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Bohmian Conditional Wave Functions (and the status of the quantum state)

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Abstract. The de Broglie - Bohm pilot-wave theory – uniquely among realistic candidate quantum theories – allows a straightforward and simple definition of the wave function of a sub-system of some larger system (such as the entire universe). Such sub-system wave functions are called "Conditional Wave Functions" (CWFs). Here we explain this concept and indicate the CWF's role in the Bohmian explanation of the usual quantum formalism, and then develop (and motivate) the more speculative idea that something like single-particle wave functions could replace the (ontologically problematical) universal wave function in some future, empirically adequate, pilot-wave-type theory. Throughout the presentation is pedagogical and points are illustrated with simple toy models.

1. Introduction

One of the central themes of the workshop was assessing the status (ontological or otherwise) of the quantum state. And, as is always the case in quantum foundations, questions about Bell's theorem and nonlocality were not far below the surface.

The pilot-wave theory of de Broglie and Bohm (a.k.a. "Bohmian Mechanics") is perhaps best known as a realistic model in which quantum non-locality is manifest. This fact is often cited, by critics, as a justification for their negative assessment of the theory. But in fact such an attitude rests on a misunderstanding of Bell's theorem. Dynamical non-locality is not the price one pays for attempting to restore determinism or "realism" to quantum theory, but is instead a necessary feature of any genuine physical explanation of the empirically-observed correlations in EPR-Bell-type experiments. That is what Bell's theorem shows, and in that sense non-locality is simply a fact of nature that we must accept. [1]

But there are actually two very different notions of "non-locality" that we should dis-entangle. One sense is the *dynamical* sense (in which "non-locality" means, roughly, "faster-than-light causal influences"). This is what I meant in the previous paragraph. But the de Broglie - Bohm pilot-wave theory also exhibits the second, more *ontological*, sense of "non-locality." In particular, the (universal) wave function – which is certainly *real* according to the pilot-wave theory – is not a local object (such as something that exists at, or assigns properties to, points or regions of three-dimensional physical space or 4-dimensional space-time). It is, instead, perhaps something like a dynamical field (obeying a kind of wave equation) – but a field that lives on an abstract high-dimensional configuration space. To use Bell's terminology, the wave function may be (for the pilot-wave theory) a "beable", but it is not a "local beable". It is, rather, a "non-local beable."


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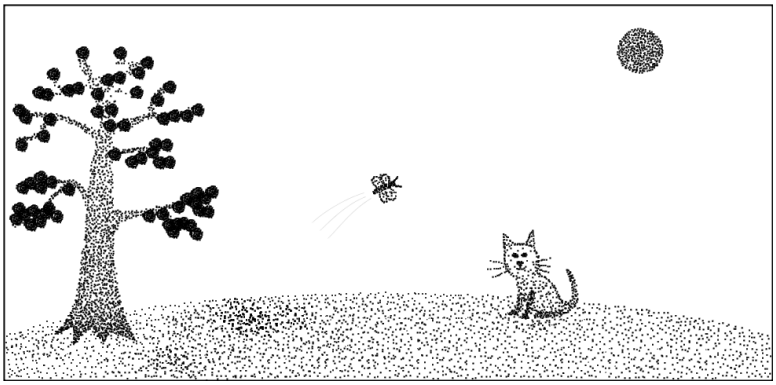

Figure 1. The familiar world of ordinary perception is, according to Bohmian Mechanics, made of the particles (whose motions are choreographed by, but which are not the same thing as, the wave function Ψ). It is a happy and comfortable world in which one can find things like butterflies and unambiguously-alive cats.

These two senses of "non-locality" are not entirely unrelated. For example, some authors have suggested that a field on configuration space is natural and appropriate for a dynamically-nonlocal theory, and/or that in some sense the theory is (dynamically) nonlocal *because* the wave function lives on configuration space. [2] On the other hand, it is certainly possible to have a theory that contains exclusively local beables but which is nonlocal in the dynamical sense (think Newtonian gravity), or vice versa. So, conceptually, it is possible (and, I think, clarifying) to distinguish the two notions.

