# EPR paradox, quantum nonlocality and physical reality

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**Abstract.** Eighty years ago Einstein, Podolsky and Rosen demonstrated that instantaneous reduction of wave function, believed to describe completely a pair of entangled physical systems, led to EPR paradox. The paradox disappears in statistical interpretation of quantum mechanics (QM) according to which a wave function describes only an ensemble of identically prepared physical systems. QM predicts strong correlations between outcomes of measurements performed on different members of EPR pairs in far-away locations. Searching for an intuitive explanation of these correlations John Bell analysed so called local realistic hidden variable models and proved that correlations consistent with these models satisfy Bell inequalities which are violated by some predictions of QM and by experimental data. Several different local models were constructed and inequalities proven. Some eminent physicists concluded that Nature is definitely nonlocal and that it is acting according to a law of nonlocal randomness. According to these law perfectly random, but strongly correlated events, can be produced at the same time in far away locations and a local and causal explanation of their occurrence cannot be given. We strongly disagree with this conclusion and we prove the contrary by analysing in detail some influential finite sample proofs of Bell and CHSH inequalities and so called Quantum Randi Challenges. We also show how one can win so called Bell's game without violating locality of Nature. Nonlocal randomness is inconsistent with local quantum field theory, with standard model in elementary particle physics and with causal laws and adaptive dynamics prevailing in the surrounding us world. The experimental violation of Bell-type inequalities does not prove the nonlocality of Nature but it only confirms a contextual character of quantum observables and gives a strong argument against counterfactual definiteness and against a point of view according to which experimental outcomes are produced in irreducible random way.

## 1. Introduction

Eighty years ago Einstein, Podolsky and Rosen (EPR) [1] demonstrated that an instantaneous reduction of a wave function, describing a couple of entangled physical systems, led to so called EPR paradox.

The paradox disappears in a statistical interpretation of QM according to which a wave function describes only an ensemble of identically prepared physical systems [2–5]. When values of physical observables are measured probabilities of different outcomes depend strongly on experimental context. Quantum probabilities are not degrees of belief of some intelligent agents but are objective properties of physical phenomena and experiments. Whether these probabilities can be deduced from some more detailed description of quantum phenomena is an open question.

In order to reproduce quantum statistics *"sub-quantal"* descriptions have to introduce supplementary parameters which are called hidden variables. Fifty years ago John Bell [6, 7], trying to explain strong correlations between spin polarization projections of two physical systems prepared in a spin singlet state, analyzed a large class of so called local realistic hidden variable models (LRHV). He found that correlations predicted by these models obeyed Bell inequalities (BI) which were violated by some correlations predicted by QM. Five years later John Clauser et al. [8] derived CHSH inequalities which were particularly suited for experimental testing. Using LRHV or stochastic hidden variables models (SHV) several other inequalities were proven and shown to be violated by QM and by data of several ingenious experiments [9–15].

In this paper we concentrate on spin polarization correlation experiments (SPCE) with photons but some of our conclusions apply also to recent experiment with electrons by Hensen *et al.* [15].

In SPCE a source is sending two correlated signals to distant polarization beam splitters (PBS) and detectors. The outcomes on far away detectors, registered in some carefully chosen time windows, are 1=polarization up, -1= polarization down and 0=no count. Outcomes seem to appear randomly but distant time series are correlated more strongly than it is permitted by LRHV and SHV. These correlations are consistent with predictions of QM. One may only conclude that a sub-quantal description of quantum phenomena cannot be based on LRHV or SHV.

However large majority of quantum information community inspired by papers of John Bell, concluded that a local and a causal explanation of correlations observed in SPCE is impossible and that *Nature is nonlocal*.

The violation of Bell type inequalities is believed to imply a violation of local realism (LR) as defined par example by Richard Gill [16]: *"local realism = locality + realism, is closely related to causality"...* " events have causes (realism); cause and effect are constrained by time and space (locality)".

This belief is incorrect because the assumption of *local realism* or *Bell locality* used in different proofs it is not LR defined above.

Nevertheless *quantum nonlocality* is considered to be a mysterious property of Nature. Apparently a mystery has a seductive power and not only excellent fiction writers but also distinguished scientists allow themselves to over-exaggerate quantum paradoxes. Entangled "photon pairs" are compared to pairs of fair dices which in each trial give perfectly matching outcomes. This is of course impossible. Rolling of fair dices produces random outcomes which cannot be perfectly correlated.

In his recent book Nicolas Gisin [17] explains in a pedagogical way *quantum nonlocality* and reviews results of many experimental and theoretical papers on the subject. He claims that quantum correlations observed in SPCE cannot be explained by causes belonging to the common past: *"Nature does not satisfy the continuity principle ... Nature is nonlocal"*. He advocates a new law of Nature: *"We must accept ... nonlocal randomness, an irreducible randomness that manifests itself in several widely separated places without propagating from one point of space to next"*. At the same time randomly produced events are strongly correlated creating what means inexplicable mystery. In [17] one finds also statements such as: *"A particle passes through two neighboring slits at the same time. Therefore, an electron is indeed both here and a meter to the right of here"*. This is not what QM says!

When promoting his new law of *nonlocal randomness* Gisin doesn't care that quantum field theory (QFT) and a standard model in particle physics are local theories. He seems to forget that all biological phenomena point towards adaptive dynamics and local causality.

For example in their fall migration tiny birds Bar-tailed Godwit (Limosa lapponica baueri) fly from Alaska to New Zealand 11 000 km in about eight days over the open Pacific Ocean, without stopping to rest or refuel. How could it be possible if *nonlocal randomness* was a law

of Nature?

The *nonlocal randomness* is not needed in order to understand long range correlations in SPCE. The assumption which is used in LRHV is not LR, as defined above. LRHV use the assumption of *counterfactual definiteness* (CFD) according to which values of quantum observables, including incompatible ones, are predetermined before a measurement and recorded passively by measuring instruments [18].

