

Leptonic decay constants of the pion and its excitations in holographic QCD

Gastão Krein

Instituto de Física Teórica, Universidade Estadual Paulista
Ria Dr. Bento Teobaldo Ferraz, 271 - Bloco II, 01140-070 São Paulo, SP, Brazil
E-mail: gkrein@ift.unesp.br

Abstract. We present results of a recent study of the structure of excited pions within a chiral holographic model for QCD. In particular, we prove that the leptonic decay constants of the excited states of the pion vanish in the chiral limit when chiral symmetry is dynamically broken. Comparisons are made with corresponding results obtained in light-front holography.

1. Introduction

There is a remarkable prediction of QCD [1] that the leptonic decay constants of the excited states of the pion are dramatically suppressed relative to the decay constant of the ground-state pion — in the chiral limit, the decay constants of the excited pions are exactly zero. Although within a quark model perspective a suppression of a leptonic decay constant for excited states is expected, as it is proportional to the wave-function at the origin, there is, however, no a priori reason, within such a perspective, for the dramatic suppression predicted by QCD. Our work [2] is partially motivated by the failure of light-front holography (LFH) [3, 4] in this matter: while the experimental values of the masses of the lowest radially and orbitally excited states of the pion are well reproduced, the leptonic decay constants of the excited states do not vanish in the chiral limit [5]. Like in other instances of hadron structure, in particular regarding chiral corrections of hadron properties [6, 7, 8, 9, 10, 11], dynamical chiral symmetry breaking (DCSB) in QCD along with the (pseudo) Goldstone boson nature of the ground-state pion is the key feature behind the suppression of the decay constants of the excited states.

The quark mass dependence of the leptonic decay constants of the pion has been studied with lattice simulations. Ref. [12] finds for the pion's first radial excitation $f_{\pi^1}/f_{\pi^0} \sim 0.08$ when extrapolated to the chiral limit; the experimental value extracted in Ref. [13] gives $f_{\pi^1}/f_{\pi^0} < 0.064$ — we denote the decay constant of the n -th excited state by f_{π^n} and that of the ground state by f_{π^0} . Another lattice collaboration [14] finds a value consistent with zero for f_{π^1} when extrapolated to the chiral limit. A very recent [15] lattice simulation reports for the three lowest excited states the following results: f_{π^1} shows almost no suppression, f_{π^2} is significantly suppressed, and $f_{\pi^3} \simeq f_{\pi^1}$.

In the next section, we present a brief discussion on the role of DCSB in QCD in the vanishing of the leptonic decay constants of the excited states of the pion. In Section 3, we present some details of our study [2] in the context of a hard wall holographic chiral model [16, 17]. Our numerical results are presented in Section 4 and Section 5 contains our Conclusions.



2. The QCD prediction

In QCD, the vanishing of the leptonic decay constants of the excited states of the pion in the chiral limit follows from a generalized GOR relationship [1]:

$$f_{\pi^n} m_{\pi^n}^2 = 2m_q \rho_{\pi^n}, \quad (1)$$

where m_{π^n} is the mass of the pion's n -th excited state, $m_q = m_u = m_d$ (we work in the approximation of isospin symmetry) and ρ_{π^n} is the gauge-invariant residue at the pole $P^2 = -m_{\pi^n}^2$ in the pseudoscalar vertex function. The vertex function is related to the matrix-valued Bethe-Salpeter wavefunction $\chi_{\pi^n}^a(P, q)$ via

$$i\rho_{\pi^n} \delta^{ab} := \int \frac{d^4k}{(2\pi)^4} \text{Tr} \left[t^a \gamma_5 \chi_{\pi^n}^b(q, P) \right], \quad (2)$$

with the $SU(2)$ generators t^a , $a = 1, 2, 3$, normalized as $2 \text{Tr} (t^a t^b) = \delta^{ab}$. It is important to note that while m_q and ρ_{π^n} are scale dependent, the product $m_q \rho_{\pi^n}$ is renormalization group invariant. For the ground-state pion, DCSB implies [18]

