated in [4]. The analysis of the adjoint scalar field is completely analogous to the gluon case, since only one nontrivial momentum function may be associated with both scalar and gluon two-point functions, despite the very different tensorial structure. On the other hand, Dirac fermions have a spinorial nature and their two-point function has two independent scalar momentum functions, each of them giving rise to a different spectral density. This is physically related to the fact that a single Dirac fermion field describes 2 types of degree of freedom, associated with different parities.

One can consistently show, as discussed in detail in [3], that there are two positivity conditions to be satisfied by Dirac fermions:

$$\Delta_v(t) \geq 0\,, -\partial_t \Delta_v(t) \geq \Delta_s(t)\,, \tag{13}$$

where the Schwinger functions $\Delta_I(t)$ are related to the vectorial ($I = v$) piece of the propagator ($\propto \gamma_\mu$) and the scalar one ($I = s$). Those conditions are both violated by our model tree-level propagator using the values of the parameters that fit lattice quark data (cf. Fig. 2), as was shown in [3].

4.3. Physical spectrum of matter boundstates

We have seen that the proposed model encodes, through the violation of reflection positivity, the absence of physical asymptotic particles associated with the colored matter fields. The other side of the confinement phenomenon that regards the spectrum is the generation of colorless boundstates out of these nonphysical fundamental excitations.

Considering the case of QCD, one has to address the question about whether this model is capable of producing a physical spectrum of mesons and baryons compatible with experiments and lattice. It is well-known that chiral symmetry aspects of QCD have a great influence on the spectrum structure so this is a crucial test of the compatibility between our color confinement model and another intricate non-perturbative phenomenon in gauge-matter theories. In [3], it has been shown that a formulation of our proposal exists that displays the right chiral properties. As a consequence, the existence of the pion as a pseudo-Goldstone boson with the adequate quantum numbers follows directly. Besides the pion, the formation of another mesonic boundstate was studied. The ρ meson composite correlator was shown to develop a pole with the right spectral properties to be interpreted as a physical particle. Reasonable estimates of its mass and decay constant were obtained without resorting to any hadronic input; the only parameters used have been extracted from lattice fits of the quark and gluon propagators.

5. Summary and outlook

We have briefly reviewed a proposal that extends the infrared effective actions obtained through the incorporation of the effect of Gribov copies in order to describe also the confinement of matter. A universal scenario of color confinement in gauge matter theories is constructed, with several consistent predictions obtained: (i) propagators in line with lattice data for Dirac fermions and adjoint scalars, (ii) positivity violation, indicating confinement, (iii) the existence of physical boundstates, qualitatively compatible with the chiral symmetry structure in QCD. This same proposal has been applied to describe the Gribov problem in supersymmetric confining theories, with results consistent with other (exact) non-perturbative approaches [14, 15].

Other promising applications of the current proposal are underway. The thermodynamics of a confining quark model defined by the leading order approximation of the model here discussed, has been investigated. All thermodynamic quantities were shown to be non-trivial and well-behaved, despite the unphysicalness (positivity violation) of the confined fundamental degrees of freedom (cf. [16] and B. Mintz contribution for these proceedings).

Acknowledgments

The author thanks M. Capri, D. Dudal, D. Fiorentini, D. Granado, M. Guimaraes, I. Justo, B. Mintz, and S. Sorella for collaboration. L.F.P. is supported by a BJT fellowship from the brazilian program *Ciência sem Fronteiras* (grant number 301111/2014-6).

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