About what can be witnessed by a Leggett–Garg inequality test: modeling its violation

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Abstract. The Leggett–Garg inequality is a widely used test of the "quantumness" of a system, and involves correlations between measurements realized at different times. According to its widespread interpretation, a violation of the Legget–Garg inequality disproves macroscopic realism and non-invasiveness. Nevertheless, recent results point out that macroscopic realism is a model dependent notion and that one should always be able to attribute to invasiveness a violation of a Legget–Garg inequality. This opens some natural questions: how to provide such an attribution in a systematic way? How can apparent macroscopic realism violation be recast into a dimensional independent invasiveness model? The present work answers these questions by introducing an operational model where the effects of invasiveness are controllable through a parameter associated with what is called the *measurability* of the physical system. Such a parameter leads to different generalized measurements that can be associated with the dimensionality of a system, to measurement errors or to back action.

1. Introduction

The Leggett–Garg inequality (LGI) was proposed by Leggett and Garg [1] in the 1980s as a witness to the "quantumness" in a macroscopic system. Its violation, thus, would imply nonclassicality, defined by the rejection of the two assumptions bellow:
(i) Macroscopic Realism (MR): a macroscopic system with two or more macroscopically distinct states available to it will at all times be in one of those states.
(ii) Noninvasive measurability (NIM): it is possible, in principle, to determine the state of the system with arbitrarily small perturbation to its subsequent dynamics.
This inequality involves measurements of observables that can be associated with macroscopic properties of a system at different times, defined here as $\hat{Q}(t_i)$. By defining $C_{kl} \equiv \langle \hat{Q}(t_k)\hat{Q}(t_l)\rangle$, we can write the following LGI [2]:

$$-2 \leq K_{LG} \equiv C_{12} + C_{23} + C_{34} - C_{14} \leq 2. \quad (1)$$

Nonetheless, there has been a recent debate in the literature [3, 5] about if a violation of the LGI allows one to reject both the assumptions. Maroney and Timpson [3] showed that what a violation of a LGI primarily witnesses is whether the measurement is invasive or not. MR is a model-dependent notion, as, for instance, the notion of quantum superposition is not incompatible with realism [4]. However, it is also discussed in Ref. [3] with the aid of the *ontic models* framework introduced by Spekkens [6] that despite the fact that what is most immediately witnessed by a violation of a LGI is invasiveness of measurements, this violation allows one to reject a specific notion of MR, the 'operational eigenstate mixture macrorealism'. This corresponds to a given preparation such that, for certain measurements, would return a given outcome with probability 1. As discussed by the authors, our experience tell us that when observation of macroscopic properties takes place, the system is put into an operational eigenstate of those properties, such that if a particular value for a macroscopic property is obtained, then we must obtain that same value if we look at the system immediately afterwards. Therefore, operational eigenstates can provide a characterisation of the behaviour of macroscopic systems when observed, and the violation of the LGI would permit us to reject MR in these terms.

2. Modeling invasiveness of measurements in LGIs

We propose, in [7], a model to test a LGI using positive operator valued measurements (POVM). In our model, invasiveness can be associated with the resolution and efficiency of a measurement apparatus. These parameters, according to the experimental situation, may, or not, be associated with the dimensionality of the system. The introduced POVMs provide a physically sound and operational interpretation of what is actually being tested by the LGI.

The system considered consists of a arbitrary spin j prepared in a initially maximally mixed state of the eigenstates m of the z-component, $\hat{J}_z$, which dynamics is given by a precession around the x-component, $\hat{J}_x$ - as discussed above, this state is not supposed to violate the LGI, what ensures that nonclassicality will arise from the system's dynamics and the measurement processes. In order to measure the correlations C_{kl} in Eq. 1, we define a two-valued POVM, which allows us to introduce parameters defined as the *measurability*, σ, and the *resolution*, Δm_μ, μ being the specific m's that can always be faithfully determined by the measurement apparatus. Therefore, for the particulars $m = \mu$, one is always able to associate with them a well-defined parity once they are perfectly determined by the measurement apparatus, whilst for $m \neq \mu$, the interplay between the parameters σ and Δm_μ will determine how well the correct parity can be associated with these outcomes. For a fixed resolution, Δm_μ, invasiveness can be associated with the measurability, σ, and this later will then control the violation of the LGI. We refer the reader to Ref. [7] for all the details.

This scheme is quite general and can be associated with different physical origins, for instance the increase of dimensionality of a system and the classical disturbance created by a the measurement process. The interpretation of each parameter depends on the physical system one uses to test a LGI, the role of each parameter clearly identified to invasiveness, whatever its physical origin is. Our model can be used to help understanding and interpretation of LGI tests and can be tested experimentally in a number of physical systems, such as Stern–Gerlach-like experiments with inhomogeneous fields and the orbital angular momentum of photons.

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