In any case, it is the specifically ontological notion of "non-locality" that we will focus on here. After briefly reviewing, in Section 2, the standard formulation of the pilot-wave theory (in which the universal wave function seems to have the status of a "non-local beable"), we introduce, in Section 3, the Bohmian Conditional Wave Function (CWF) and highlight some of its intriguing properties, especially *vis-a-vis* measurement. Section 4 briefly explains how the CWF can be used as part of a rigorously-derivable semi-classical approximation scheme, and then Section 5 introduces a related (so-called "Bohmian Double Semi-Quantum", BDSQ) approximation scheme. Finally, in Section 6, we discuss how something like the BDSQ could perhaps be a useful jumping-off point for producing a (dynamically non-local) theory of exclusively local beables.

2. Bohmian Mechanics

According to Bohmian Mechanics, a complete description of the physical state of the universe involves the (universal) wave function $\Psi = \Psi(\vec{x}_1, \vec{x}_2, ..., \vec{x}_N, t)$ as well as the configuration $Q(t) = \left\{\vec{X}_1(t), \vec{X}_2(t), ..., \vec{X}_N(t)\right\}$ of N particles. In the simple case of non-relativistic, spinless particles the wave function evolves in accordance with Schrödinger's equation

$$i\hbar\frac{\partial\Psi}{\partial t} = \sum_{i=1}^{N}\frac{-\hbar^2}{2m_i}\vec{\nabla}_i^2\Psi + V(\vec{x}_1, ..., \vec{x}_N)\Psi. \tag{1}$$

The particle positions evolve under the influence of the wave function as follows:

$$\frac{d\vec{X}_i}{dt} = \frac{\hbar}{m_i}\,\mathrm{Im}\left(\frac{\vec{\nabla}_i\Psi}{\Psi}\right)\Bigg|_{\vec{x}_j=\vec{X}_j\ \forall\ j}. \tag{2}$$

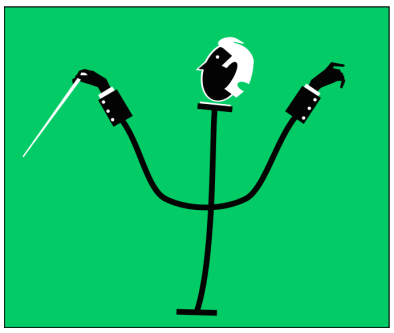

Figure 2. The wave function of the universe (artist's conception).

Note in particular that those two equations completely define the dynamics of the theory, and they apply exceptionlessly. Unlike in textbook/orthodox quantum mechanics, for example, there are no special provisions for violations of the usual dynamical evolution during "measurements."

How should one think about "physical reality" according to the pilot-wave theory? To begin with, it is crucial to appreciate that ordinary material objects (like tables, trees, cats, planets, and pieces experimental measuring equipment) are made of the particles. The particles are (in Bell's terminology) "local beables" and function (in the terminology of Goldstein *et al.*) as the "primitive ontology" of the theory. [3] The wave function is also real, according to the theory, but is by contrast not a "local beable" and not part of the theory's "primitive ontology". Unlike the particles, Ψ is a spooky, invisible, ethereal object which, so to speak, lurks offstage and orchestrates the motion of the particles according to Equation (2). These two components of the theory's posited ontology are illustrated in Figures 1 and 2, respectively.

The greatest virtue of Bohmian Mechanics is the simplicity with which one can understand, in terms of the posited local beables (namely, particles with definite positions), the emergence of a familiar macroscopic world of three dimensional objects. This is what's really illustrated in Figure 1. And in so far as one regards the theory's *dynamical* non-locality as a necessary feature (rather than a bug), the theory's only vice would seem to be that it is very difficult to understand what kind of thing, exactly, the universal wave function Ψ is supposed to be. As a "non-local beable", it does not in any straightforward sense exist in ordinary physical space, and yet it influences the particles in physical space. How does that work? And what does it even mean for a physically real thing to not exist in physical space? Is there some other space where it does exist? Physically? Questions of this sort have led some to suggest that Ψ is not a "thing" at all, but is instead more like a "law". [4]

Our ultimate goal here is to propose an alternative possibility. But first we must put some technical groundwork in place.

3. Bohmian Conditional Wave Functions

It is important to appreciate that the Ψ which appears in Equation (1) is the wave function of the universe. This is not the wave function one normally deals with in quantum mechanics, which is always the wave function for some small system which is (say) later to be *measured* by means of an appropriate interaction with some other object (a measuring device, say) outside the small system of interest. So it is not immediately obvious how to relate the axioms of Bohmian Mechanics to textbook quantum mechanics.