In SHV it is assumed that results of measurements in distant laboratories are obtained in irreducibly random way what destroys all non-trivial correlations created by a source.

If these assumptions are not used various Bell type inequalities cannot be proven [19–50] and their violations give neither information about the locality of Nature nor about the completeness of QM.

Various probabilistic models used to prove inequalities are not consistent with experimental protocols used in SPCE [31, 37, 44]. If contextual character of quantum observables is properly taken into account correlations may be explained in intuitive way [28-33, 40-47]. Moreover many experiments in quantum optics and in neutron interferometry can be simulated event by event in a local and causal way [51–54].

Unfortunately it seems that these results are ignored or not well understood. This is why in this paper we give a detailed critical analysis of some influential finite sample proofs of Bell type inequalities and we hope that it will finally close the issue.

This paper is organized as follows.

In section 2 we recall EPR paradox and Bohm's spin version of it (EPR-B) and the explanation given by a statistical and contextual interpretation of QM.

In section 3 we explain long range correlations in SPCE and their dependence on how pairing of distant outcomes is done.

In section 4 we analyze finite sample proofs of CHSH and Bell given by Richard Gill [16, 55] and by Sacha Vongher [56] and we discuss impossible quantum Randi challenges proposed by them

In section 5 we show how one may win so called Bell's game discussed in [17] without invoking *nonlocal randomness*.

In section 6 we show how long range correlations in SPCE can be explained in a local and causal way if contextual character of quantum observables is correctly taken into account.

In section 7 we present few results from our paper written with Hans De Raedt [57] showing that *sample homogeneity loophole* was not closed in experiments testing various Bell type inequalities.

In section 8 we present our point of view on the Physical Reality and on its abstract description provided by QT.

The last section contains conclusions.


## 2. EPR paradox and statistical interpretation

Let us start with few axioms of QM which were believed to be true before the publication of EPR paper:

- A1: Any pure state of a physical system is described by a specific <u>unique</u> wave function $\Psi$.

- A2: Wave function reduction: any measurement causes a physical system to jump instantaneously into one of eigenstates of the dynamical variable that is being measured. This eigenstate becomes a new wave function describing a state of the system.

- A3: A wave function $\Psi$ provides a <u>complete</u> description of a pure state of an <u>individual</u> physical system.

EPR considered two particular individual systems I+II in a pure quantum state, which interacted in the past, separated and evolved freely afterwards [1]. Using A2 they concluded that:

- A single measurement performed on one of the systems, for example on the system I, gives instantaneous knowledge of the wave function of the system II moving freely far away.

- By choosing two different incompatible observables to be measured on the system I it is possible to assign two different wave functions to the system II (same physical reality: the second system after the interaction with the first).

Since a measurement performed in a distant location on the system I does not disturb in any way the system II thus according to A1 and A3 the system II should be described by a unique wave function not by two different wave functions. Moreover these wave functions are eigenstates of two non-commuting operators representing incompatible physical observables what allows to deduce indirectly the values of these incompatible physical observables for the system II without disturbing it in any way what contradicts Heisenberg uncertainty relations.

Bohr [58] promptly reacted to EPR paper and pointed out that it was not possible to assign two different wave functions to the <u>same</u> reality (the second system after the interaction with the first) since the different wave functions could be assigned to the system II only in two different incompatible experiments in which both systems were exposed to different influences before the measurement on the system I was performed.

Bohr's arguments show that different eigenfunction expansions of the same wave function $\Psi$ provide probabilistic predictions for the behavior of identically prepared physical systems in different mutually excluding (complementary) experimental contexts. Therefore $\Psi$ is not an attribute of a single couple of these systems but only a mathematical tool used to deduct statistical regularities in various quantum experiments performed on identically prepared couples of these systems.

In 1936 Einstein [2] noticed that EPR paradox disappeared if purely statistical interpretation of QM was used: "*$\Psi$ function does not, in any sense, describe the state of one single physical system. Reduced wave functions describe different sub-ensembles of the systems*" [2]. The statistical interpretation has been promoted with success by Leslie Ballentine [3, 4]: "*the habit of considering an individual particle to have its own wave function is hard to break ... though it has been demonstrated strictly incorrect*".

According to the statistical contextual interpretation of QM (SCI) [4, 5, 31, 40, 41]:

(i) A state vector $\Psi$ is not an attribute of a single electron, photon, trapped ion, quantum dot etc. A state vector $\Psi$ or a density matrix $\rho$ describe only an ensemble of identical state preparations of some physical systems.

(ii) A mysterious wave function reduction is neither instantaneous nor non-local. In EPR experiment a state vector describing the system II obtained by the reduction of the entangled state of two physical system I+II describes only an sub-ensemble of systems II being partners of those systems I for which a measurement of some observable gave the same specific outcome. Different sub-ensembles are described by different reduced state vectors.

(iii) A value of a physical observable, such as a spin projection, is not a predetermined attribute of a system but is created in the interaction of a system with a measuring instrument [5, 36, 44].

Let us recall below EPR-B paradox [4, 59] in which a source produces pairs of particles in a spin singlet state.

According to the orthodox interpretation of QM each pair of photons is described by a state vector

$$\Psi = (|+\rangle_P |-\rangle_P - |-\rangle_P |+\rangle_P)/\sqrt{2} \tag{1}$$

where $|+\rangle_P$ and $|-\rangle_P$ are state vectors corresponding to photon states in which their spin is "up" or "down" in the direction **P** respectively. If we measure a spin projection of a photon I on the direction **P** we have an equal probability to obtain a result "1" or "−1". If we obtain "1" a reduced state vector of the photon II is $|-\rangle_P$, if we obtain "−1" a reduced state vector of the photon II is $|+\rangle_P$. By choosing a direction **P**, for the measurement to be performed on

the photon I, when " photons are in flight and far apart" we can assign different incompatible reduced state vectors to the same photon II. In other words: we can predict with certainty, and without in any way disturbing the second photon, that the **P**-component of the spin of the photon II must have the opposite value to the value of the measured **P**-component of the spin of the photon I. Therefore for any direction **P** the **P**-component of the spin of the photon II has unknown but predetermined value what contradicts QM.

