

# Charmless Three-Body B-decays: final state interaction and CP violation

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**Abstract.** We obtain an explicit expression for the charge-parity violation (CPV) considering final state interactions (FSI) and the  $\rho(770)$  and  $f_0(980)$  resonances in the  $B^\pm \rightarrow \pi^\pm \pi^+ \pi^-$  decay. In addition, we investigate the channel coupled by the strong interaction  $B^\pm \rightarrow \pi^\pm K^+ K^-$ , dictated by the CPT constraint. We use our model to fit experimental data of such decays [1]. For the interfering resonant contributions to the CP asymmetry, we show that locally CPT constraint seems to be valid in the  $B^\pm \rightarrow \pi^\pm \pi^+ \pi^-$  channel. Our work suggests, in agreement with the CPT constraint, that the CP asymmetry in coupled channels are related and have opposite signs. Our formula for the CP asymmetry fairly fit the LHCb data improving our understanding of the interplay between the FSI and CP violation. For more complete and detailed studies for the channels  $K^\pm K^+ K^-$ ,  $K^\pm \pi^+ \pi^-$ ,  $\pi^\pm K^+ K^-$  and  $\pi^\pm \pi^+ \pi^-$  see Ref. [2].

## 1. Introduction

CP violation (CPV) in charmless three-body  $B$  decays is related with interferences between the weak and strong phases. One type is based in the BSS model [3] produced through the interference in the same intermediate state of the tree and penguin amplitudes. Other is the FSI coupling different final states with distinct weak phases and this interference between channels producing CPV, where a relevant property is the presence of the CPT constrain [4]. Finally, interferences in the Dalitz plot between neighbor resonances that share the same phase space region [5, 6], can also generate CPV. Induced CPV by interference of scalar and vector resonances brings important contributions in charmless  $B$  decays, as we can see in Ref. [7, 8].

When CPV induce different individual partial decay widths, the CPT invariance implies other channels with the same amount of CPV with opposite sign. The mechanism to couple different channels distributing the CPV is the final state interaction (FSI). Usually, it is not common the explicit consideration of CPT invariance in the study of CP violation, assuming the coupling between multiple channels. However, in previous work [4] it was shown that for charmless three-body  $B$  decays the coupling is between particular channels not being necessary to consider many channels. An important process to consider between 1 and 1.6 GeV is the re-scattering process related with the S-wave  $\pi\pi \rightarrow KK$  amplitude [9], which can be used to couple specific channels.



Based in experimental evidences, the two-body re-scattering can contribute to CP violation in the strongly coupled channels  $\pi\pi$  and  $KK$ .

Here we use CPT constraint, final state interaction and resonances in order to construct a CP asymmetry formula for leading order in strong interaction, i.e. in scattering matrix. The ingredients considered to build the CP formula are the  $\rho$  and  $f_0(980)$  resonances plus a non-resonant background. By writing the CP asymmetry in a convenient way, we identify each type of interference in terms of a free parameter to fit the LHCb data [1]. We have considered only two-body scattering contributions in this work, discarding three-body re-scattering effects. However, some analysis of the Dalitz plot of the  $D^+ \rightarrow K^- \pi^+ \pi^+$  decay suggests that this contribution must be important [10]. Our study considered the  $B^\pm \rightarrow \pi^\pm \pi^+ \pi^-$  decay, separating the phase space in two regions as presented in the experimental results. With the convenient parameters obtained performing the fit, we plot the strongly coupled channel  $\pi^\pm K^+ K^-$  comparing our result with the experimental data.

## 2. Deriving the CP asymmetry formula

Initially, we want to explicitly build the decay amplitude and then the CP asymmetry considering as key elements the CPT constraint, the coupling between channels by the strong interaction through FSI and, finally, the resonances associated with the decay.