As it turns out, though, Bohmian Mechanics does allow one to fully understand – to derive – the textbook QM formalism, including even the infamous collapse postulate. To see this, notice that Bohmian mechanics provides a natural (and, among candidate quantum theories, unique)

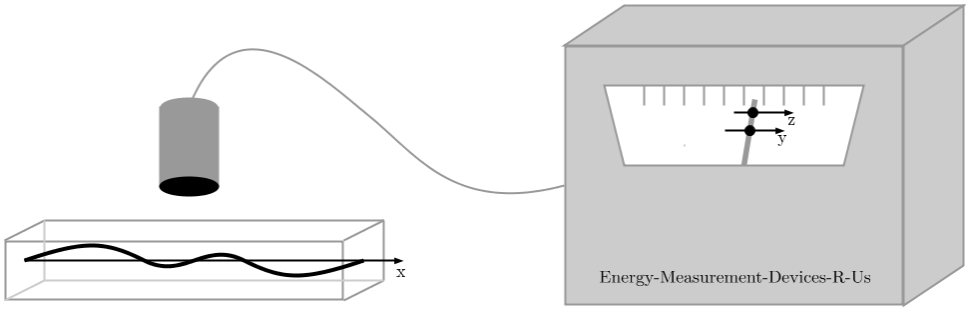

Figure 3. The energy of a "particle-in-a-box" is measured, and the outcome is registered in the final position of the particles composing the pointer.

way of defining the wave function of a sub-system. Consider, for example, a particular particle with degree of freedom x. We can divide the generic configuration point $q = \{x, y\}$, where y now denotes the coordinates of *other* particles, i.e., degrees of freedom from outside our sub-system of interest. We then define the conditional wave function (CWF) for our particle as follows:

$$\phi(x,t) = \Psi(q,t)|_{y=Y(t)} = \Psi\left[x, Y(t), t\right]. \tag{3}$$

That is, the CWF for one particle is simply the universal wave function, evaluated at the actual positions $Y(t)$ of the *other* particles.

Note that the "guidance equation" (according to which the wave function "orchestrates" the motion of the particles) for each particle can be re-expressed in terms of that particle's CWF

$$\frac{d\vec{X}}{dt} = \frac{\hbar}{m}\,\mathrm{Im}\left(\frac{\vec{\nabla}\phi(x,t)}{\phi(x,t)}\right)\Bigg|_{x=X_j(t)} \tag{4}$$

so that the CWF can be understood as having precisely the dynamical significance, for the associated particle, that one perhaps normally imagines to be involved in a "pilot-wave theory".

It is fairly straightforward to see that, in the case where the particle in question is suitably decoupled from its environment, the associated CWF will obey the obvious single-particle Schrödinger equation that one would have expected from the point of view of textbook quantum mechanics.

The more interesting and surprising thing is that, in the kind of case that we would normally describe as a *measurement*, the CWF *collapses* according to the usual textbook rule, even though nothing like a "collapse postulate" appears in the basic axioms of the theory. Let us illustrate with a simple toy model, in which the quantum system is a one-dimensional "particle-in-a-box" and the energy-measurement apparatus is treated very schematically. In particular, we explicitly track just the horizontal positions of two particles in the apparatus pointer – see Figure 3 – which are taken to interact with the particle-in-a-box according to the interaction Hamiltonian

$$\hat{H}_{int} = \lambda \hat{H}_x(\hat{p}_y + \hat{p}_z) \tag{5}$$

where $\hat{H}_x$ is just the energy of the PIB, and $\hat{p}_x$ and $\hat{p}_y$ are the momentum operators for the pointer particles. (The total Hamiltonian includes not only $\hat{H}_{int}$ and $\hat{H}_x$ but also kinetic energy terms for the two pointer particles.) The interaction is such that, if the initial quantum state is

