Hadrons in AdS/QCD models

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Abstract. We review some recent results on hadrons using holographic models for strong interreactions inspired in the AdS/CFT correspondence, also known as AdS/QCD models.

1. Introduction

The AdS/CFT correspondence relates string theory in 10 dimensions with Yang-Mills theories in 4 dimensions. In particular, the string theory can be defined on $AdS_5 \times S^5$ space, where AdS_5 is a 5 dimensional anti-de Sitter space, and S^5 is a 5 dimensional hypersphere. If we choose a situation where the S^5 part of the model is not excited, one has an effective AdS_5/CFT_4 correspondence, where CFT_4 is a 4 dimensional conformal field theory. Actually, this CFT_4 is a superconformal Yang-Mills theory in 4 dimensions. Since we want to describe strong interactions, we need a non-conformal field theory in 4 dimensions. So, one has to break the conformal symmetry of this theory. One simple way to do this is to consider the AdS_5 space, given by the metric:

$$ds^2 = \frac{R^2}{z^2} \left(dz^2 + d\vec{x}^2 - dt^2 \right) \tag{1}$$

and restrict the radial coordinate z to the interval $[0, z_{max}]$. This is known as the hardwall model proposed in refs. [1, 2, 3, 4], since it has a hard cutoff at $z = z_{max}$. Antoher holograhic model is the softwall model where a dilaton field is introduced in the action integral [5]:

$$\mathcal{I} = \int d^5x \, e^{-kz^2} \, \mathcal{L} \tag{2}$$

and plays the role of a soft cutoff. The great appeal of this model is that it implies *linear* Regge trajectories. For instance, for scalar glueballs the predicted trajectory in this model is given by [6]

$$m^2 = k(4n + 8) \qquad (n = 0, 1, 2, ...) \tag{3}$$

although linear the comparison with glueball masses is not good. Phenomenologically, if one introduces anomalous dimensions for the scalar glueball operators in the softwall model [7] the results for the masses are better, but the trajectories are no longer linear.

Other holographic models use the geometry of hypermembranes with Dirichlet boundary condition, or simply D-branes. This is also useful because it gives flavor to the models [8]. In particular, very interesting models are the D3-D7 [9] and the D4-D8 [10] ones. For an excellent review see [11].

2. Some results from the Hardwall Model

Besides the initial proposal of the Hardwall model [1, 2, 3, 4], we will describe shortly other results that were obtained from this holographic model.

2.1. Glueballs in the Hardwall model and the Pomeron

In ref. [12] we discussed how to obtain the glueballs masses for the states J^{PC} with even J and $P = C = +1$, using Dirichlet and Neumann boundary conditions on the cutoff at $z = z_{max}$. In particular, the best results were found for the Neumann boundary condition, which gives the following Regge trajectory:

$$J = (0.26 \pm 0.02)\text{GeV}^{-2}m^2 + 0.80 \pm 0.40 \tag{4}$$

in good agreement with the Pomeron trajectory. In Figure 1 we show the Regge trajectories obtained from the Hardwall model for J^{++} states associated with the pomeron.
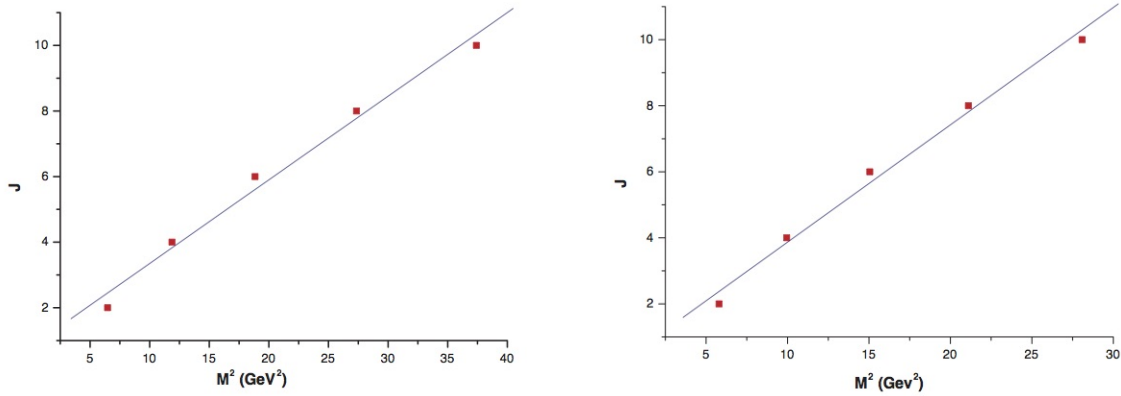
Figure 1. Approximate linear Regge trajectories for Neumann (left panel) and Dirichlet (right panel) boundary conditions for the states $2^{++}, 4^{++}, 6^{++}, 8^{++}, 10^{++}$, associated with the pomeron.