The solution of EPR-B paradox given by SCI is simple the wave function reduction is not instantaneous and a reduced one particle state $|+\rangle_P$ describes only an ensemble of partners of the particles I which were found to have "spin down" by a spin polarization analyzer pointing in the direction **P**. For various directions **P** it is a different sub ensemble of particles II. Strong correlations between distant outcomes in EPR experiments are due to various conservation laws. More detailed discussion of EPR and EPR-B paradoxes may be found for example in [40].

## 3. Long distance correlations in SPCE and their interpretation

For singlet state (1) QM predicts strong correlations between spin projections (denoted A and B) measured in different directions characterized by angles $\theta_A$ and $\theta_B$ namely:

$$E(AB|\Psi) = -\cos(\theta_A - \theta_B) \tag{2}$$

Equation (2) seems to predict strict anti-correlation of all **P**-components of the spin when the coincidence measurements are performed in the same direction **P** on both photons in far-away locations.

Since a choice of a direction **P** can be made when "photons are in flight" it seems impossible to keep strict anti-correlations of spin projections on all possible directions unless QM is incomplete and spin projections on all possible directions are predetermined by a source. In this case we have a mixed statistical ensemble of photon pairs characterized by correlated and predetermined spin projections on all possible directions which are recognized by polarization analysers and registered by the detectors. This CDF assumption motivates a use of joint probability distributions in LRHV [22, 23, 37, 44].

Correlations which one may obtain using LRHV or SHV always satisfy Bell type inequalities [6, 7]. These inequalities are for some directions violated by quantum correlations [2] and by experimental data [9–15].

This is why some scientists started to consider supra-luminal influences between members of the photon pairs or between distant experimental settings and when these speculations failed *quantum nonlocality* became a mystery of Nature.

Let us show below that, in spite of equation (2), QM predicts only imperfect correlations for SPCE and a causal explanation of observed correlations is possible.

(i) In SPCE a pulse from a laser hitting the nonlinear crystal produces two correlated signals propagating in opposite directions. Clicks on the distant detectors are correlated.

(ii) Bohr strongly insisted on a wholeness of quantum phenomena. We do not see pairs of photons when they are created and when they travel across the experimental set-up. We only register clicks on the detectors.

(iii) Quantum field theory tells us only that a photon is a one photon state of quantized electromagnetic field and how it can be used in quantum calculations without giving us any intuitive picture of photons or virtual photons.

(iv) QM mechanics does not predict strict anti-correlations [36, 39–41] because directions of spin polarization analyzers are not sharp and can only be defined by some small intervals $I_A$ and $I_B$ containing angles close to $\theta_A$ and $\theta_B$, respectively. Therefore even if detection

efficiencies were perfect and if we dealt with a perfect singlet spin state a correct prediction of QM for expectation values in SPCE is:

$$E(AB|\Psi) = -\int_{I_A}\int_{I_B}\cos(\theta_1-\theta_2)\,d\rho_A(\theta_1)\,d\rho_B(\theta_2) \tag{3}$$

(v) In order to estimate correlations one has to define specific time windows in distant locations and define a pairing of clicks observed. There are many cases when a click is observed only in one location or no clicks are observed at all.

(vi) To explain outcomes of SPCE, instead of a singlet state, more complicated mixed quantum states have to be used and strict anti-correlations are not predicted even for sharp directions. For more details see a paper by Köfler *et al.* [60].

(vii) LRHV models fail because they neglect a contextual character of QT: "*The measuring instruments must always be included as part of the physical situation from which our experience is obtained*" [61].

In SPCE we have two far away laboratories performing experiments $x$ and $y$ on two physical signals $S_1$ and $S_2$ produced by some source $S$. Outcomes of experiments $(x, y)$ in each particular time window are $(a, b)$ where $a = \pm 1, 0$ and $b = \pm 1, 0$. The outcomes form two ordered samples of data: $S_A = \{a_1, a_2, \ldots a_n \ldots\}$ and $S_B = \{b_1, b_2, ..bn \ldots\}$. These outcomes are observations of two time series of random variables $\{A_1, A_2 \ldots A_n \ldots\}$ and $\{B_1, B_2 \ldots B_n \ldots\}$ called sampling distributions. If all $A_i$ are independent and identically distributed (i.i.d) as some random variable $A$ and all $B_i$ are i.i.d as some random variable $B$ then outcomes of experiments $x$ and $y$ can be completely described by conditional generalized joint probability distributions (GJPD): $P(A = a, B = b|x, y, S_1, S_2)$.

GJPD, describing outcomes of distant experiments, have different properties than standard joint probability distributions of a multivariate random variable and they strongly depend on how pairing of distant outcomes is made [44].

For example let us define two pairings:

- Systematic pairing: $S_{AB}(1k) = \{(a_1, b_k), (a_2, b_{k+1}), (a_3, b_{k+2}) \ldots\}$
- Random pairing: $S_{AB}(R) = \{(a_s, b_t)|s \leq t \text{ and } s \text{ and } t \text{ are chosen at random}\}$.

If $S_A = \{-11 - 11 - 11 - 1 \ldots\}$ and $S_B = \{1 - 11 - 11 - 11 \ldots\}$ then for k-odd paring we have perfect anti-correlations, for k-even pairing we have perfect correlations and for random pairing there are no correlations at all...

In SPCE we have two synchronized clocks in both laboratories, time windows are chosen in function of them and an appropriate systematic pairing is used. There are several difficulties and uncertainties related to this procedure, called the coincidence-time loophole, which have to be closed [62].

It is important to underline that in statistics:

(i) Correlations do not prove any causal relation between $x$ and $y$.

(ii) No communication or direct influence between $x$ and $y$ is needed for their existence.