$$\rho_{\pi^0} = -\frac{1}{f_{\pi^0}} \langle \bar{q}q \rangle, \quad (3)$$

where $\langle \bar{q}q \rangle = \langle \bar{u}u \rangle = \langle \bar{d}d \rangle$ is the quark condensate. When Eq. (3) is used in Eq. (1), one obtains the well known GOR relationship

$$f_{\pi^0}^2 m_{\pi^0}^2 = 2m_q |\langle \bar{q}q \rangle|. \quad (4)$$

Now, the key result follows via the following sequence of arguments [1]:

- (i) The existence of excited states entails finite matrix-valued $\chi_{\pi^n}(P, q)$ wavefunctions;
- (ii) The ultraviolet behavior of the QCD quark-antiquark scattering kernel [19] guarantees that the integral in Eq. (2) is finite;
- (iii) Then,

$$\rho_{\pi^n}^0 := \lim_{m_q \rightarrow 0} \rho_{\pi^n} = \text{finite}, \quad (5)$$

- (iv) Since, by hypothesis, $m_{\pi^n}^2 \neq 0$ in the chiral limit, Eq. (1) implies $f_{\pi^n} = 0$ for $m_q = 0$.

While the GOR relationship, Eq. (4), which is a key result of DCSB, is obtained in holographic QCD in a rather straightforward way, the vanishing of f_{π^n} , $n > 0$, is more subtle, as we discuss in the following.

3. The calculation in holographic QCD

The hard-wall model is defined on a slice of 5-d Anti-de-Sitter spacetime (with unit radius)

$$ds^2 = \frac{1}{z^2} \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2 \right), \quad 0 < z \leq z_0, \quad (6)$$

where $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$ is the metric of 4-d flat spacetime, and $z_0 = 1/\Lambda_{\text{QCD}}$ corresponds to an infrared mass gap in the 4-d gauge theory. The AdS/CFT correspondence maps 4-d field theory operators $\mathcal{O}(x)$ to 5-d fields $\phi(x, z)$; in the present case the relevant operators for describing DCSB are the left and right handed currents $J_{L\mu}^a = \bar{q}_L \gamma_\mu t^a q_L$, $J_{R\mu}^a = \bar{q}_R \gamma_\mu t^a q_R$, corresponding to the $SU(N_f)_L \times SU(N_f)_R$ chiral flavor symmetry and the quark bilinear operator $\bar{q}_R q_L$ related to DCSB. In the dual theory, these 4-d operators correspond

to 5-d gauge fields $L_m^a(x, z)$, $R_m^a(x, z)$ and a 5-d bifundamental scalar field $X(x, z)$, both living in an AdS slice described by Eq. (6).

DCSB was first implemented in holographic QCD in Refs. [16, 17]; here we follow the conventions and notation of Ref. [16]. The action for the fields $L_m^a(x, z)$, $R_m^a(x, z)$ and $X(x, z)$ is given by

$$S = \int d^5x \sqrt{|g|} \text{Tr} \left[(D^m X)^\dagger (D_m X) + 3|X|^2 - \frac{1}{4g_5^2} (L^{mn} L_{mn} + R^{mn} R_{mn}) \right], \quad (7)$$

where

$$D_m X := \partial_m X - i L_m X + i X R_m, \quad (8)$$

$$L_{mn} := \partial_m L_n - \partial_n L_m - i [L_m, L_n], \quad (9)$$

$$R_{mn} := \partial_m R_n - \partial_n R_m - i [R_m, R_n]. \quad (10)$$

We restrict the discussion to the quark flavors u and d . The classical solution that describes chiral symmetry breaking is given by

$$L_m^0 = R_m^0 = 0, \quad 2X_0 = \zeta M z + \frac{\Sigma}{\zeta} z^3, \quad (11)$$

with $\zeta = \sqrt{N_c}/2\pi$, and

$$M = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \sigma_u & 0 \\ 0 & \sigma_d \end{pmatrix}. \quad (12)$$

The AdS/CFT dictionary leads to identification of the coefficients M and Σ with the 4-d quark mass and chiral condensate terms responsible for the explicit and dynamical breaking of chiral symmetry.