2.2. Odd spin glueballs in the Hardwall model and the Odderon

In ref. [13] it was shown that the Hardwall model can give the odd spin glueball masses that can be related to the Odderon Regge trajectories. In particular, we obtained the trajectories:

$$J = -(0.83 \pm 0.40) + (0.22 \pm 0.01)m^2 \; ; \qquad J = (0.34 \pm 0.37) + (0.17 \pm 0.01)m^2 \,. \tag{5}$$

The experimental trajectories for the Odderon are not known. Any way, these trajectories are in agreement with the ones predicted by relativistic and non-relativistic models for the Odderon. See ref. [13] for details. In Figure 2 we show some of these trajectories for the Odderon.

Besides the experimental quest for the Odderon, there is also a question if the state 1^{--} belongs or not to the Odderon Regge trajectories. The hardwall approach to this problem could not solve this debate, since we could reproduce the expected Odderon properties with and without this state.
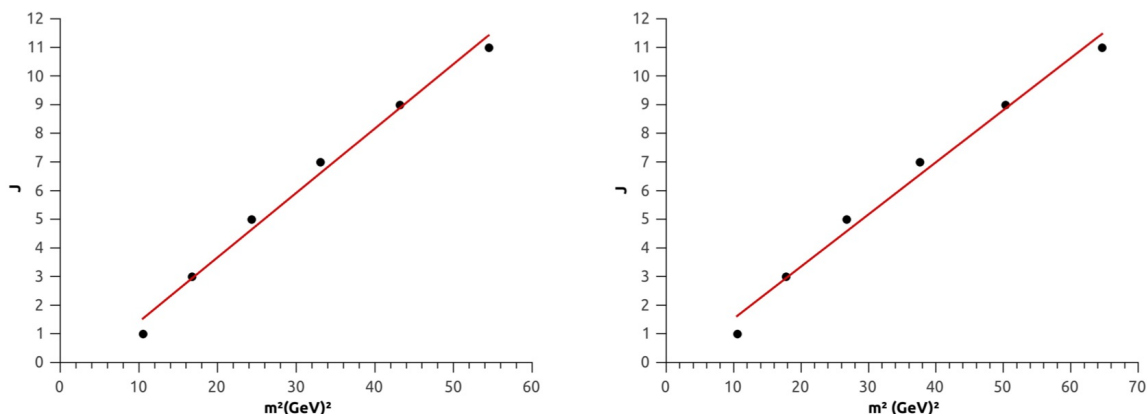
Figure 2. Glueball masses for the states 1^{--}, 3^{--}, 5^{--}, 7^{--}, 9^{--}, 11^{--} from the holographic hardwall model using Dirichlet (left panel) and Neumann (right panel) boundary conditions. We also plot an approximate linear Regge trajectory, representing the odderon.

2.3. Quark-antiquark interaction and the Cornell potential from Wilson loops

Maldacena [14] and Rey and Yee [15] calculated Wilson loops for static strings in the $AdS_5 \times S^5$ space: $< W(C) > \sim e^{-\text{Area}} \sim e^{-TE(L)}$, where $E(L)$ is the energy of the quark-antiquark pair. The result is $E(L) \sim -1/L$ which means a non-confining potential in agreement with a conformal field theory.