### 2.1. CPT constraint and FSI

Applying the  $CPT$  operator in a hadron state we get  $CPT|h\rangle = \chi|\bar{h}\rangle$ , where  $|h\rangle$  is a hadron state,  $\bar{h}$  is the charge conjugate state and  $\chi$  a phase. Due to the fact that CPT is a fundamental symmetry of the strong and weak interactions, then,  $(CPT)^{-1} H CPT = H$ , where  $H$  can be the weak ( $H_w$ ), or the strong Hamiltonian ( $H_s$ ). The use of CPT constraint in the weak matrix element for a hadron decay, i.e.  $\langle\lambda_{out}|H_w|h\rangle$ , gives

$$\langle\lambda_{out}|H_w|h\rangle = \chi_h \chi_\lambda \langle\bar{\lambda}_{in}|H_w|\bar{h}\rangle^* . \quad (1)$$

where the presence of the final state interaction related with the distortion from the strong force is included in  $\lambda_{out}$ . We can rewrite Eq. (1) by using the hermiticity of the weak Hamiltonian and the completeness relation of the strong Hamiltonian eigenstates:

$$\langle\lambda_{out}|H_w|h\rangle = \chi_h \chi_\lambda \sum_{\bar{\lambda}'} S_{\bar{\lambda}',\bar{\lambda}} \langle\bar{\lambda}'_{out}|H_w|\bar{h}\rangle^* , \quad (2)$$

where  $S_{\bar{\lambda}',\bar{\lambda}} = S_{\lambda',\lambda} = \langle\bar{\lambda}'_{out}|\bar{\lambda}_{in}\rangle$  is the S-matrix element.

By considering in Eq. (2) the equality between the sum of partial decay widths of all decay channels of  $h$  and  $\bar{h}$ , and also the hermiticity of  $H_w$ , we get

$$\sum_{\lambda} |\langle\lambda_{out}|H_w|h\rangle|^2 = \sum_{\bar{\lambda}} \left| \sum_{\bar{\lambda}'} S_{\bar{\lambda}',\bar{\lambda}}^* \langle\bar{\lambda}'_{out}|H_w|\bar{h}\rangle \right|^2 = \sum_{\bar{\lambda}} |\langle\bar{\lambda}_{out}|H_w|\bar{h}\rangle|^2 . \quad (3)$$

The decay amplitude can be written in two pieces, namely,  $\mathcal{A}^\pm = A_\lambda + B_\lambda e^{\pm i\gamma}$ . Here, the complex amplitudes  $A_\lambda$  and  $B_\lambda$  are invariant under CP. Furthermore, CP transformation only change the sign of the weak phase  $\gamma$ . As we know by the BSS mechanism [3], the CP-violating phase enters linearly at lowest order in the hadron decay amplitude. We can now identify the following relations:  $\mathcal{A}^- = \langle\lambda_{out}|H_w|h\rangle$ , and  $\mathcal{A}^+ = \langle\bar{\lambda}_{out}|H_w|\bar{h}\rangle$ , which means that  $A_\lambda$  and  $B_\lambda$  contains the strongly interacting final-state channel. With these considerations in the CPT constraint, Eq. (2), the decay amplitude can be written as

$$A_\lambda + e^{\mp i\gamma} B_\lambda = \chi_h \chi_\lambda \sum_{\lambda'} S_{\lambda',\lambda} \left( A_{\lambda'} + e^{\pm i\gamma} B_{\lambda'} \right)^* , \quad (4)$$

which implies a relation between  $A_\lambda$  or  $B_\lambda$  with their respective complex conjugates.

The strong scattering amplitude of  $\lambda' \rightarrow \lambda$ , namely,  $t_{\lambda',\lambda}$ , is useful in a decay with channels coupled by rescattering and can be related with the S-matrix through  $S_{\lambda',\lambda} = \delta_{\lambda',\lambda} + i t_{\lambda',\lambda}$ .