One can obtain the meson spectrum from the kinetic part of the action in Eq. (7). This action is obtained by expanding up to quadratic order in the fields $V_m = V_m^a t^a$, $A_m = A^a t^a$ and the fluctuation π^a [2]:

$$S^{\text{Kin}} = \int d^4x \int \frac{dz}{z} \left\{ \frac{v^2}{2z^2} \left[-(\partial_z \pi^a)^2 + (\partial_{\hat{\mu}} \pi^a - \partial_{\hat{\mu}} \phi^a)^2 + (A_{\hat{\mu}}^{\perp, a})^2 \right] - \frac{1}{4g_5^2} \left[-2(\partial_z V_{\hat{\mu}}^a)^2 + (v_{\hat{\mu}\hat{\nu}}^a)^2 - 2(\partial_z A_{\hat{\mu}}^a)^2 - 2(\partial_z \partial_{\hat{\mu}} \phi^a)^2 + (a_{\hat{\mu}\hat{\nu}}^{\perp, a})^2 \right] \right\}, \quad (13)$$

where

$$v_{mn}^a := \partial_m V_n^a - \partial_n V_m^a, \quad a_{mn}^a := \partial_m A_n^a - \partial_n A_m^a, \quad (14)$$

and

$$v(z) := \zeta m_q z + \frac{\sigma_q}{\zeta} z^3. \quad (15)$$

We distinguish vectorial Minkowski indices $\hat{\mu}$ and vectorial AdS indices $\mu = (\hat{\mu}, z)$. To obtain the Lagrangian of Eq. (13), the vector field $A_{\hat{\mu}}^a$ was decomposed into $A_{\hat{\mu}}^a = A_{\hat{\mu}}^{\perp, a} + \partial_{\hat{\mu}} \phi^a$, with $\partial_{\hat{\mu}} A_{\perp, a}^{\hat{\mu}} = 0$, and gauge invariance was used to fix the gauge to $V_z^a = A_z^a = 0$.

Next, we make a Kaluza-Klein expansion of the fields, in that the 5-d fields are expanded in an infinite discrete set of modes, each mode being the product of a wave function in the radial coordinate z and a meson field depending on the Minkowski coordinates x :

$$V_{\hat{\mu}}^a(x, z) = g_5 \sum_{n=0}^{\infty} v^{a,n}(z) \hat{V}_{\hat{\mu}}^{a,n}(x), \quad (16)$$

$$A_{\hat{\mu}}^{\perp,a}(x, z) = g_5 \sum_{n=0}^{\infty} a^{a,n}(z) \hat{A}_{\hat{\mu}}^{a,n}(x), \quad (17)$$

$$\pi^a(x, z) = g_5 \sum_{n=0}^{\infty} \pi^{a,n}(z) \hat{\pi}^{a,n}(x), \quad (18)$$

$$\phi^a(x, z) = g_5 \sum_{n=0}^{\infty} \phi^{a,n}(z) \hat{\pi}^{a,n}(x). \quad (19)$$

The wave functions $\phi^{a,n}(z)$ and $\pi^{a,n}(z)$ are not independent; the relation between them can be obtained from the 5-d field equations —for details, see Ref. [2]. Replacing these into the action of Eq. (13), one obtains the following 4-d action:

$$S^{\text{Kin}} = \sum_{n=0}^{\infty} \int d^4x \left\{ \frac{1}{2} [\partial_{\hat{\mu}} \hat{\pi}^{a,n}(x)]^2 - \frac{1}{2} m_{\pi^{a,n}}^2 [\hat{\pi}^{a,n}(x)]^2 - \frac{1}{4} [\hat{v}_{\hat{\mu}\hat{\nu}}^{a,n}(x)]^2 + \frac{1}{2} m_{V^{a,n}}^2 [\hat{V}_{\hat{\mu}}^{a,n}(x)]^2 \right. \\ \left. - \frac{1}{4} [\hat{a}_{\hat{\mu}\hat{\nu}}^{a,n}(x)]^2 + \frac{1}{2} m_{A^{a,n}}^2 [\hat{A}_{\hat{\mu}}^{a,n}(x)]^2 \right\}, \quad (20)$$