In ref. [16] we re-analized this problem in the hardwall model, which is a slice of the AdS_5 space (we assume that the S^5 degrees of freedom are not excited and then are not relevant here). There are three typical situations for the static string in this case, shown in Figure 3. In the general case, the static string is located between r_1 and r_2, the positions of the branes defining the AdS slice. If we consider the first brane located at $r_1 = nR$ in the limit $n \to \infty$ and the second at $r_2 = R$, then the quark-antiquark interaction energy coincides with the Cornell potential (see Figure 4):

$$E(L) = -\frac{4}{3}\frac{a}{L} + \sigma L + \text{const.} , \qquad (6)$$

that means a non-confining potential at short distances plus a confining linear term for large L, capable of reproducing the phenomenology of quarkonium systems.

2.4. Wilson loops at finite temperature in an AdS slice

We also considered the calculation of Wilson loops for static strings in an AdS slice at finite temperature [17]. The finite temperature situation is described by a an AdS-Schwazrschild space which corresponds to a black hole in the AdS space. The temperature of the string is the Hawking temperature of the black hole. This situation allows us to describe the confinement/deconfinement phase transition in large N Yang-Mills theories. There are three typical situations related to the deconfined phase, the transition and the confined phase. See Figures 5, 6 and 7. For large values of the quark-antiquark separation L the leading asymptotic behavior of the interaction energy is given by

$$E \sim \frac{L}{2\pi\alpha'} \sqrt{1 - \frac{r_T^4}{R^4}} \qquad (r_T < R) \qquad (7)$$
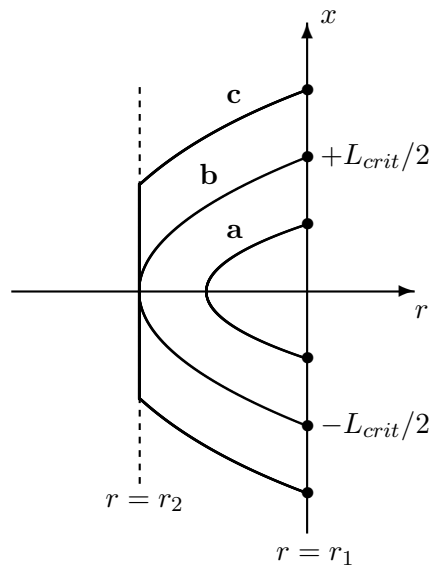
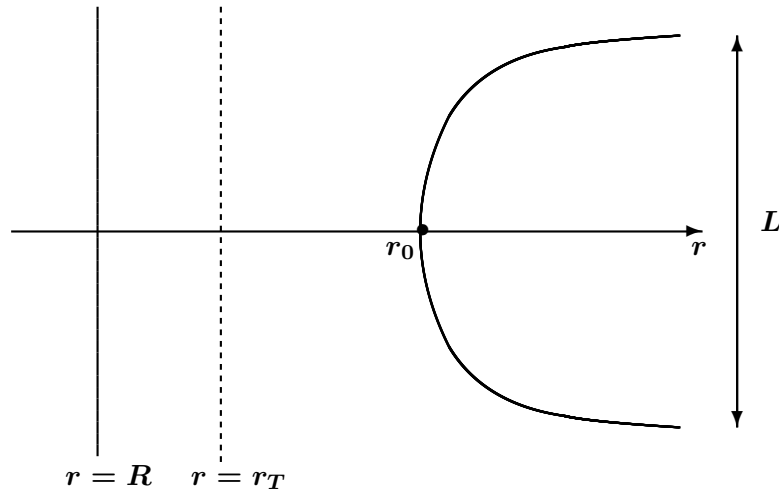**Figure 3.** Schematic representation of geodesics in an AdS slice space. Curve **a** corresponds to a geodesic with $L < L_{crit}$, curve **b** to $L = L_{crit}$ and **c** to $L > L_{crit}$.
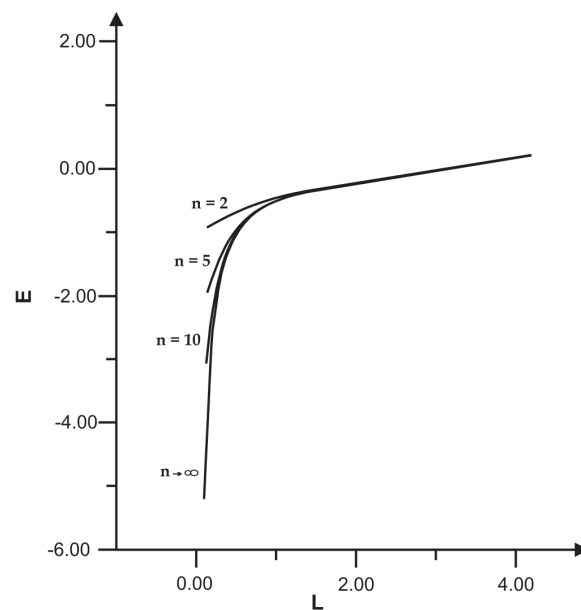