In the decay amplitudes  $A_\lambda$  and  $B_\lambda$ , there are effects from the FSI distortion. In order to take into account only the leading order of such effects, we rewrite the decay amplitudes as  $A_\lambda \rightarrow A_{0\lambda}$  and  $B_\lambda \rightarrow B_{0\lambda}$ . Since we will introduce resonances in the  $\pi\pi$  channel, like the  $\rho(770)$  or  $f_0(980)$ , let us consider a different notation to decompose in angular momentum states the source decay amplitudes  $A_{0\lambda}^J$ ,  $B_{0\lambda}^J$  and the scattering matrix  $t_{\lambda',\lambda}^J$ . The decomposed decay amplitude, retaining terms up to leading order (LO) in  $t_{\lambda',\lambda}$  is

$$\mathcal{A}_{LO}^\pm = \sum_J (A_{0\lambda}^J + e^{i\gamma} B_{0\lambda}^J) + i \sum_{\lambda',J} t_{\lambda',\lambda}^J (A_{0\lambda'}^J + e^{i\gamma} B_{0\lambda'}^J), \quad (5)$$

where the relations  $A_{0\lambda} = \chi_h \chi_\lambda A_{0\lambda}^*$  and  $B_{0\lambda} = \chi_h \chi_\lambda B_{0\lambda}^*$  were used.

With the amplitude in Eq. (5) it is possible to compute the CP asymmetry  $\Delta\Gamma_\lambda = \Gamma(h \rightarrow \lambda) - \Gamma(\bar{h} \rightarrow \bar{\lambda})$  as

$$\Delta\Gamma_\lambda = 4(\sin \gamma) \sum_{JJ'} \text{Im}\{(B_{0\lambda}^J)^* A_{0\lambda}^{J'} + i \sum_{\lambda'} [(B_{0\lambda}^J)^* t_{\lambda',\lambda}^{J'} A_{0\lambda'}^{J'} - (B_{0\lambda'}^{J'} t_{\lambda',\lambda}^{J'})^* A_{0\lambda}^J]\}, \quad (6)$$

where the external sum of  $\lambda'$  represents each channel coupled by the strong interaction to  $\lambda$  separately. The first term in Eq. (6) is related to the interference between two CP conserving amplitudes without FSI and, as a consequence of the CPT constraint, must satisfy  $\sum_\lambda \text{Im} [(B_{0\lambda}^J)^* A_{0\lambda}^J] = 0$ . The ‘‘compound’’ CP asymmetry, related to the second and third terms in the imaginary part, has the important property of canceling each other when summed to all FSI.

## 2.2. Resonances

In order to consider the formation of resonances in the partonic process, it is useful to separate the source amplitudes in resonant  $R$  and non-resonant  $NR$  parts as

$$A_{0\lambda}^J = A_{0\lambda NR}^J + \sum_R A_{0\lambda R}^J \quad \text{and} \quad B_{0\lambda}^J = B_{0\lambda NR}^J + \sum_R B_{0\lambda R}^J. \quad (7)$$

The resonant parts will consider all the resonances in the  $\lambda$  channel, but it is not considering yet the two-hadron rescattering process and are read as bare amplitudes. The next step to put resonances in Eq. (6) is identify the terms with Breit-Wigner amplitudes as

$$(1 + i t_{\lambda\lambda}^J) A_{0\lambda R}^J \rightarrow a_0^R F_{R\lambda}^{BW} P_J(\cos \theta) \quad \text{and} \quad (1 + i t_{\lambda\lambda}^J) B_{0\lambda R}^J \rightarrow b_0^R F_{R\lambda}^{BW} P_J(\cos \theta), \quad (8)$$

where  $J$  is the spin of the resonance and  $P_J(\cos \theta)$  is the Legendre polynomial with  $\theta$  being the angle between the equally charge particles in the  $B^\pm \rightarrow \pi^\pm \pi^+ \pi^-$  decay in the  $\pi^+ \pi^-$  frame. We denote the unpaired pion as  $\pi'^\pm$ .