where appropriate normalization conditions have been used for the wave functions $v^{a,n}(z)$, $a^{a,n}(z)$, $\pi^{a,n}(z)$ and $\phi^{a,n}(z)$, and the masses are obtained from the following equations

$$\frac{\beta(z)}{z} [\pi^{a,n}(z) - \phi^{a,n}(z)] = -\partial_z \left[\frac{1}{z} \partial_z \phi^{a,n}(z) \right], \quad (21)$$

$$\beta(z) \partial_z \pi^{a,n}(z) = m_{\pi^{a,n}}^2 \partial_z \phi^{a,n}(z), \quad (22)$$

$$-\partial_z \left[\frac{1}{z} \partial_z v^{a,n}(z) \right] = \frac{m_{V^{a,n}}^2}{z} v^{a,n}(z), \quad (23)$$

$$\left[-\partial_z \left(\frac{1}{z} \partial_z \right) + \frac{1}{z} \beta(z) \right] a^{a,n}(z) = \frac{m_{A^{a,n}}^2}{z} a^{a,n}(z), \quad (24)$$

where $\beta(z)$ is given by

$$\beta(z) := \frac{g_5^2}{z^2} v(z)^2 = g_5^2 \left(\zeta m_q + \frac{\sigma_q}{\zeta} z^2 \right)^2. \quad (25)$$

We solve the above equations imposing Dirichlet boundary conditions at $z = \epsilon$:

$$\pi^{a,n}|_{z=\epsilon} = v^{a,n}|_{z=\epsilon} = a^{a,n}|_{z=\epsilon} = 0, \quad (26)$$

and Neumann boundary conditions at $z = z_0$:

$$\partial_z \pi^{a,n}|_{z=z_0} = \partial_z v^{a,n}|_{z=z_0} = \partial_z a^{a,n}|_{z=z_0} = 0. \quad (27)$$

Note that one can omit the flavor index a , as there is no flavor mixing implied by the equations and boundary conditions.

The leptonic decay constants can be extracted by considering the holographic currents. The currents can be identified from boundary terms at $z = \epsilon$ of the variation of the action. Specifically, after using the Kaluza-Klein expansions in the boundary terms, the currents are given by (omitting the flavor index a) [2]:

$$\langle J_V^\mu(x) \rangle = \sum_{n=0}^{\infty} \left[\frac{1}{g_5 z} \partial_z v^n(z) \right]_{z=\epsilon} \hat{V}_n^\mu(x), \quad (28)$$

$$\langle J_A^\mu(x) \rangle = \sum_{n=0}^{\infty} \left[\frac{1}{g_5 z} \partial_z a^n(z) \right]_{z=\epsilon} \hat{A}_n^\mu(x) + \sum_{n=0}^{\infty} \left[\frac{1}{g_5 z} \partial_z \phi^n(z) \right]_{z=\epsilon} \partial^\mu \hat{\pi}^n(x), \quad (29)$$

$$\partial_\mu \langle J_A^\mu(x) \rangle = - \sum_{n=0}^{\infty} \left[\frac{\beta(z)}{g_5 z} \partial_z \pi^n(z) \right]_{z=\epsilon} \hat{\pi}_n(x). \quad (30)$$

From these, one can read off the decay constants for the vector mesons (g_{V^n}), the axial-vector mesons (g_{A^n}), and the pions (f_{π^n}):

$$g_{V^n} = \left[\frac{1}{g_5 z} \partial_z v^n(z) \right]_{z=\epsilon}, \quad g_{A^n} = \left[\frac{1}{g_5 z} \partial_z a^n(z) \right]_{z=\epsilon}, \quad f_{\pi^n} = \left[-\frac{1}{g_5 z} \partial_z \phi^n(z) \right]_{z=\epsilon}. \quad (31)$$