Now let us describe a plausible contextual model, consistent with local causality, able to describe imperfect long range correlations observed in SPCE.

(i) Signals $S_1$ and $S_2$, correlated at the source, when arriving to respective experimental settings $x$ and $y$, are described by supplementary parameters $\lambda_1 \in \Lambda_1$, $\lambda_2 \in \Lambda_2$ and $P(\lambda_1, \lambda_2)$.

(ii) A choice of a setting $x$ has no influence on measurements performed using a setting $y$ in a distant location (no signaling, parameter independence).

(iii) Experimental settings in each location can be chosen randomly or in a systematic way and observed correlations do not depend on how a choice is made.

(iv) Measuring devices, as perceived by incoming signals, are described by supplementary parameters $\lambda_x \in \Lambda_x$, $\lambda_y \in \Lambda_y$, $P_x(\lambda_x)$ and $P_y(\lambda_y)$.

(v) To preserve a partial memory of correlations created by a source outcomes $a$ and $b$ have to be produced in a local and deterministic way in function of local supplementary parameters $\lambda = (\lambda_1, \lambda_2, \lambda_x, \lambda_y)$ describing signals and measuring devices in successive time windows. Namely $a = A_x(\lambda_1, \lambda_x)$ and $b = B_y(\lambda_2, \lambda_y)$ where $A_x$ and $B_y$ are functions equal $\pm 1$ or $0$.

(vi) Correlations in this local and causal model are described by:

$$E(AB|x,y) = \sum_{\lambda \in \Lambda_{xy}} P(\lambda)\, A_x(\lambda_1, \lambda_x) B_y(\lambda_2, \lambda_y) \tag{4}$$

where $P(\lambda) = P(\lambda_1, \lambda_2)P_x(\lambda_x)P_y(\lambda_y)$ and $\Lambda_{xy} = \Lambda_1 \times \Lambda_2 \times \Lambda_x \times \Lambda_y$ depend on (x, y).

Let us draw a causal graph representing how successive results $(a, b)$ are produced:

$$x{\to}\Lambda_x{\to}\lambda_x{\to}a{\leftarrow}\lambda_1{\leftarrow}\Lambda_1{\leftarrow}S_1{\leftarrow}S{\to}S_2{\to}\Lambda_2{\to}\lambda_2{\to}b{\leftarrow}\lambda_y{\leftarrow}\Lambda_y{\leftarrow}y. \tag{5}$$

It is important to underline that assuming a free will a choice of experimental settings $(x, y)$ does not depend on supplementary parameters $\lambda = (\lambda_1, \lambda_2, \lambda_x, \lambda_y)$ but of course parameters $(\lambda_x, \lambda_y)$ strongly depend on the choice of the settings. The reasoning based on the symmetry which tries to prove that the assumption of a free will implies that $\lambda$ do not depend on the choice of settings is simply incorrect.

If model (4) is used different Bell type inequalities may not be proven because there is no common probability space for all settings $(x, y)$ [44].

One does not need to evoke *nonlocal randomness* and a quantum magic to give an intuitive causal interpretation of Hansen *et al.* [15] experiment. We base our discussion below on a pedagogical description of this experiment given by Howard Wiseman [63]. Alice and Bob in successive time windows create entangled states of their electrons with photons. Photons are sent to Juanita's laboratory. Alice and Bob randomly choose settings for measurements of their respective electrons. They obtain their measurement outcomes and Juanita performs a joint measurement of photons sent by Alice and Bob.

If Juanita registers undistinguishable photons it means that the electrons in distant locations were prepared in particular physical states in which outcomes of measurements of their spin polarisation projections are correlated. In this experiment the cause of correlations is not a partial memory of a common past or instantaneous communications coming from Juanita but similar physical conditions created independently in far-away locations when "successful" measurements were performed. Outcomes of Juanita's experiments give only information whether such conditions were created or not in a given time-window. The decisions which measurement outcomes are post-selected to estimate correlations depend on rare "successful" Juanita's results.

A detailed discussion of an intimate relation of experimental protocols and probabilistic models and why (4) cannot be derived by partial integration from some larger common probability space was given in [44]. Let us mention here an important paper by Andrei Khrennikov [33] who constructed a rigorous Kolmogorov model for SPCE in which CHSH could not be proven. In the next section we examine some finite sample proofs of Bell and CHSH inequalities and show that they contain flaws and use assumptions not valid for SPCE.

## 4. Finite sample proofs of Bell type inequalities and quantum Randi challenges

Finite samples due to statistical fluctuations may violate Bell and CHSH inequalities even if a corresponding probabilistic model never violates them. Assuming particular experimental protocols Richard Gill [16, 55] found probabilistic bounds on possible violations of CHSH in function of a sample size N.

In his first influential paper [16] he studies an experimental set-up of five computers: $O$, $x$, $y$, $R_A$ and $R_B$.

- A computer O, called a source, sends two correlated messages: strings of the length N containing approximately 50% of 0s and 1s.

- Computers x and y are "measurement stations" producing outputs $\pm 1$ in function of the messages received. Produced strings contain approximately 50% of $-1$ and 1.

- Computers $R_A$ and $R_B$, called randomizers, send randomly (as they were results of independent fair coin tosses) setting labels "1" or"2" for each pair of outcomes produced by $x$ and y. <u>This is the only source of randomness.</u>

- Regrouping pairs of outputs corresponding to four possible choices of pairs of labels: (1,1), (1,2), (2,1) and (2,2 ) we obtain 4 samples of sizes $\approx N/4$ formed as by a random pairing $S_{AB}(R)$ of distant outcomes $\pm 1$ produced by $x$ and y. As we saw such pairing destroys pre-existing correlations between messages as in SHV [44] and remaining correlations satisfy CHSH inequality.

Using this idealized model of SPCE he proves that a probability, of finding a violation of CHSH, so large as predicted by QM and reported by Weihs *et al.* [11], is smaller than $10^{-32}$.