Now we are able to develop the asymmetry formula and use it to fit the experimental data. The relevant sources to consider in the  $B^\pm \rightarrow \pi^\pm \pi^+ \pi^-$  decay, at low invariant  $\pi^+ \pi^-$  mass, are the non-resonant amplitude, the vector and scalar resonances, namely,  $\rho(770)$  and  $f_0(980)$ , and the coupling between  $\pi\pi$  and  $KK$  (two strongly interacting channels). The amplitudes with such features are

$$\mathcal{A}_{0\lambda}^\pm = a_0^\rho F_\rho^{BW} k(s) \cos \theta + a_0^f F_f^{BW} + a_{0\lambda}^{nr} + b_{0\lambda}^{nr} e^{\pm i\gamma} + [b_0^\rho F_\rho^{BW} k(s) \cos \theta + b_0^f F_f^{BW}] e^{\pm i\gamma}, \quad (9)$$

where the factor  $k(s) = \sqrt{1 - \frac{4m_\pi^2}{s}}$  is used to consider the threshold behavior of the  $p$ -wave. Since in this case we have a  $J = 1$  particle  $\cos\theta$  vary from  $-1$  to  $+1$ .

We only consider the inelastic process, namely,  $KK \rightarrow \pi\pi$  in the part of the CP asymmetry, Eq. (6), with the scattering matrix, written as  $t_{\pi\pi, KK}^{J=0}$  in this case. We approximate the kaon-kaon S-wave phase-shift as  $\delta_{KK} \approx \delta_{\pi\pi}$  between 1 and 1.6 GeV, using  $t_{\lambda', \lambda}^{J=0} = \sqrt{1 - \eta^2} e^{i(\delta_{\pi\pi} + \delta_{KK})} \approx \sqrt{1 - \eta^2} e^{2i\delta_{\pi\pi}}$ , where the parametrizations for the pion-pion phase-shift and inelasticity are taken from [11].

By expanding the CP asymmetry with all the considerations discussed above, and writing the parameters in a convenient form to proceed with the fitting to the experimental data, it is possible to use some relations with complex numbers to write

$$\begin{aligned} \Delta\Gamma_\lambda &= \mathcal{B}\sqrt{1 - \eta^2(s)} \cos[2\delta_{\pi\pi}(s)] + |F_\rho^{\text{BW}}(s)|^2 k(s) \cos\theta \left\{ \mathcal{D}(m_\rho^2 - s) + \mathcal{E}m_\rho\Gamma_\rho(s) \right\} + \\ &+ |F_\rho^{\text{BW}}(s)|^2 |F_f^{\text{BW}}(s)|^2 k(s) \cos\theta \times \\ &\times \left\{ \mathcal{F}[(m_\rho^2 - s)(m_f^2 - s) + m_\rho\Gamma_\rho(s)m_f\Gamma_f(s)] + \mathcal{G}[(m_\rho^2 - s)m_f\Gamma_f(s) - m_\rho\Gamma_\rho(s)(m_f^2 - s)] \right\}, \end{aligned} \quad (10)$$

where all the parameters  $\mathcal{B}$ ,  $\mathcal{D}$ ,  $\mathcal{E}$ ,  $\mathcal{F}$  and  $\mathcal{G}$  are related only with the partonic amplitudes  $A_{0\lambda}^J$  and  $B_{0\lambda}^J$  and with the phase  $\gamma$ . The expression in Eq. (10) contains terms involving the non-resonant background interfering with the resonances  $\rho(770)$  and  $f_0(980)$ , and with the S-wave  $KK \rightarrow \pi\pi$  amplitude. It also presents interference between the two resonances. We also remark that all the terms that locally violate CPT was completely ignored and, as in the previous work [4], the phase inside the cosine function of the  $KK \rightarrow \pi\pi$  amplitude was chosen to be zero.

The expression that we have for the CP asymmetry in Eq. (10) is not ready yet to be compared with the experimental data [1] because one Dalitz variable still need to be eliminated by integration. The experimental data is given in terms of the low invariant mass  $\sqrt{s} = m_{\pi^+\pi^-}$  of the paired pions, and the phase space is separated in two regions namely, that for  $\cos\theta < 0$  and  $\cos\theta > 0$ . It is not hard to perform the integration  $\Delta\Gamma(s) \equiv \int \Delta\Gamma_\lambda(s, m_{\pi^+\pi^+}^2) dm_{\pi^+\pi^+}^2 = \Delta\Gamma(s)^{(\cos\theta < 0)} + \Delta\Gamma(s)^{(\cos\theta > 0)}$  eliminating the dependence in the mass pair with the bachelor pion  $m_{\pi^+\pi^+}^2$  and separating the phase space.