$$\Psi(x,y,z,0) = \psi_n(x)G_0(y)G_0(z) \tag{6}$$

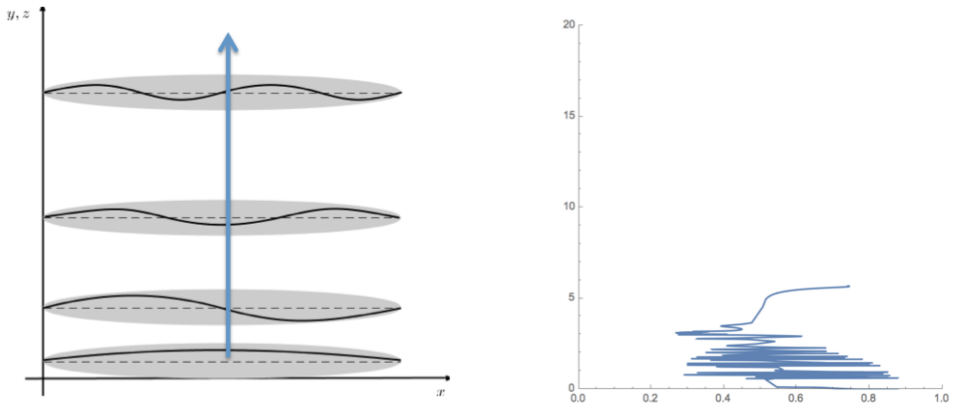

Figure 4. The left panel indicates the evolution of the wave function $\Psi(x, y)$ in configuration space – the z coordinate is suppressed here – while the right panel indicates the evolution of the actual particle configuration $(X(t), Y(t))$ for a random initial condition $(X(0), Y(0))$. Note that the final pointer position $Y(t) \approx 6$ is in the support of the $n = 2$ "branch" of Ψ and so we would say, in this case, that the energy measurement had outcome E_2.

(where ψ_n is an eigenstate of $\hat{H}_x$ with eigenvalue E_n and G_0 is a Gaussian wave packet centered on zero) the final state involes (pseudo-) Gaussian wave packets that have moved, to the right, by a distance proportional to E_n:

$$\Psi(x, y, z, t) = \psi_n(x) e^{-iE_n t/\hbar} G_t(y - \lambda E_n t) G_t(z - \lambda E_n t). \tag{7}$$

It then follows from the linearity of the Schrödinger equation that, if instead the PIB begins in a superposition of energy eigenstates

$$\Psi(x, y, z, 0) = \left(\sum_n c_n \psi_n(x) \right) G_0(y) G_0(z) \tag{8}$$

the final state is the entangled superposition

$$\Psi(x, y, z, t) = \sum_n c_n \psi_n(x) e^{-iE_n t/\hbar} G_t(y - \lambda E_n t) G_t(z - \lambda E_n t). \tag{9}$$

From the orthodox perspective (in which one regards the wave function itself as somehow describing or corresponding to physical reality) this last equation reflects the so-called measurement problem. But from the Bohmian point of view (in which it is the particles, not the wave function, which are supposed to correspond to the observable material world) there is no such problem. The particles move in some particular way and, in particular, the particles composing the pointer end up in some particular location which we interpret as indicating the (totally unambiguous and unproblematic and actual) outcome of the experiment. See Figure 4.

But then consider the time-evolution of the CWF of the particle-in-the-box. At $t = 0$, when the quantum state factorizes, the CWF is just (proportional to) the energy-superposed initial state of that particle: $\phi(x, 0) \sim \sum_n c_n \psi_n(x)$. Then, as the quantum state begins to separate into branches, the CWF evolves in a very complicated non-linear way. But once the branches have ceased to overlap appreciably in configuration space – see the left panel of Figure 4 – the CWF settles down into one of the energy eigenstates $\psi_n(x)$. For example, for the particular

initial particle positions illustrated in the Figure (with the final pointer position $Y(T)$ in the support of the $n = 2$ branch of Ψ), it is clear that

$$\phi_1(x, T) = \Psi\left[x, Y(T)\right] \sim \psi_n(x). \tag{10}$$

So the conditional wave function has "collapsed," as a result of the interaction with the energy measuring device, from a superposition of many energy eigenstates, to the one particular energy eigenstate that corresponds to the actual outcome of the experiment. (Some video animations of the particle-in-a-box's CWFs evolving – i.e., "collapsing", but in a perfectly continuous way – can be found in the supplementary materials.) This is the sense in which the pilot-wave theory *explains* the collapse postulate, and in general the rules that are instead postulates for ordinary QM. Although, according to the pilot-wave theory, the "big" (i.e., universal) wave function always evolves in accordance with the Schrödginer equation (and hence never collapses) the "small" wave functions associated with sub-systems obey a sub-system Schrödinger equation when (according to ordinary QM) they should, but also collapse when (according to ordinary QM) they should.