Figure 4. Energy in GeV as a function of string end-points separation L in GeV^{-1} for AdS slices with $r_1 = nR$ and $r_2 = R$. For $n \to \infty$ the energy behaves as the Cornell potential Eq. (6).

which is the confined phase. Note that for $r_T \geq R$ a phase transition occurs. This is represented in Figure 8. The obtained critical temperature in our model is $T \sim 230$ MeV, which is of the order of the QCD scale.

Figure 5. Schematic representation of a U-shaped geodesic with minimum r_0 far from the brane and from the horizon. This kind of geodesic appears for small quark anti-quark separation.
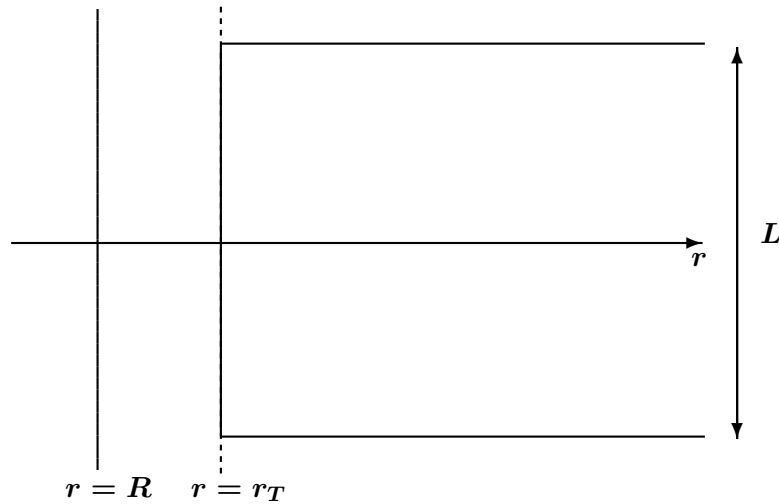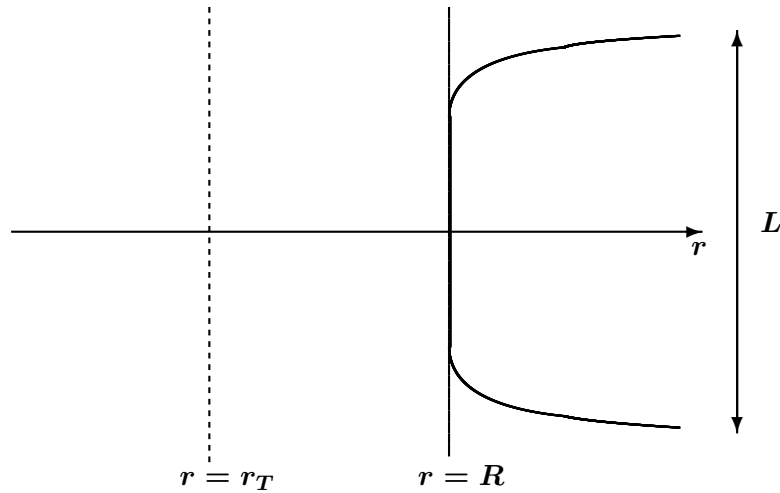