The protocol described above can be modified in a way that a sample from some joint probability distribution of the outcomes $(\pm 1, \pm 1; \pm 1, \pm 1)$ corresponding to 4 possible experimental settings is prepared. From this sample, using the labels sent by randomizers, marginal samples for particular settings are extracted.

This protocol is described in detail in [55]. An idealized SPCE with N subsequent *"photon pairs"* is analyzed. There are no losses and 4 possible experimental settings are chosen in a random way.

For clarity of an argument we replace N by 4N. Possible outcomes $\pm 1$ are values of 4 random variables $A$, $A'$, $B$, $B'$. Assuming that *"measurements which were not done also have outcomes; actual and potential measurement outcomes which are independent of the measurement settings actually used by all the parties"* Gill describes $4N$ subsequent pairs using $4N \times 4$ spreadsheet of numbers $\pm 1$. The rows are labelled by an index $j = 1, 2, \ldots, 4N$ and columns by $A$, $A'$, $B$, and $B'$.

This spreadsheet defines a random sample of size $4N$ drawn from some joint probability distribution of 4 random variables $(A, A', B, B')$ and marginal expectation values $E(AB)$ values can be estimated by $\langle AB \rangle = \frac{1}{4N} \sum_{j=1}^{4N} A_j B_j$. Similarly one finds estimates $\langle A'B \rangle$, $\langle AB' \rangle$ and $\langle A'B' \rangle$. Since for any row we have $A_j B_j + A_j B'_j + A'_j B_j - A'_j B'_j = \pm 2$ thus for each value of $N$ we obtain CHSH inequality:

$$\langle AB \rangle + \langle AB' \rangle + \langle A'B \rangle - \langle A'B' \rangle \leq 2. \qquad (6)$$

In SPCE settings are chosen at random thus Gill constructs finite samples of expected size N for each experimental setting in the following way: "Suppose *that for each row of the spreadsheet, two fair coins are tossed independently of one another, independently over all the rows. Suppose that depending on the outcomes of the two coins, we either get to see the value of A or A', and either the value of B or B'. We can therefore determine the value of just one of the four products AB, AB', A'B, and A'B', each with equal probability 1/4, for each row of the table. Denote by $< AB >_{obs}$ the average of the observed products of A and B ("undefined" if the sample size is zero). Define $< AB' >_{obs}$, $< A'B >_{obs}$ and $< A'B' >_{obs}$ similarly. When N is large one*

*would expect $< AB >_{obs}$ to be close to $< AB >$ and the same for the other three averages of observed products. Hence the equation (6) should remain approximately true when we replace the averages of the four products over all 4N rows with the averages of the four products in each of four disjoint subsamples of expected size N each*".

Following this construction Gill finds some probabilistic bound on the violation of (6) and makes a conjecture:

$$\Pr\left(\langle AB\rangle_{obs} + \langle AB'\rangle_{obs} + \langle A'B\rangle_{obs} - \langle A'B'\rangle_{obs} \geq 2\right) \leq \frac{1}{2} \tag{7}$$

He proposes also a Quantum Randi Challenge (QRC): "*Construct 4Nx4 spreadsheet deduce from them 4 marginal samples of the expected size N, find the expectation values: $\langle AB\rangle_{obs}$, $\langle A'B\rangle_{obs}$, $\langle AB'\rangle_{obs}$, and $\langle A'B'\rangle_{obs}$ and check the CHSH inequality. If the program reproducibly, repeatedly (significantly more than half the time) violates CHSH, then the creator has created a classical physical system which systematically violates the CHSH inequalities, thereby disproving Bell's theorem.*

Of course this QRC is impossible since finite samples of expected size N, extracted as above, from a counterfactual 4Nx4 spreadsheet may not, as it was proven, violate CHSH significantly and more than half the time. It does not mean that CHSH may not be consistently violated by finite experimental samples or samples generated using a specific local contextual model (4).

CFD is also used by Sacha Vongher [56]. He considers 800 tennis balls with instructions written on them. Each ball results in a measurement 0 or 1 according to the angle it encounters and instructions (HV) it carries. Angles at the measuring stations and HV may change randomly for each pair. Vongher uses angles $\alpha = a\pi/8$ ($a = 0$ or $3$) and $\beta = b\pi/8$ ($b = 0$ or $2$). The instructions on each ball tell what to output for any choice of the angles it encounters: $(A_0, A_3)$ and $(B_0, B_2)$ for Alice's and Bob's balls respectively.

For each pair of balls we can fill one row of $800 \times 4$ counterfactual spreadsheet but now instead of $-1$ we put 0. Vongher adds an additional constraint the strict anti-correlation for $a = b = 0$ what reduces degrees of freedom to 3 and he chooses for remaining independent variables: $A_3$, $B_0$ and $B_2$. Different settings are labelled by $d = |b-a|$ and $N_d \approx 200$ for $d = 0, 1, 2, 3$. Outcomes are counted by 8 counters $N_d(X)$ where $X = E$(equal) or $X = U$(unequal). In his model $N_0(E) = 0$ and $N_0(U) \approx 200$ because of imposed strict anti-correlations. Using this notation and the $800 \times 4$ spreadsheet Vongher proves a finite version of Bell inequality:

$$N_1(U) \leq N_2(E) + N_3(U) \tag{8}$$

He simulates his experiment with tennis balls running a computer program 1000 times. Results are revealing statistical fluctuations present in any finite sample and are very instructive.

- No violation of Bell and CHSH inequalities was observed for 800 pairs.
- For second model, in which strict anti-correlations for $d = 0$ were kept but some pairs were not prepared, Bell inequality (8) and CHSH were violated 50% of the time in 1000 runs.
- For third model, in which anti-correlations occurred only 87% of time, Bell inequality was violated 87% of time and CHSH still only 50% of time.
- For simulations based on the idealized quantum model (2) Bell inequality was violated 91% and CHSH was violated 99% of time in 1000 runs.