4. Bohmian Semi-Classical Techniques

In addition to helping us understand how, from the Bohmian pilot-wave point of view, the usual textbook quantum formalism can be understood to *emerge*, naturally, from a set of simple and unambiguous axioms, CWFs are also relevant to understanding and justifying semi-classical approaches. Let's discuss this in terms of a simple toy-model system: two interacting one-dimensional simple harmonic oscillators. The total Hamiltonian will be

$$\hat{H} = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2}m\omega^2 x^2\right) + \left(-\frac{\hbar^2}{2M}\frac{\partial^2}{\partial y^2} + \frac{1}{2}m\omega^2 y^2\right) + \lambda xy. \tag{11}$$

If (say) $M \gg m$ we might treat the system semi-classically, with the wave function $\psi(x, t)$ of the ("light") quantum system obeying

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + \frac{1}{2}m\omega^2 x^2\psi + \lambda xY\psi \tag{12}$$

with a contribution from the ("heavy") classical system whose position $Y(t)$ obeys

$$\ddot{Y} + \omega^2 Y + \frac{\lambda}{M}\mathcal{X} = 0. \tag{13}$$

In the usual ("orthodox") semi-classical treatment

$$\mathcal{X} = \langle\psi|\hat{x}|\psi\rangle \tag{14}$$

is the "quantum average position" of the x-system.

In the pilot-wave theory, though, we have the resources to let the x-system influence $Y(t)$ via the *actual*, Bohmian position, that is

$$\mathcal{X} = X(t) \quad \text{satisfying} \quad \frac{dX}{dt} = \frac{\hbar}{m}\,\mathrm{Im}\left(\frac{\nabla\psi}{\psi}\right)\bigg|_{x=X}. \tag{15}$$

One anticipates that, in situations where the "quantum average position" is very different from any of the individual quantum possibilities (e.g., there is a scattering event, with a transmitted packet and a reflected packet, and so the average position is nowhere near any of the possible

positions) the uniquely Bohmian semi-classical technique should yield more accurate results. This is indeed borne out by some preliminary studies, although this issue has not yet received the attention it probably deserves. [6, 7, 8, 9]

Note also that, whereas the orthodox semi-classical approach can only really be motivated in a somewhat hand-wavy way, it is possible to give a more rigorous derivation of the uniquely Bohmian semi-classical (BSC) approximation. To begin with, the full Bohmian description of the two-particle system involves the Schrödinger equation

$$i\hbar\frac{\partial\Psi(x,y,t)}{\partial t} = \left[\frac{-\hbar^2}{2m}\nabla_x^2 + \frac{-\hbar^2}{2M} + V(x,y)\right]\Psi(x,y,t) \tag{16}$$

as well as the guidance equations for the positions of the two particles:

$$\frac{dX}{dt} = \frac{\hbar}{m}\mathrm{Im}\left(\frac{\nabla_x\phi}{\phi}\right)\Big|_{x=X(t)} \tag{17}$$

(where ϕ is the CWF of the x-system) and

$$\frac{dY}{dt} = \frac{\hbar}{M}\mathrm{Im}\left(\frac{\nabla_y\Psi}{\Psi}\right)\Big|_{x=X(t),y=Y(t)}. \tag{18}$$

As is reasonably well-known, if we take a time derivative of this last guidance equation, it can be re-expressed in 2nd order form:

$$M\ddot{Y} = -\nabla_y\left(V+Q\right)|_{x=X(t),y=Y(t)} \tag{19}$$

where Q is the so-called "quantum potential". When (the gradient of) Q is negligible compared (the gradient of) V, we arrive at the semi-classical equation with the Bohmian-type feedback, i.e., Equation (13) with $\mathcal{X} = X(t)$.

It is then also possible to show that, as a consequence of the "big" wave function Ψ obeying Equation (16), the CWF ϕ satisfies the pseudo-Schrödinger equation

$$i\hbar\frac{\partial\phi(x,t)}{\partial t} = \frac{-\hbar^2}{2m}\nabla_x^2\phi(x,t) + V[x,Y(t)]\phi(x,t) + i\hbar\frac{dY}{dt}\left.\frac{\partial\Psi}{\partial y}\right|_{y=Y(t)} + \frac{-\hbar^2}{2M}\left.\frac{\partial^2\Psi}{\partial y^2}\right|_{y=Y(t)}. \tag{20}$$

This makes it clear that, when the last two terms on the right hand side can be ignored (i.e., roughly, if M is large and dY/dt is small) then we have exactly the semi-classical dynamical equation for the quantum half – a Schrödinger equation in which the particle is influenced by the "classical" position Y of the other particle through the interaction potential.