Figure 6. Schematic representation of a Box-shaped geodesic which reaches the horizon but not the brane. This is a typical situation at high temperature ($r_T > R$) and large quark anti-quark separation L.

*2.5. Deep inelastic structure functions from supergravity at small Bjorken parameter*
Deep inelastic structure functions can be calculated from supergravity when the Bjorken parameter x satisfies $x > 1/\sqrt{gN}$. Early studies of this process using gauge/string duality were presented in [3]. We consider a gauge theory with very large ´ t Hooft coupling gN in order to investigate the region $x << 1$. In this case the center of mass energy is large enough to increase the number of hadronic constituents of the final state. We calculate the structure functions in terms of the number of final hadronic constituents. At small x we find a scaling law similar to geometric scaling but with $\gamma_s = 0.5$ and $\lambda = 1$. The obtained cross section is given by [18]

$$\sigma(q^2, x) \sim (q^2 x^\lambda)^{\gamma_s^{-1}} , \tag{8}$$

Figure 7. Schematic representation of a degenerated U-shaped geodesic with minima along the brane. This corresponds to the confined phase.
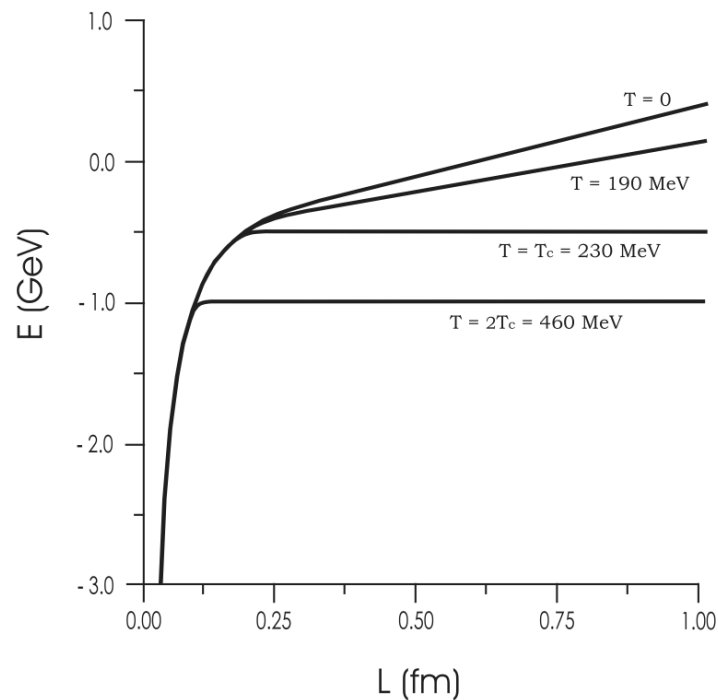
Figure 8. Energy as a function of string end-points separation for different temperatures. The critical temperature is $T_c \sim 230$ MeV. Below this temperature the system is confined and above it is deconfined. This is described by Eq. (7).

corresponding to the structure function

$$\frac{F_2(q^2, x)}{q^2} \sim (q^2 x^\lambda)\gamma_s^{-1} \,. \tag{9}$$

2.6. Other results

We have studied quasinormal modes associated with glueballs and vector mesons in the softwall model [19, 20]. We also studied vetor mesons and baryons within the D4-D8 Sakai-Sugimoto model [21, 22, 23]. Scalar and vetor mesons in models defined over conifold metrics were discussed in [24, 25]. Other DIS studies using holographic models include [26, 27, 28, 29, 30].

Acknowledgments

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