QRC proposed by Vongher is the following. Write a computer program preserving the idea of CDF and strict anti-correlations for $d = 0$, simulate 1000 finite samples and show that Bell and CHSH inequalities are violated so consistently as by samples generated using the quantum model (2). Since finite samples constructed Gill and Vongher are similar to those drawn from the populations described by probabilistic models for which Bell, CHSH and Eberhard inequalities are satisfied therefore their QRC are impossible challenges.

In SPCE 4Nx4 spreadsheets do not exist. Outcomes are not predetermined by a source and in some time-windows there are no counts or only single counts are detected. Thus random variables $A$, $A'$, $B$ and $B'$ take 3 values $\pm 1$ and 0 and a post-selection is needed in order to extract interesting finite samples containing the outcomes obtained when particular experimental settings are used. Correlations estimated in different incompatible experimental settings do not need to satisfy Bell, CHSH or CH inequalities [20–50].

As we explained in the discussion following the equation (3) strict anti-correlations are neither predicted by QM nor observed in SPCE. "Photons" are neither tennis balls with the instruction written on them (LRHV) nor perfect dices (SHV).

A strong argument against *nonlocal randomness* was given by Hans de Raedt and Kristel Michielsen et al. [51–54] who simulated event by event in a local and causal way several experiments in quantum optics and in neutron interferometry.

One could try to reproduce outcomes of various SPCE using a particular contextual model (4) but it was not done.


## 5. How to win Bell's game without nonlocal randomness

In order to explain in a pedagogical way *quantum nonlocality* Nicolas Gisin discusses a particular Bell's game [17].

Alice and Bob have two identical boxes each equipped with a joystick and a screen. If the joystick is pushed to the left or to the right from a vertical (neutral position) a result appears on a screen. The results are binary: 0 or 1. Alice and Bob synchronize their watches and move some distance apart. Starting at 9am every minute they push the joysticks on their boxes and record joystick positions and the results on the screen. Alice does not know the choice of the position of the joystick made by Bob and vice versa.

Let $x = 0$ or 1 denote positions of the joystick on the Alice's box and $a = 0$ or 1 displayed results. Let $y = 0$ or 1 denote positions of the joystick on the Bob's box and b=0 or 1 displayed results.

The rules of Bell's game are the following:

- Settings (positions) $x$ and $y$ are chosen randomly.
- Outcomes $a$ and $b$ are determined locally in function of $x$ and $y$.
- If $(x, y) = (1, 1)$ and $a \neq b \rightarrow 1$ point gained.
- If $(x, y) \neq (1, 1)$ and $a = b \rightarrow 1$ point gained.
- Otherwise no point is gained.
- The game is won if the average score is greater than 3.

These rules can be summarized by a simple equation: $[a + b]_2 = xy$ ( a sum modulo 2 of $a$ and $b$ is equal to a normal product of $x$ and $y$ ). A point is gained if the equation is obeyed. A claim is made that $S$ is always smaller than 3 thus the winning of Bell's game is impossible.

Let us analyze a proof given in [17]. Since the results $a$ and $b$ are determined locally in function of $x$ and $y$ thus each box may use only 4 available local programs which are denoted $i$ and $j$.

- $i = 1$: $a = 0$ for all $x$      $j = 1$: $b = 0$ for all $y$
- $i = 2$: $a = 1$ for all $x$      $j = 2$: $b = 1$ for all $y$
- $i = 3$: $a = x$                 $j = 3$: $b = y$
- $i = 4$: $a = 1 - x$      $j = 4$: $b = 1 - y$

We have 16 combinations of programs $(i, j)$. Programs can change at each minute. Programs $(i, j)$ determine outcomes $(a, b)$ for settings $(x, y)$. Gisin displays all possible outcomes for 16

| $(i, j)$ | $(a,b)$ for $(0,0)$ | $(a,b)$ for $(0,1)$ | $(a,b)$ for $(1,0)$ | $(a, b)$ for $(1,1)$ | $S$ |
|---|---|---|---|---|---|
| $(1,1)$ | $(0,0)$ | $(0,0)$ | $(0,0)$ | $(0,0)$ | 3 |

**Table 1.** Counterfactual calculation of scores of Bell's game in [17]

| Time | $(i,j)$ | $(x,y)$ | $(a,b)$ | $S$ |
|---|---|---|---|---|
| 1 | $(1,1)$ | $(0,0)$ | $(0,0)$ | 1 |
| 2 | $(2,2)$ | $(0,1)$ | $(1,1)$ | 1 |
| 3 | $(4,3)$ | $(1,1)$ | $(0,1)$ | 1 |
| 4 | $(3,4)$ | $(1,0)$ | $(1,1)$ | 1 |

**Table 2.** Calculation of scores in 4 consecutive minutes of Bell's game

possible pairs of programs $(i, j)$ in a 16 row table. In Table 1 we reproduce the first row of Gisin's table (on page 17 of his book) using slightly different notation.

For 3 first settings $a = b$ thus 1 point is gained, no point is gained for the setting $(1, 1)$ for a total score $S = 3$. This is the maximal score possible. For example if $(i, j) = (1, 2)$ then $a = 0$ and $b = 1$ for all settings and a total score is $S = 1$, Thus the average $< S >$ of scores obtained, in this way, for any set of lines is necessarily smaller than 3.

The reasoning given above is counterfactual and incorrect since programs chosen on each side may change every minute. In Table 1 it is assumed that the same couple $(1, 1)$ of programs is used for all possible 4 settings chosen one after another. This is not what is happening, minute after minute, during Bell's game thus one cannot use scores of each line in order to estimate $< S >$ for the game.

We display below in Table 2 possible scores in 4 consecutive minutes of Bell's game

Even if there are no common causes responsible for choices of settings one can, for a particular finite sample, obtain $< S > \approx 4$ consistent with no-signalling. If $(i, j)$ and $(x, y)$ were chosen at random we would expect to obtain $< S > \approx 2$ in a long run.. Therefore it is clear that in order to win Bell's game consistently outcomes are neither predetermined as in Table 1 nor produced randomly.