Thus, not only does the pilot-wave theory afford a somewhat novel version of the semi-classical treatment (which should and seems to work better than standard approaches in at least some cases), it also allows a more rigorous derivation and hence a better and more explicit understanding of when and why a semi-classical treatment might be appropriate. And it does this, in part, because the pilot-wave theory allows this very natural definition of "the wave function of a sub-system", the CWF.

5. The Bohmian Double Semi-Quantum (BDSQ) Approximation

The last section described the Bohmian approach to breaking a compound system apart into a quantum part and a classical part. It is interesting to note, though, that one could also break a compound system apart and treat both parts in a quantum way, with influences on each sub-system occuring via the Bohmian position of the particle in the other system, as in the

BSC treatment. I call this the "Bohmian Double Semi-Quantum" (or BDSQ) approximation scheme, and the idea centers around the Bohmian CWF. As noted in the previous section, for a two-particle system whose ("big") wave function Ψ obeys the Schrödinger equation, the CWF for particle 1 will obey

$$i\hbar\frac{\partial\phi_1(x,t)}{\partial t} = -\frac{\hbar^2}{2m_1}\frac{\partial^2\phi_1}{\partial x^2} + V[x,Y(t)]\phi_1 + i\hbar\frac{dY}{dt}\left.\frac{\partial\Psi}{\partial y}\right|_{y=Y(t)} - \frac{\hbar^2}{2m_2}\left.\frac{\partial^2\Psi}{\partial y^2}\right|_{y=Y(t)} \tag{21}$$

and similarly the CWF for particle 2 will obey

$$i\hbar\frac{\partial\phi_2(y,t)}{\partial t} = -\frac{\hbar^2}{2m_2}\frac{\partial^2\phi_2}{\partial y^2} + V[X(t),y]\phi_2 + i\hbar\frac{dX}{dt}\left.\frac{\partial\Psi}{\partial x}\right|_{x=X(t)} - \frac{\hbar^2}{2m_1}\left.\frac{\partial^2\Psi}{\partial x^2}\right|_{x=X(t)}. \tag{22}$$

And of course we also have the particle positions $X(t)$ and $Y(t)$, evolving under the influence of their respective CWFs. Dropping the last two terms in both of the previous two equations gives a simple, closed system of two one-particle Bohmian systems (i.e., particles moving under the influence of a pilot-wave which in turn evolves according to a Schrödinger-type equation) which interact: the position of each particle affects the evolution of the other particle's pilot-wave via the interaction potential.

Note that this scheme will exactly reproduce the ordinary Bohmian particle trajectories (and hence the empirical predictions of ordinary quantum mechanics) if the two particles are (and remain!) unentangled. (In this case, the "big" wave function Ψ is a product, and so each particle's Bohmian CWF is simply proportional to the state it would be assigned in ordinary QM.) So this is in some sense a "small entanglement approximation" (SEA). [10]

I illustrate this here with the simple example of the two interacting 1-dimensional SHOs mentioned before. This system is nice because it can be solved exactly. For example, if $\Psi(x,y,0) = \psi_1(x)\psi_0(y)$ (i.e., particle 1 is in the first excited state, $n = 1$, while particle 2 is in the ground state, $n = 0$), the exact solution of Schrödinger's equation is

$$\Psi(x,y,t) = \cos\left(\frac{\lambda t}{\hbar\omega}\right)\psi_1(x,t)\psi_0(y,t) - i\sin\left(\frac{\lambda t}{\hbar\omega}\right)\psi_0(x,t)\psi_1(y,t). \tag{23}$$

That is, the system oscillates back and forth between the initial state and the state in which the one quantum of energy is instead held by particle 2.

If we treat the system using the BDSQ approximation scheme, the wave function of each particle can be written as in, for example,

$$\phi_1(x,t) = a_0(t)\psi_0(x,t) + a_1(t)\psi_1(x,t). \tag{24}$$

To solve the system numerically, we must track, for particle 1, $a_0(t)$, $a_1(t)$ and of course $X(t)$, and then three similar degrees of freedom for particle 2. See Figure 5 for graphs showing the results from this simple example.