Now, taking the divergence of (29), one obtains

$$f_{\pi^n} m_{\pi^n}^2 = -\frac{1}{g_5} \left[\frac{\beta(z)}{z} \partial_z \pi^n(z) \right]_{z=\epsilon}, \quad (32)$$

where we made use of the on-shell equation for the pion field, $\partial^2 \hat{\pi}^n(x) = -m_{\pi^n}^2 \hat{\pi}^n(x)$. Moreover, using this result into Eq. (30), the divergence of the axial current takes the form of an extended PCAC relation

$$\partial_\mu \langle J_A^\mu(x) \rangle = \sum_{n=0}^{\infty} f_{\pi^n} m_{\pi^n}^2 \hat{\pi}_n(x). \quad (33)$$

Now, using Eq. (32) one obtains a generalized GOR relationship in the form of Eq. (1), when making the identification

$$2m_q \rho_{\pi^n} := -\frac{1}{g_5} \left[\frac{\beta(z)}{z} \partial_z \pi^n(z) \right]_{z=\epsilon}. \quad (34)$$

As will be shown in the next section, the function ρ_{π^n} is finite as $m_q \rightarrow 0$, independently of the mode number n . This, like in QCD, allows to predict the behavior of f_{π^n} close to the chiral limit.

4. Numerical Results

The free parameters of the model are z_0 , m_q , and σ_q . The hard-wall cutoff z_0 can be fixed (in the limit of exact isospin symmetry) by the mass of the ρ meson, namely $z_0 = (322.5 \text{ MeV})^{-1}$, and this essentially sets the scale of all hadronic masses. The values of the other parameters are fixed by fitting the ground-state pion mass $m_{\pi^0} = 139.6 \text{ MeV}$ and leptonic decay constant $f_{\pi^0} = 92.4 \text{ MeV}$, which leads to $m_q = 8.31 \text{ MeV}$ and $\sigma_q = (213.7 \text{ MeV})^3$.

Fig. 1 displays the results for the m_q dependence of the masses of the ground-state pion and its excitations up to $n = 3$ —for the solutions for $n > 3$ up to $n = 6$, the m_q dependence is similar to that shown in Fig. 1. We note that the mass of the ground-state pion can be fitted as $m_{\pi^0} \sim m_q^{1/2}$ near the chiral limit, which is consistent with the GOR (4). The masses of

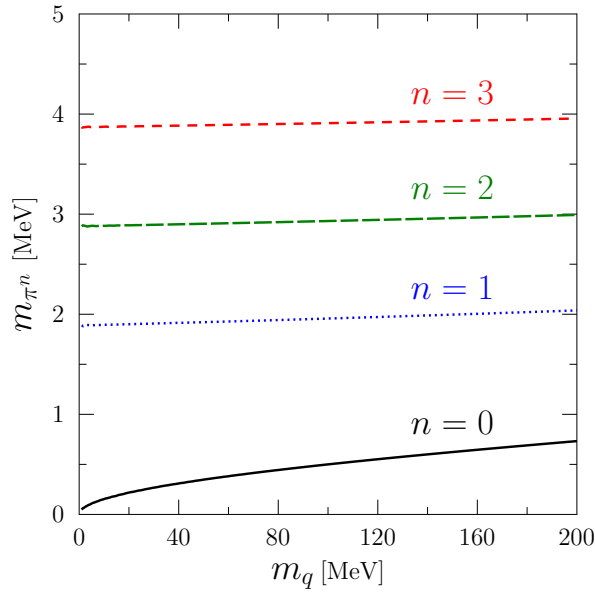


Figure 1. Quark mass dependence of the pion masses.