The experiment described above is called Bell's game because it resembles an idealised SPCE for which QM, using (2), predicts an average score $< S > \approx 3.41$. Let us notice that if outcomes for each setting are predetermined the experimental protocol of Bell's game is similar to the protocol used by Vongher [56] in which $(i, j)$ are instructions written on pairs of tennis balls.

If we reject *nonlocal randomness* we have to use a principle of a common cause, local causality and contextuality in order to explain how a protocols $(i, j)$ may depend on settings chosen randomly in distant locations.

We follow the same logic which led us to equations (4) and (5). Every minute settings are chosen randomly and their microstates are described by two local parameters $(\lambda_x, \lambda_y)$ drawn from $\Lambda_x \times \Lambda_y$. Two correlated signals, described by parameters $(\lambda_1, \lambda_2)$ drawn from $\Lambda_1 \times \Lambda_2$, arrive to the Alice's and Bob's boxes. The outcomes $(a, b) = (i(x), j(y))$ where $i = f(\lambda_1, \lambda_x)$ and $j = g(\lambda_2, \lambda_y)$. Thus protocols used by distant boxes are correlated and correlations depend on settings and on correlated signals arriving to boxes.

Calculations in the equation $[a + b]_2 = xy$ cannot be done locally what means that neither Alice nor Bob can find both values $a$ and $b$ having only their local information. It does not mean that Nature does not satisfy the continuity principle and is nonlocal. The outcomes $(a, b)$ are not results of magical nonlocal calculations but they are produced in locally causal interactions of correlated signals with experimental devices in different experimental contexts.

## 6. Sample homogeneity loophole and violation of Bell type inequalities

Bell-CHCH-CH inequalities cannot be proven if a contextual character of physical observables is correctly taken into account [22–47]. Therefore it is not strange for us that they are violated in SPCE, in experiments from different domains of science [31, 32] and even in classical mechanics [42].

After several ingenious experiments and after closing several experimental loopholes [9–15] there seems to be strong experimental evidence that Bell type inequalities are violated in Nature.

However there are two problems troubling us. Both of them are related to finite statistics.

(i) As Vongher demonstrated when a particular local realistic model was used to generate 1000 samples of size 800 it turned out that Bell inequality was violated 87% of time and CHSH 50% of time. To compare in the experiment of Weihs *et al.* [11] two long runs were analysed and in only one of them a significant violation of CHSH was observed. In Giustina *et al.* [13] and in Christenson *et al.* [14] experiments only one long run was analysed and a significant violation found. In Hensen *et al.* [15] we have only one sample containing 245 data items.

(ii) Statistical analysis made in these experiments assumed that studied samples were homogeneous. Recently we reported with Hans de Raedt [57] a dramatic breakdown of statistical inference due to sample inhomogeneity. Therefore if sample homogeneity is not checked results of significance tests may not be trusted and we say that the *sample homogeneity loophole* (SHL) was not closed [64].

In order to show how detrimental SHL can be we quote here one example from our paper [57]. We simulated a random experiment in which a measuring device, operating according to some specific internal protocol, was outputting one of 6 possible discrete values. We generated 100 runs (each run containing $10^5$ data items). Using these large samples we made a significance test of a null hypothesis $H_0 : 1 - B \geq 0$. When three runs 25, 50 and 75 were used the inequality was violated for each run by more than 2000 SEM (standard error of the mean) and one could with a great confidence reject the null hypothesis. When we performed the average over 100 runs ($10^7$ data items) $1 - B = +0.95$ SEM and of course the null hypothesis could not be rejected. The reason was that samples produced by our device were not homogeneous.

Let us now comment on the experiment of Giustina *et al.* [13]. In this experiment a source is sending polarization-entangled photons which after passing by one of four possible polarization measuring settings $(\alpha_i, \beta_j)$ are sent to two detectors one operated by Alice and another by Bob. Clicks on the detectors are registered and the coincidence counts determined.

Each setting defines a different random experiment and in order to check Eberhard's inequality one estimates a value of a random variable $J$ being a particular combination of values of 6 random variables (4 coincidence counts and 2 single counts) deduced from data gathered in all four experimental settings. If the local realism (CFD) was true $J$ should be always positive thus the null hypothesis tested is $H_0 : J \geq 0$.

Data gathered during 300 seconds of recording per setting were divided into 30 bins and a sample S of 30 different $J$-values was obtained. From this sample a value of the mean $< J >$ together with its standard mean error SEM$= s/n^{0.5}$ ($n = 30$, $s$=sample standard deviation) were estimated and $67\sigma$ (67 SEM) violation of Eberhard's inequality was reported.

Usually we use this terminology if we are convinced that the central limit theorem can be used for a finite sample and $< J >$ is normally distributed. Since the normality of this distribution cannot be proven Khrennikov *et al.* [65] used Chebyshev inequality and concluded that the null hypothesis can be still rejected at the confidence level of 99.95%.

However this null hypothesis test is based on only one (1) sample containing 30 data items. Since a whole set of data contains outcomes from 4 different random experiments it would be surprising if it was homogeneous. A sample of observed values of $J$ could only be homogeneous

if data sets obtained for each fixed setting were homogeneous and it was not tested carefully enough.

Giustina *et al.* [13] closed the " fair-sampling loophole". Köfler *et al.* [61] showed that the experiment was immune to the "production-rate loophole" and that the results were consistent with quantum theory. Larsson *et al.* [62] proved that the experiment was not vulnerable to the "coincidence-time loophole". It is clear that it is an excellent experiment performed with a great scrutiny. Nevertheless unless the additional sample homogeneity tests are performed and SHL is not closed its impact on conclusions of significance tests is unknown.

Similar opinion has Andrei Khrennikov [66] who postulates that the open access data-base for, e.g., Bell tests should be created and points out that the statistical analysis of data, which was reported as violating Bell's inequality, suffers of a number of problems.