### 3. Using the CPV formula for $B^\pm \rightarrow \pi^\pm\pi^+\pi^-$ and $B^\pm \rightarrow \pi^\pm K^+K^-$ decays

Now we can perform the fit of the experimental data for the  $B \rightarrow \pi\pi\pi$  presented in Ref. [1] with integrated Eq. (10). It is important to note that our decay amplitude was not symmetrized and this should be necessary due to the presence of identical particles in the final state. The experimental data for the different regions  $\cos\theta < 0$  and  $\cos\theta > 0$  present different patterns and the reason is that the symmetrical terms must interfere and affect more significantly the CPV in the  $\cos\theta < 0$  region. It is clear to see if we look to the Dalitz plot for the  $B \rightarrow \pi\pi\pi$  decay in Ref. [1] or if we draw the decay diagrams for  $\cos\theta < 0$  and  $\cos\theta > 0$ .

The masses and widths used here for the  $\rho(770)$  and  $f_0(980)$  resonances were taken from the E791 experiment [12] and from the particle data group [13]. All free parameters used in our model are compatible with GeV units.

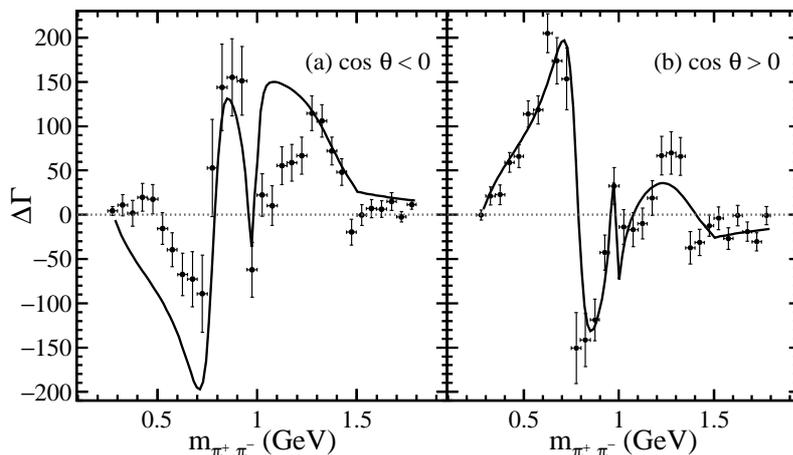
#### 3.1. $B \rightarrow \pi\pi\pi$ decay

Since our expression does not consider symmetrization, we perform the fit for  $\cos\theta > 0$ , region with less interference from the symmetrical term, and use the parameters obtained to plot the CP asymmetry for  $\cos\theta < 0$ . We also use the value of the parameter  $\mathcal{B}$  obtained from the fit, and related to the  $\pi^+\pi^- \rightarrow K^+K^-$  amplitude, to plot the coupled channel  $B^\pm \rightarrow \pi^\pm K^+K^-$ .

This is consistent with the CPT constraint for the coupled  $\pi\pi\pi$  and  $\pi KK$  channels above the  $KK$  threshold, which says that  $\Delta\Gamma_{\pi\pi\pi} = -\Delta\Gamma_{\pi KK}$ .

We have some ingredients in our formula to the CP asymmetry and it is important to know which amplitudes are more relevant in the  $B \rightarrow \pi\pi\pi$  decay. For such a decay, we have a large effect due to the  $\rho$  resonance in comparison with the  $f_0(980)$  one, but both are present in the Dalitz plot. The best fit for this region was found by using the amplitude with the parameter  $\mathcal{D}$ , related to the interference between the real part of the  $\rho$  resonance and the non-resonant partonic amplitude. In other words, the real part of the  $\rho$  resonance is related to the real part of the Breit-Wigner function. For the region around 1 GeV, the interference between the  $\rho(770)$  and  $f_0(980)$  resonances is important. For this reason, the asymmetry term with the  $\mathcal{G}$  parameter, that represents this effect, is also present in Eq. (10). Above the  $KK$  threshold, the  $KK \rightarrow \pi\pi$  rescattering predominates, and term with the  $\mathcal{B}$  parameter encompasses such an effect.