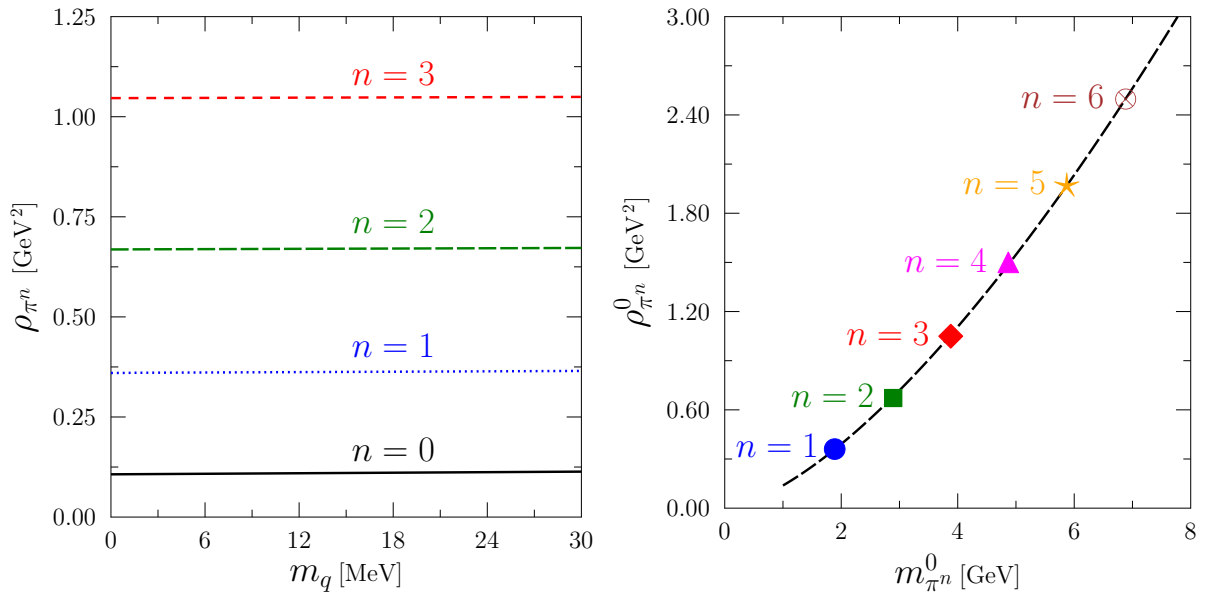


Figure 2. Left Panel: Quark mass dependence of ρ_{π^n} . Right panel: The function $\rho_{\pi^n}^0$ for the first six excited states; the dashed line is a fit to the discrete eigenvalues.

the excited states, on the other hand, can be fitted as $m_{\pi^n} = m_{\pi^n}^0 + a_n m_q$, where $m_{\pi^n}^0$ are the corresponding masses in the chiral limit.

In the left panel of Fig. 2 we display the quark mass dependence of the function ρ_{π^n} . Clearly, the curves show that ρ_{π^n} is finite as $m_q \rightarrow 0$, a very important feature for establishing that $f_{\pi^n} = 0$ for $n > 0$. In addition, one can see that ρ_{π^n} is approximately independent of m_q for $0 \leq m_q \leq 30$ MeV for all the values of n investigated. On the right panel of Fig. 2, we display the chiral limit of ρ_{π^n} , $\rho_{\pi^n}^0$, defined in Eq. (5), as a function of the masses of the excited pions in the chiral limit, $m_{\pi^n}^0$. While, as remarked, the ρ_{π^n} are approximately independent of m_q , the

masses m_{π^n} present a slight dependence on m_q . In particular, one can fit the function $\rho_{\pi^n}^0$ as

$$\rho_{\pi^n}^0 = \gamma \left(m_{\pi^n}^0\right)^{3/2}, \quad n \geq 1, \quad (35)$$

with $\gamma = 4.375 \text{ MeV}^{1/2}$. Eq. (32) then implies that the generalized GOR relationship takes the form

$$f_{\pi^n}^0 := \lim_{m_q \rightarrow 0} f_{\pi^n} = \gamma \frac{2m_q}{\sqrt{m_{\pi^n}^0}}, \quad n \geq 1. \quad (36)$$