One could wrongly understand that we do not believe in the experimental evidence indicating the violation of Bell type inequalities. As we already told above it is just the opposite.


## 7. Physical reality

All our science is built on the assumption that there exists an <u>objective</u> external world governed by some laws of Nature which we want to discover and to harness [67–70].

In Physics we construct idealized mathematical models in order to explain in qualitative and quantitative way various phenomena which we observe or we create in our laboratories.

Our perceptions are biased by our senses and by our brain and depend on the location from where they are made. For example motions of the planets are complicated when observed from the Earth and it took many centuries to explain them in a simple way using the heliocentric system and Newton's equations of motion. Stars and planets are perceived by a naked eye as clinking points on a sky. These images are created by our brain when the light hits the retina of our eyes but of course there is something real behind the scenes causing these perceptions, The stars and planets existed before the life existed on the Earth thus statements such as: "A Moon does not exist if we don't look at it", made in order to impress a general public, are incorrect and misleading.

Similarly in quantum phenomena, which we observe and create, there should be something behind the scenes which is causally responsible for statistical regularities we discover. In our opinion quantum probabilities neither correspond to irreducible propensities of individual physical systems nor to beliefs of some human agents but they are objective properties of quantum phenomena as a whole [4, 5, 31, 37, 67, 68, 70]. Necessity of probabilistic description is due to impossibility to see and control what is going behind the scenes.

Even in macroscopic physics we are unable to describe all observed phenomena. For example we neither can explain all ripples on a surface of a lake after passing of a boat nor motions of successive water droplets in a waterfall. In quantum phenomena we see some "ripples" but not "a lake"?

In contrast to classical physics measuring instruments do not register, in general, pre-existing values of physical observables characterizing physical systems. Value of a physical observable is obtained as a result of an incontrollable interaction of a physical system with a measuring device and any attempt to provide more details about what is going on behind the scenes has to include a description of a microscopic state of a measuring device in a moment of measurement.

Taking into account enormous successes of QM and QFT a more detailed <u>unambiguous</u> description of quantum phenomena does not seem to be needed or possible. However telling that there is nothing behind the scenes is in our opinion naïve, unproven and unproductive.

The efforts to prove that QM is an emergent theory and to understand better its foundations may help to dismiss the quantum magic and speculations that Nature is non local.

If locality of interactions and causality were violated in the micro-world how could we have locally causal macro-world and how could we reconcile QM and QFT with the General

Relativity?

Let us finish this section with three remarks.

- QM and QFT describe statistical properties of quantum phenomena in a way consistent with Einsteinian locality.
- According to fathers of QM a question what is a wave function of the Universe or a question by which slit electron passes in two slit interference experiment are meaningless. Therefore a statement that an electron is at the same time here and a meter from here is nonsense.
- *Quantum teleportation* is an implementable experimental protocol in quantum information but statements that Alice has two entangled photons, sends one of them to Bob and after makes a joint measurement of a remaining photon with a photon to be teleported are incorrect. Three photons cannot be manipulated as described above. Such statements have nothing to do with QM and create a confusion among general public.

The quantum magic is born if exotic interpretations of QM are adopted or if naïve or incorrect models of sub-quantum phenomena are constructed and examined.

When we start to evoke magic it is the end of physical explanation and an admission that we will never understand. Fortunately some papers presented at this conference and at previous Emergent Quantum Mechanics conferences show that we can get some intuitive insight on what might be going under the scenes. Let us cite here for example a paper by Grössing *et al.* [71].

## 8. Conclusions

Paradoxes are created if incorrect interpretation of QT is adopted or if inappropriate sub-quantal description of quantum phenomena is used. In our opinion a statistical contextual interpretation of QM reconciles the ideas of Bohr and Einstein and is free of paradoxes.

QM, QFT and a standard model in elementary particle physics are consistent with Einsteinian locality. The proofs of various Bell type inequalities are based on CFD or on the assumption that the experimental outcomes are produced in irreducibly random way. If these assumptions are not valid Bell-type inequalities cannot be simply proven.

Therefore paraphrasing Howard Wiseman [63] the violation of these inequalities observed in several experiments hammers, in our opinion, the final nail in the coffin of *counterfactual definiteness* and *irreducible randomness*.

In order to explain long range correlations observed in SPCE we do not need to postulate a new law of Nature called *nonlocal randomness*. An intuitive, contextual and causal explanation may be given: signals are correlated by a source, keep partial memory of it when flying to distant laboratories and outcomes are produced in locally deterministic way in function of micro-states of the signals and of measuring devices at the moment of the measurement.

Therefore the violation of Bell type inequalities gives an additional argument in favor of a point of view that quantum probabilities might emerge from some underlying more detailed and causal description of quantum phenomena to be discovered.

We have recently demonstrated with Hans de Raedt [57] that significance tests may dramatically break down if studied samples are not homogeneous. We do not doubt that various Bell type inequalities are violated in SPCE however the results of various significance tests may not be trusted since sample homogeneity was not or could not be tested carefully enough.

Concerning the Physical Reality we strongly believe that there exists an external world whose existence does not depend on whether it is observed or not. This external world is governed by laws of Nature which we try to discover. Quantum phenomena which we create depend on the devices used to probe this external world and on detailed contexts of our experiments. The information we get is contextual and complementary but quantum probabilities are objective properties of quantum phenomena and not beliefs of some human agents.

An additional argument in favor of the idea that QM might be an emergent theory would be given

by a discovery of some fine structures in experimental time series of data which were not predicted by the theory. It would not only prove that QM may not provide the most complete description of the individual physical systems but it would also prove that QM is not predictably complete [35, 40, 72, 73].

Let us finish this article with words of Einstein [2]:"*Is there really any physicist who believes that we shall never get any insight into these important changes in the single systems, in their structure and their causal connections... To believe this is logically possible without contradiction; but, it is so very contrary to my scientific instinct that I cannot forego the search for a more complete description*".

## Acknowledgments

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