The comparison of the integrated asymmetry (integrated Eq. 10) with the experimental data is presented in Fig. 1b. The same fitted parameters obtained in the fit procedure of Fig. 1b were used to plot the full curve of Fig. 1a for the  $\cos\theta < 0$  case. The experimental data presented here were extracted from Fig. 4c of Ref. [1].



**Figure 1.** Integrated CP asymmetry (full curves) of the  $B^\pm \rightarrow \pi^\pm\pi^+\pi^-$  decay, compared with experimental data (symbols) taken from Ref. [1]. Results for (a)  $\cos\theta < 0$  and (b)  $\cos\theta > 0$ .

The terms considered in Eq. (10) for the CP asymmetry vanishes after their integration in  $\cos\theta$  and sum over the  $\cos\theta > 0$  and  $\cos\theta < 0$  cases. This is not true, however, for the term related to the  $KK \rightarrow \pi\pi$  amplitude. This term is related to the region between 1 and 1.6 GeV of the invariant mass pair  $m_{\pi^+\pi^-} = \sqrt{s}$  and cancels the corresponding asymmetry term of the coupled channel  $\pi^\pm K^+ K^-$ .

We remind that we are using here a empirical parametrization for the pion-pion phase-shift and inelasticity from Ref. [11] with no consideration of the errors, but only using the central values given there. These must be a large source of uncertainty in the region between 1 and 1.6 GeV. Other approximation that must affect our calculations is the one we made for the  $KK$  phase-shift, namely,  $\delta_{KK} \approx \delta\pi\pi$ . This should be improved in the future in order to become our description even more realistic.

Other clear difference between our model and the experimental data in Fig. 1a is seen below the region of the  $\rho$  mass resonance. It is possible to relate such a difference to the missing symmetrization. Other way to improve our model in that region is to consider the S-wave  $\pi\pi$  elastic scattering, since at the region below  $\rho$  mass, the  $f_0(600)$  resonance, namely,  $\sigma$ , will affect directly the results [14].

### 3.2. $B \rightarrow \pi KK$ decay

In this decay, we are only treating the region between 1 and 1.6 GeV, related to the  $KK \rightarrow \pi\pi$  amplitude. We have to derive the CP asymmetry for the coupled channel, but again, we will discard terms that do not obey CPT locally. The only term related to the coupled channels is that representing the inelastic scattering interfering with the non-resonant amplitude. The integration in  $\cos\theta$  of such a term gives

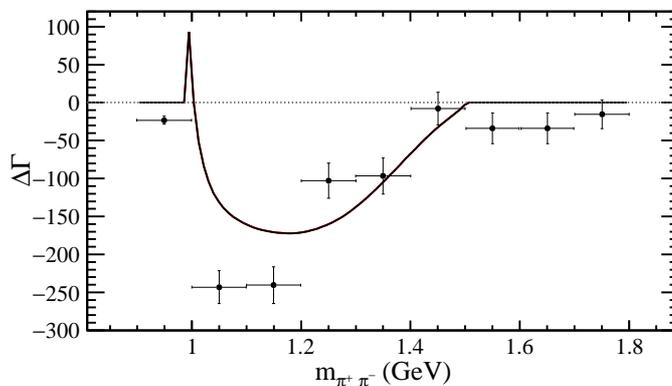
$$\Delta\Gamma(s) = -\frac{2\mathcal{B}\sqrt{1-\eta^2(s)}\cos[2\delta_{\pi\pi}(s)]}{a'(s)\sqrt{s-4m_K^2}}, \quad (11)$$

where the integration brings the following kinematical factor,

$$a'(s) = \frac{1}{(s-4m_K^2)^{1/2} \left[ \frac{(M_B^2 - m_\pi^2 - s)^2}{4s} - m_\pi^2 \right]^{1/2}}, \quad (12)$$

with  $\sqrt{s-4m_K^2}$  being the kaon momentum in the rest frame of the  $KK$  subsystem. This is necessary to make valid the relation  $\Delta\Gamma_{\pi\pi\pi} = -\Delta\Gamma_{\pi KK}$ , with  $\Delta\Gamma_{\pi\pi\pi}$  being the asymmetry in Eq. (10) after the integration in  $\cos\theta$  and the sum in  $\cos\theta > 0$  and  $\cos\theta < 0$ , and  $\Delta\Gamma_{\pi KK}$  given in Eq. (11).