One sees that the BDSQ approach reproduces the qualitative, oscillatory behavior of the exact fully-quantum solution, but that the details (such as the period of the oscillations) are not exactly correct. It is of course not surprising that the calculations do not precisely reproduce the exact dynamics; recall that the last two terms in Equations (21) and (22) have been simply ignored. But, I want to suggest, it is interesting that we get oscillations here at all, because these arise from the specifically Bohmian character of the approximation scheme. In particular, at $t = 0$, $\phi_1 = \psi_1$ (the first excited state) and $\phi_2 = \psi_0$ (the ground state). The interaction, $\hat{H} = \lambda\hat{x}\hat{y}$, would seem to be impotent if one replaces $\hat{y} \to \langle\psi_0|\hat{y}|\psi_0\rangle = 0$. But of course, in the Bohmian semi-quantum approximation, one instead has $\hat{y} \to Y$ which can be (and typically is) different from zero even when, for example, ϕ_2 is the perfectly symmetric ground state.

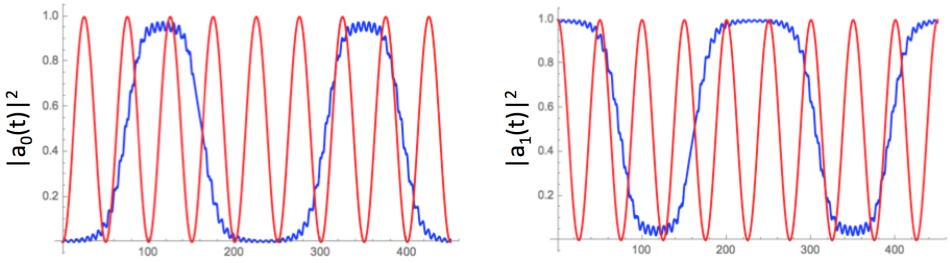

Figure 5. The blue curves show the results of a numerical calculation of the time-dependent weights of the $n = 0$ and $n = 1$ states in ϕ_1. The red curves show the corresponding quantum probabilities from the exact solution. One sees that while the overall qualitative behavior (namely, the back-and-forth oscillations) is basically correct, the details are not quite right. This is not too surprising since the BDSQ is a rather uncontrolled approximation scheme. Still, the result is interesting in comparison to a corresponding non-Bohmian approach, in which each sub-system is influenced only by the quantum average position of the other system. In this case one would never see even the qualitatively appropriate oscillatory behavior seen here. The oscillation, that is, arises specifically from the fact that each quantum half-system feels a force (so to speak) from the Bohmian position of the other half system, and this can be different from zero even when the quantum average would be zero.

Note that we can, in principle, go beyond the BDSQ approximation by including the neglected terms by introducing new fields, for example

$$\phi_1'(x,t) = \left.\frac{\partial \Psi(x,y,t)}{\partial y}\right|_{y=Y(t)} \tag{25}$$

and

$$\phi_1''(x,t) = \left.\frac{\partial^2 \Psi(x,y,t)}{\partial y^2}\right|_{y=Y(t)} \tag{26}$$

which will obey their own dynamical Schrödinger-type equations (e.g.)

$$i\hbar\frac{\partial \phi_1'(x,t)}{\partial t} = -\frac{\hbar^2}{2m_1}\frac{\partial^2 \phi_1'}{\partial x^2} + V[x,Y(t)]\phi_1' + \frac{\partial V}{\partial y}[x,Y(t)]\phi_1 + i\hbar\frac{dY}{dt}\phi_1'' - \frac{\hbar^2}{2m_2}\phi_1''' \tag{27}$$

involving even higher-order fields (ϕ_1''', etc.). [11]

Or perhaps this particular re-packaging of the structure in the "big" wave function – i.e., this particular way of sorting the structure that is left out of the BDSQ – is not ideal, and some other method will work better. Sorting this out remains a work in progress. But there should certainly exist *some* sensible, converging expansion which moves the BDSQ approximation toward the exact results by introducing additional beables which capture (what is described in ordinary QM as) entanglement.

Stepping back, though, the BDSQ (and the gestured-at program of pushing beyond the BDSQ to more accurate pilot-wave approximation techniques) can be seen as a uniquely Bohmian alternative to various existing approximation techniques (for example, density functional theory and Hartree-type methods) which break apart intractable N-particle quantum systems into (something like) N single-particle systems, each responding to some appropriately-averaged version of the other $N-1$ particles. In the same way that the specifically Bohmian semi-classical approximation holds some promise of computational benefits compared to more orthodox treatments, so I think the BDSQ deserves further thought and attention.