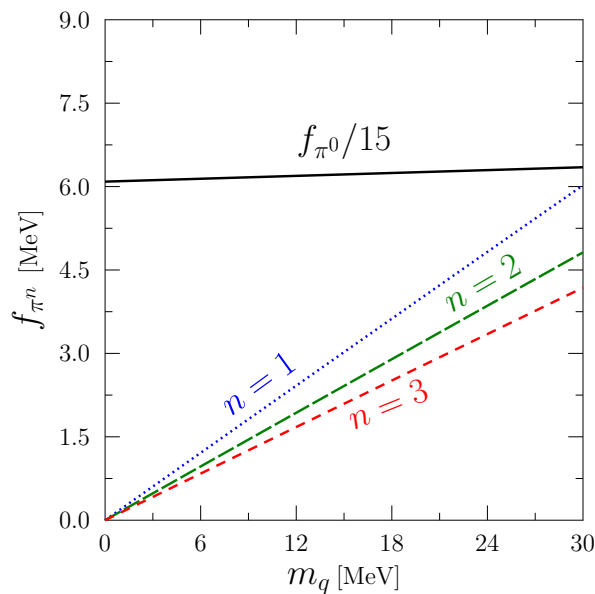


Figure 3. Quark mass dependence of f_{π^n} .

The quark mass dependence of the pion decay constants f_{π^n} near the chiral limit as displayed in Fig. 3. The results displayed clearly show that, while the ground-state pion possesses a finite leptonic decay constant f_{π^0} , the excited states have leptonic decay constants f_{π^n} that vanish in the chiral limit. The ground-state decay constant is fitted to its experimental value, $f_{\pi^0} = 92.4 \text{ MeV}$, and for the first three excited states we obtained $f_{\pi^1} = 1.68 \text{ MeV}$, $f_{\pi^2} = 1.34 \text{ MeV}$ and $f_{\pi^3} = 1.16 \text{ MeV}$. There is an experimental bound on f_{π^1} extracted from B decays [13], which is $f_{\pi^1} < 0.064 f_{\pi^0}$. Our result, $f_{\pi^1} = 1.68 \text{ MeV}$, is perfectly compatible with this bound but, compared to the lattice result of Ref. [12], it is five times smaller.

We also note that the curves for the excited states can be fitted with a linear quark mass dependence. Such a linear m_q scaling of f_{π^n} for $n \geq 1$ is precisely the one predicted in QCD [1] through the generalized GOR relationship (1):

$$f_{\pi^n} = \frac{2m_q \rho_{\pi^n}}{m_{\pi^n}^2} \sim m_q, \quad n \geq 1, \quad (37)$$

as $m_{\pi^n}^2 \sim (m_q)^0$ and $\rho_{\pi^n} \sim (m_q)^0$.

5. Conclusions

We have presented results of a recent investigation [2] on the quark mass dependence of the leptonic decay constants of the pion and its excitations in a five-dimensional holographic hard

wall model for QCD. The model is able to reproduce the QCD prediction of the vanishing of the decay constants of the excited states in the chiral limit.

The results shed light on the failure of light-front holography (LFH) in this matter. A key QCD feature captured by our approach is the obtention of the generalized GOR relationship, Eq. (1), first derived in QCD in Ref. [1]. On the other hand, in LFH the pion comes out massless in the chiral limit from the cancellation of the light-front kinetic energy and light-front quadratic confinement potential in a Schrödinger-like equation [20]. While similar cancellations occur in chiral models of QCD in Coulomb gauge as a result of DCSB — see Refs. [21, 22, 23, 24, 25] for detailed discussions on this feature —, in LFH this is not the case. In the calculation of the pion leptonic decay constants in LFH [5], the Fock-space decomposition of the light-front wave function is truncated to its lowest quark-antiquark valence component, an approximation that seems not capturing the full chiral dynamics of the pion bound state.

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