Now we are able to use the parameter related with the  $KK \rightarrow \pi\pi$  amplitude, namely  $\mathcal{B}$ , fitted by the CP asymmetry data in the  $B^\pm \rightarrow \pi^\pm\pi^+\pi^-$  decay for  $\cos\theta > 0$ , to perform the plot for the coupled channel  $\pi^\pm K^+ K^-$  and compare with the experimental data from Ref. [1]. This is shown in Fig. 2.



**Figure 2.** CP Asymmetry of the  $B^\pm \rightarrow \pi^\pm K^+ K^-$  decay given in Eq. (11) (full curve) compared with the experimental data (symbols) taken from Fig. 7b of Ref. [1].

Notice the good agreement of our model in comparison with the experimental data. However, we remind the reader on the direct influence of the approximation used for the  $KK$  phase-shift, and also the discard of the errors of the  $\pi\pi$  phase-shift and inelasticity given in [11], since we only used the central values of the parameters. This procedure generates a large source of uncertainty, as previously mentioned, and has a significant influence in our results.

The experimental data for the  $B^\pm \rightarrow \pi^\pm K^+ K^-$  decay was presented as a sum of all events, and not separated in two regions ( $\cos\theta > 0$  and  $\cos\theta < 0$ ) as we did in the  $B^\pm \rightarrow \pi^\pm\pi^+\pi^-$  case. The larger amount of events in a region of  $\pi^+\pi^-$  and  $K^+K^-$  invariant mass below 1.6 GeV in the  $B^\pm \rightarrow \pi^\pm\pi^+\pi^-$  decay is related to the same region in the  $B^\pm \rightarrow \pi^\pm K^+ K^-$  decay, as

Fig. 2 shows. Therefore, CPT is conserved both: locally, when we integrate the terms in CP asymmetry in  $\cos\theta$ , and when we sum all CP asymmetry contributions involving the coupled channels.

#### 4. Conclusions

We studied the CP violation in charmless  $B^\pm$  tree-body decays in channels coupled by the strong interaction, namely,  $\pi^\pm\pi^+\pi^-$  and  $\pi^\pm K^+K^-$ . We set to zero all CP asymmetry terms that locally violate CPT, and performed the fit for the  $\pi^\pm\pi^+\pi^-$  channel in the  $\cos\theta > 0$  region. The reason for this choice comes from the fact that such a region seems to be less affected by the Bose symmetrization of the decay amplitude, that we did not consider in this work. The  $\cos\theta < 0$  region was described by the same expression including the previous fitted parameters. The expression for the CP asymmetry was constructed here also with the inclusion of relevant resonances in the  $B^\pm \rightarrow \pi^\pm\pi^+\pi^-$  decay, namely, the  $\rho(770)$  and  $f_0(980)$  resonances.

Our model showed a good agreement with the experimental data with three terms of the derived CP formula, one for each region:  $\rho(770)$  and  $f_0(980)$  resonances and the  $KK \rightarrow \pi\pi$  amplitude. We also presented good results for the coupled channel  $B^\pm \rightarrow \pi^\pm K^+K^-$ , showing the importance of final state interactions coupling specific channels. It is worth noting two specific approaches in our model, namely, the not consideration of the Bose symmetrization of the decay amplitude, and the approximation for the  $KK$  phase shift. We should improve this points in our model in next works. The use of different interference terms in the CP asymmetry expression, and a better understanding of the microscopical amplitude, are also relevant topics we intend to explore in future investigations.

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