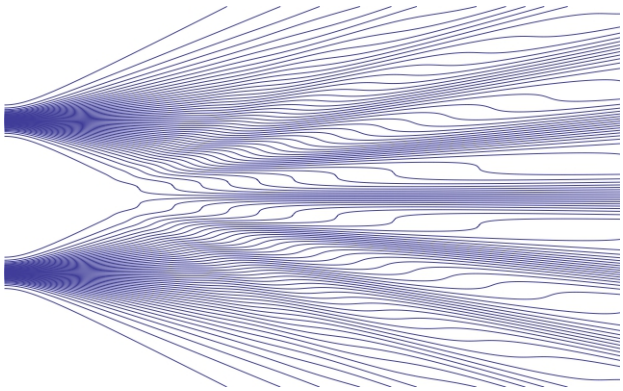

Figure 6. An ensemble of Bohmian trajectories for a double-slit experiment. Each (say) electron, in the pilot-wave picture, comprises a wave – which goes through both slits and produces an interference pattern – as well as a literal particle whose motion is "so influenced [by the interference pattern in the wave] that the particle does not go where the waves cancel out, but is attracted to where they cooperate." [12]

6. Discussion

Although I am quite interested in the potential practical applications (to computationally difficult many-particle systems) of these Bohmian approximation techniques, I also find the program outlined here interesting for the light it may shed on the questions, about the nature of the quantum state, raised at the beginning. Recall, for example, that the only really troubling feature of the de Broglie - Bohm pilot-wave theory is the involvement of the universal wave function. This is, to me, a very strange kind of thing (living as it does in the abstract configuration space) and (since its dynamics seem so comparable to the fields of classical physics) I never really got the hang of interpreting it as a 'law'.

The program outlined above, though, suggests the possibility of reformulating the pilot-wave theory in terms of single-particle pilot-wave fields (like the CWFs of ordinary Bohmian mechanics) which can be understood as normal, physical fields on 3-space. Indeed, what I described in the last section as the BDSQ – but generalized in the obvious way to a system of N particles, perhaps the entire universe – constitutes a full-fledged, well-formulated, consistent theory. Of course, we know the theory isn't *right*: its empirical predictions will only be correct for a universe without (what ordinary QM describes as) "entanglement". But still, in so far as one's goal is to think about how one might formulate an empirically viable quantum theory, in which strange ("non-local beable") objects like the universal wave function play no role, it seems like a useful jumping off point.

Let me close by quoting, perhaps somewhat out of context and in a way he might not endorse, a principle enunciated by Matt Leifer at the workshop:

> **Explanatory Conservatism:** "If there is a natural explanation for a quantum phenomenon then we should adopt an interpretation that incorporates it."

For me, this brings to mind Bell's memorable discussion of the two-slit experiment (See Figure 6):

> "Is it not clear from the smallness of the scintillation on the screen that we have to do with a particle? And is it not clear, from the diffraction and interference patterns, that the motion of the particle is directed by a wave? De Broglie showed in detail how the motion of a particle, passing through just one of the two holes in [the] screen, could be influenced by waves propagating through both holes. And so influenced that

> the particle does not go where the waves cancel out, but is attracted to where they cooperate. This idea seems to me so natural and simple, to resolve the wave-particle dilemma in such a clear and ordinary way, that it is a great mystery to me that it was so generally ignored." [12]

Surely Bell is correct that this idea – of particles being guided by waves – constitutes an exceptionally simple and natural explanation of the phenomenon of one-particle-at-a-time two-slit interference. And so, according to Leifer's principle, we should adopt an interpretation that incorporates this idea.

At present, however, it is impossible to do so, because no candidate theory fully incorporating this idea exists! The pilot-wave theory of de Broglie and Bohm, of course, is in the right ballpark, but in that theory the wave is the universal wave function – not a "local beable" at all and in particular not the kind of thing that can "propagat[e] through both holes" in the two-slit experiment. Unlike the universal wave function, though, the Bohmian conditional wave function of a single particle *is* the right kind of thing to play the role of the pilot-wave in this most-natural explanation of this most-paradigmatic quantum phenomenon.

In closing, I hope I have convinced you that the following is an interesting question: how could we formulate a realistic, empirically viable quantum theory which posited *exclusively local beables*? ... and that the pilot-wave theory and the CWF in particular seem like promising jumping-off points to try to answer it.

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